Symbolic Logic
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Lecture – 23
How to Apply the Rules of Inference in a Proof
Introduction to the Equivalence Rules

So, we are back and this is module number 23, where we are going to now learn to apply this rules of inference that we have learnt in a proof. So, that we know how to derive new lines and actually establish the validity of an argument using this procedure, I am also going to include the equivalence rules, far we have only talked about the rules of inference, but you have also need some equivalence rules. So, this module would introduce you to the equivalence rules also now what are these just reminding ourselves that we have not hand the rules of inference and. So, far we have been exposed to nine of the.

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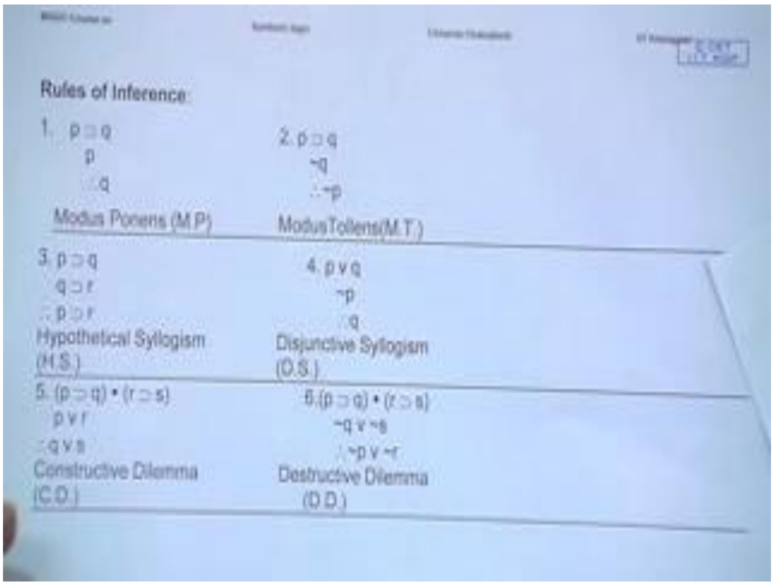

This 9 rule is going to see us through in in a long way in in a major way they are going to see us through, but it is very important and you will see soon that you need to understand how this rules apply and where you can apply which one right, but slowly we are going to take it in. So, these are the nine rules we have in front of us.

We will keep this handy and will keep on referring to these rules, but let us start talking about the formal proof ability.

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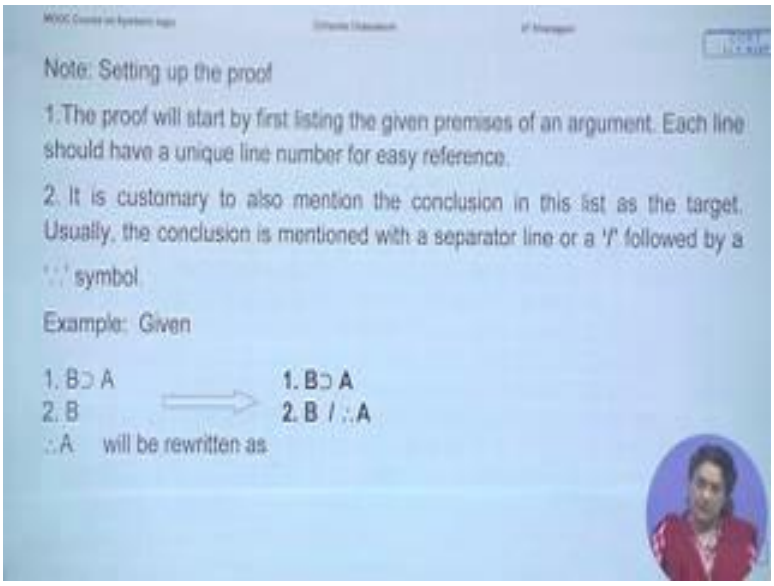

So, first how to set up the proof the setting up means that you need to start the procedure. So, first of all there will be an argument that will be given to you and you start by listing the given premises of those arguments. So, these would be given right and but each line is going to have a unique line number. So, you are going to start one two three etcetera the conclusion also will be given because the whole point is that you do not have to find the conclusion the conclusion will be given. All you have to do is to show why that conclusion can be claimed to be a valid consequence from the premises how does it validly follow from the premises given.

