Symbolic Logic

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Lecture- 22
Valid Argument Forms:
What They are, and What their Role is in Formal Proof of Validity
Rules of Inference for Formal Proof of Validity

Hello and welcome back to our session on the valid argument forms and the formal proof of validity. So, this is our module 22, where we are going to now learn have to do this formal proof of validity and I have introduced you the I have mentioned about the valid argument forms.

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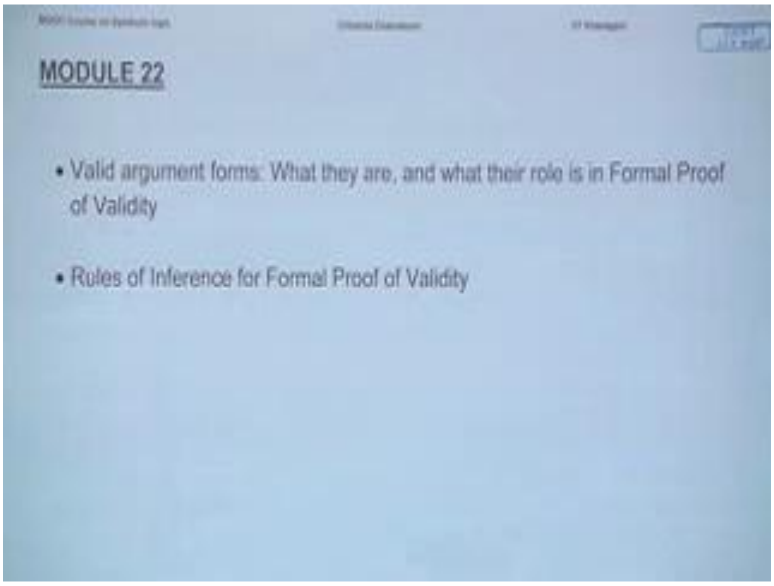

But this is where we are going to talk about the valid argument forms in detail as to where their functions are and what the roles are and then finally, we need to learn about this rules of inference. So, this is where are module 2 is module 22 is.

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