So, given premises given conclusion all you need to do is to sort of set it up. So, usually what we do is to use a separator line like. So, a slant followed by the therefore, symbol right. So, suppose you have given an argument like this then, for the proof you can set it up like this where the premises are all given numbers and this is where when the premises end that is where you put the slant line and you put the triple dot and here is the conclusion that is claimed whatever you do. Now next is not given there would be line numbers to this and. So, on, but this is how the given is set up. So, conclusion is stated here the premises are also stated here with line numbers that is your beginning point and then comes how to go further. So, the derivation will proceed by adding new lines to those new already given lines as you know.

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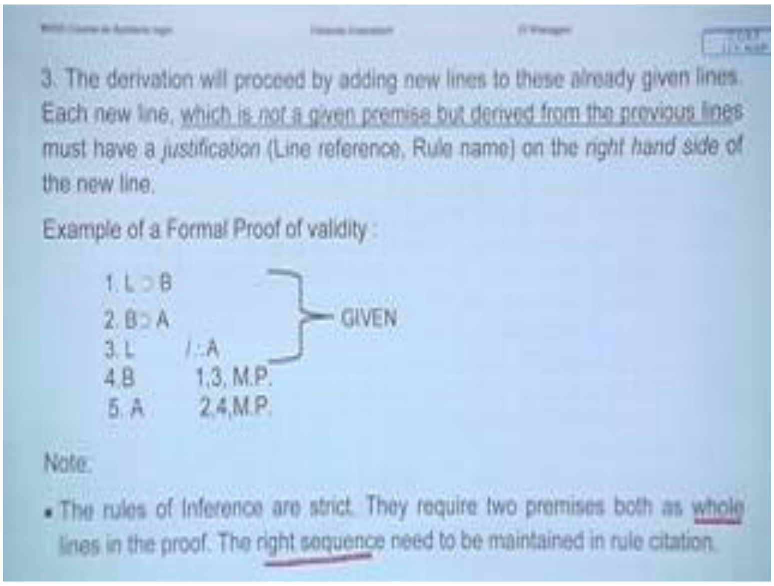

But every new line that you generate in a proof you have to justify as what well justify in terms of which line did you obtain it, from did you derive it from and which rule did you use. So, recall your knowledge of the truth tree and you will understand what we mean that every new line, that you add to the procedure you are you must be accountable you are liable to refer back to the line that was that you are using and the rule that you are using where would it show up on the right hand side of the proof.

So, we are going to now take a look in to actual examples. So, suppose we have an argument like. So, see there are three premises lined up with line numbers there is a slant line and here is the triple dot followed by the conclusion of this. Whatever you now do is to show the derivation how from these the conclusion a follows that is what the formal proof of validity is going to be like. So, how do you go about it? Now you might think that take a very good loop at the premises automatically there should be some plan forming in your mind how you can get a right now, where is a in the premises right here in the second premise which is a conditional statement and you know by now you should know there are rules that will help you to get that a out provided you have the antecedent also.

In this case antecedent is b right. So, your target is b, if can I get b then I put it together with two lines two and we can get a out. So, your now objective is how do I get b where is b here where is b here and b is also here. So, ones more, if you can have l horse shoe b and l you know you can get b out which rule helps that that is the rule called modus ponens right and you happen to have l here given here in line number three. So, if you have thought about it in this way then it is a matter of just putting it all in lines. So, you are line four would be b how did you get that from line one and three notice the sequence line one and line three because that is how the modus ponens work you need to have the horse shoe statement first and the assertion of the antecedent second.

So, one three and this commas are for separating the lines out and here comes the rule name which rule have you used the rule is modus ponens that is how you got b am I done no look at the conclusion this is where we need to go.

So, here is next line a how did you obtain it from line two and four you derived by using modus ponens. So, line 2 and 2 is not a given line four is something that you derive. But that is perfectly all right anything that you legitimately derive becomes the automatically part of the problem. So, here is a formal proof of validity that shows if it is count how many how many constants you have 1, 2, and 3. So, if you are doing by the truth table how many rows would you be requiring 8 rows minimum right. 8 rows you need you need to do the 8 rows truth table here you have done in two lines and that should be an advantage that you should remember, how do I know each line is correct or each line is true the answer is because you are using valid argument forms modus ponens.

So, all these nine rules will become the basic guarantee the that you are doing something where your truth is preserved and that is very important, I will remind you ones more what I have already said is that you are dealing with rules of inference which are rather rigid and formal in nature now they require that you pay attention to them. If they say that we need two premises then you need to have two premises as standalone statements in your proof like here we did l horse shoe b was required and l is also required before you could do the b derivation of b. So, just from l horse shoe b you cannot have b out by this rule right and notice this that is l is a standalone line it is not a part of another statement. So, that is something to also recognize and to remember when you are using the rules. So, let us revise it the rule ones more which rule did we use we use this modus ponens which looks like this that you have p horse shoe q and p then you can pull q out this is what we have used in this proof.

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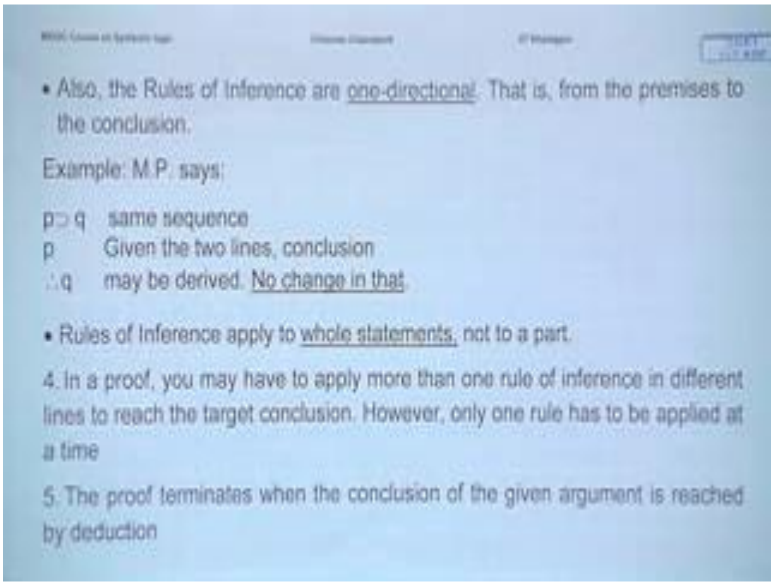

Let me, now take you to this further that rules of inferences remember are one directional that is from premises we go to the conclusion and that direction is not to be changed that is you cannot come from conclusion back to the premises that is why we call them the rules of inference you will soon. Find in a in as we introduce you the rules of equivalence that the rules of equivalence would be bidirectional you it does not matter of which side you are coming from. But rules of inferences are rather strict sequences from the given the premise you can go to the conclusion not the other way around. Now here for example, modus ponens says that you have to have p horse shoe q and p given this two lines the conclusion may be derived there is no negotiation no change is possible the second point which, we will try to understand it also with example is that rules of interference apply only to whole statements not to a part.

So, you have to have them as whole statement before you can apply which is where I have tried to explain to you, but will take this point again with an example to make this point more clear to you and the other thing is that in it is not necessary, that in a proof you can use only one rule. In fact, there are different lines as you gain experience you will see that there will be many lines in a proof and you can have a combination of various rules to reach your target as long as those are in your rule base you cannot in create a new rule or import a new rule. But it has to be within these nine rules that you have just learnt, but when you are applying remember in one step one rule at a time on the whole in the proof you can use many rules.

But in one step only one rule is permitted and then since we define what would be your starting point namely the given premises is our starting point then, when does the proof end what is the definite end of the proof the answer is when you have reached the conclusion of the given argument which is also given the conclusion is given right. So, that is where you stop. So, this is our elementary understanding of how the proof works we have a definite starting point we have a definite end point in between I have tried to explained how to derive the new lines right which would be with the valid argument forms or the rules of inference so far.

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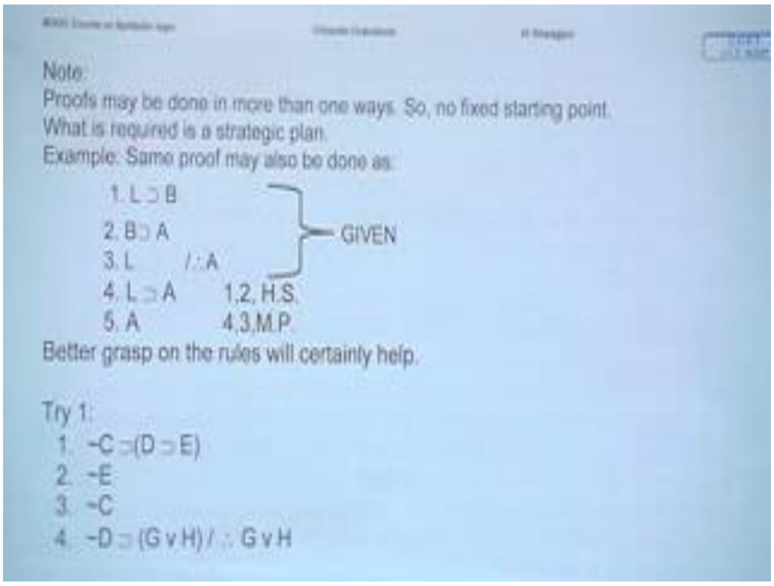

Now, comes what I would say is additional more information to process a proof may be done in more than one ways different people think in different sort of way. So, the more advance proofs you will see that there can be multiple ways of solving the same problem. So, there is no fixed starting point, if you are lucky and the problem is very simple then maybe there is a fixed starting point where you can start, but it does not have to be that way at all. So, what is required whenever you are doing a proof I suggest that you first form a strategy you sort of like the tree you try to think ahead and think a little in steps before doing the thing may be a rough planning maybe a rough work would be needed what is it that you want to do in this proof now let me just, show you that first of all that the same proof may be done in modern one ways. So, let us take our earlier argument this is what we encountered just now and this is how given.

Now, in in what we saw the example we used the modus ponens twice right, but you can do it also like this if you now see 1 and 2. You might find that there is an opportunity to apply the chain rule or the hypothetical syllogism, you have them in the right order let us visit the hypothetical syllogism what does it say that, if you have p then q you also have q and then r notice the notice the nice formation of this sour of zee or z that you have it like. So, then you can derive p horse shoe r. So, let us take a look is this what we have in the argument if. So, here is a prime opportunity to apply the h s rule and it will give you l horse shoe a, but you need to know why you are applying what will you do with l horse shoe the answer is because I already have l and I can pull a out by modus ponens. So, it might work like this also.

So, this is another way of doing the same proof we are using another rule the more rules that you are you have commands over the more rules that you have grasp over the better. Because sometimes the rules can save you safe sometime rules can you about instead of using one rule if you use the other one you might have a more efficient proof. So, get acquainted very, very well with the rule base the nine rules that we have learnt try to get it shall we try our new problem together.

So, here is a given argument can you construct a formal proof of validity this one is four premise and here is a conclusion g wedge h what is the first point this is always set up like. So, you are going to add new lines here, but before that what is required absolutely essential is a plan a strategy plan how are you going to go. So, that this would be the last line of your proof how can you derive g wedge h from all of this take a good look at the premises and try to form a plan then those of you who are already keen or you know a hobby of puzzle solving you have already known to starting namely; that means, g wedge is right is here.

So, if you can have tilde d then pulling g wedge h is not a problem because you know modus ponens will help you there, but not d is not given anywhere which means that not d has to be somehow generated from all of this. So, now, our next question is how can we get now d out where is d d is right here d is here, but what we need is not d can you see any opportunity of getting a not d well if you recall your rules very well this is why we say that you need to know your rules really well you know that if you have you you have d horse shoe e if that can be somehow separated out pulled out and if you also have not e then there is a rule that will help you to get not d which rule is that let us take a look into a rule base and we find that the rule is called modus tollens. Modus tollens or empty what is it say that if you have p horse shoe q and not q then you can pull the negation of the antecedent out. So, in our case this modus tollens application here would be possible provided we have tilde e do we have tilde e yes we have here, but can we apply it here because we said the rules of inference are all applied to whole statement.

So, it cannot apply just to this part. So, therefore, we need to think about what to do about this tilde c, well we find that one and three combined tilde c is already given. So, modus ponens will help you to get the d horse shoe e out as a standalone statement correct ones you have that you can plug in tilde e use modus tollens, to get not d the moment you get not d you can get g wedge h am I making sense here if I am that is very good if not listen to me slowly, but what is required is a plan that was my plan how am I going to do this, but you work with me. So, that you have your plan how to work on problems such as this now let us plug it all together and construct the formal probability.

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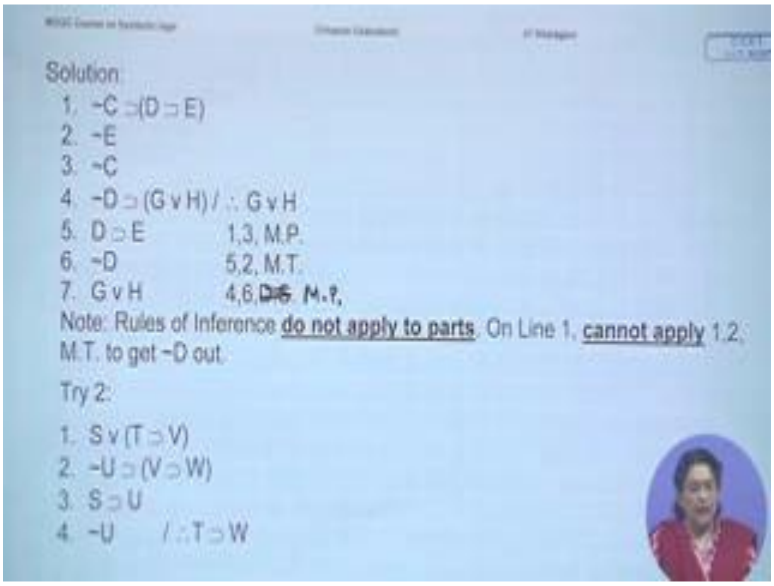

And your answer is going to sort of look like this. So, where do we start we start where we have learnt together putting in together. So, where do we start we start where we want it namely we are going to use one and three modus ponens and pull out d horse shoe e why d horse shoe e. So, that we can use the tilde e in line to get not d out, that is our next stage we are pulling not d out by this rule called modus tollens.

This is modus ponens this is modus tollens and here comes then the simple idea of g wedge h and by the rule that is you are using four and six and this is your classic modus ponens right. So, that is what will give you this result fine simple again how many constants can we find 1, 2, 3, 4, 5. So, it would have been a 2 to the power, 5 truth table for you to establish the validity, but here you have done it only by adding three lines right that is the formal probability and each of them is a valid argument form which ensures that every state the truth has been preserved, if you started with truth then you landed in truth right. So, that is how it goes.

So, here probably is a chance to also appreciate when we said the rule of inference does not apply to parts. So, on line for example, you cannot apply line two directly on to this part d horse shoe e why not because d horse shoe is a part of our whole sentence. So, until you have sort of operated on this horse shoe this horse shoe is not accessible to you all right. So, this is why we have to do the first get the d horse shoe e as a standalone line before we can do the modus tollens on line number five and get tilde d out.

So, that is how the rule rules of inference work are we ready to try another example. So, let us see here this is an argument and where you need to construct the formal proof of validity again what is required is some set of planning do not going in to proof or with random steps because then you are slightly that you will be laid as stray or not finding your way and. So, on let us have a definite plan before you actually start the proof.

So, what is required is t horse shoe w this is what we have to reach where is t w t horse shoe w anywhere nowhere it is given is there a chance to get t horse shoe w anywhere can you see that can you spot that look at your premises those are your friends. So, look at your premises and try to locate the possibility of having t horse shoe w and if you are keen and closely watching the premises you might see that you do not have t horse shoe w, but what you have is close enough possible in a in line number one and two where here you have t horse shoe v and here you have v horse shoe w and it that should ring a bell in your mind that here is a chance. If we can get t horse shoe v and v horse shoe w out then the chain rule of the hypothetical syllogism will give a t horse shoe w. So, the point is how do I get v horse shoe w separate standalone how do I get t horse shoe v separately out.

Now, for v horse shoe w the problem is easy why because look at your premise line number two and four combined application of modus ponens will give you v horse shoe w separate got it, but t horse shoe v is still not very clear, how do I get t horse shoe v out well you remember this s wedge t horse shoe v that is a wedge right. If you somehow can get not s then there is a rule that will allow you to get t horse shoe v that rule is called the disjunctive syllogism this is why you need to know your rules thoroughly, this is the rule that we are referring to if you have a wedge and you also have the negation of the first disjunct then, this rule will allow you to get the second disjunct out p wedge q and not p together you can derive q right. So, this is the rule which we are plugging in here, s wedge t horse shoe v and if we can get not s then we can use the d s rule to get the t horse shoe v out.

So, how do I get not s well that should be easy is not it 3 and 4 line, 3 and 4 just take a good look do you see how you can get not s out you are right. So, this is going to be a modus tollens, opportunity to get the not s out modus tollens is this rule ones more p horse shoe q not q therefore, not p right. So, this is what we do all right now that sounds like a good plan to work on let us do that. So, are going to do it or shall I do it.

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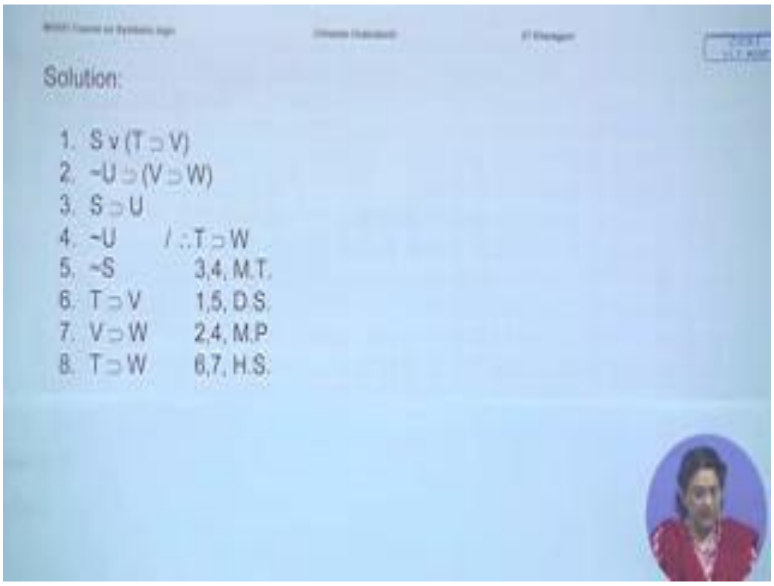

So, let us see what it looks like when we have done it. So, here is the proof that is your beginning point and then you start wherever you want to start I started out with getting the not s out how from line three and four by modus tollens.

So, I already have the not s out and that will give me t horse shoe v on line one and five by d s and then v horse shoe w. We got from 2 and four and this is my modus ponens right. So, we have it now what we wanted and line number 8 should be immediately clear to you t horse shoe w from 6 and 7 the order is important. So, 6 and 7 and h s the order by order I mean the order in the rule. So, the rule has a certain way of the sequences given and you follow that also in your citation of the lines. So, this is a proof that we have ready this is how, the rules of inferences are applied time has come to join or bring more rules in to our rule base.

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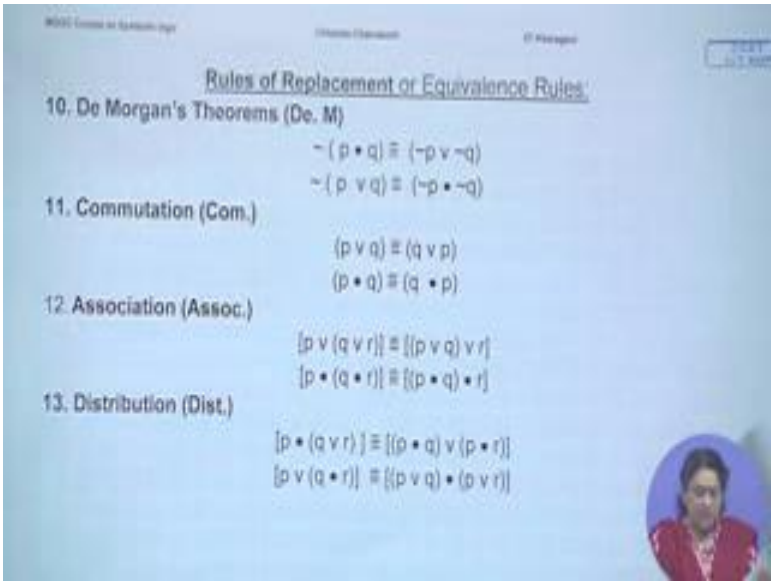

So, this is our rules of replacement equivalence rules first we will introduce you the rules what they are some of them are already known to you I do not have to explain, but still we are all beginners. So, will go like this.

First set is De Morgan's theorems and we have it like this if you have a situation. Where negation of the dot you can re write it as a wedge with negation attached to each of these. So, what you have done you have a negation of a conjunction you can re write, it as this junction, but there will be tilde attached to each of this components remember this is a triple bar this is an equivalent rules. So, you can come from the left hand side to the right hand side or from right hand side to the left hand side also either way. So, equivalence rules are going to be bidirectional.

So, this is corollary of that if you have negation of wedge then you can convert it into a conjunction, but negations will attached to the components ones more from left to right from right to left this is commutation this is the role that you needed to change the position so, but if you have wedge then you can right p wedge q in place of it you can replace it by q wedge p does not matter.

Similarly, if you are p dot q you can re write it has q dot p. So, what you are doing you are switching the position without changing the actual connective this is the equivalence rule that is known as association this is commutation will refers to is at com this is association will refer to it as, what does it do if you have throughout wedges then what it allows you is to regroup it take a look this is p or q or r this is still all the wedges are in place, but the grouping has changed now p wedge q or r the emphasis has changed same goes for the dots if you have throughout dot you can change the grouping like.

So, by the association rule we are going to revisit this rules more in our next modules, but let us go through this this is distribution rule this is known as if you have p dot q it sort of works like a multiplication. So, if you have p dot q wedge r distributed it becomes p dot q wedge p dot r if you want to understand mechanically the inside connective become outside the outside connective becomes inside and you just multiply it through. Here is p wedge q dot r it will become p wedge q and p wedge r and you can come back from both directions. So, these are just 4 of this rules of replacement more there are more rules of replacement, but we will visit them in our next module and we will talk a little bit more about the equivalence rules how to apply them and so on. But this is where we will end this module here.

Thank you very much.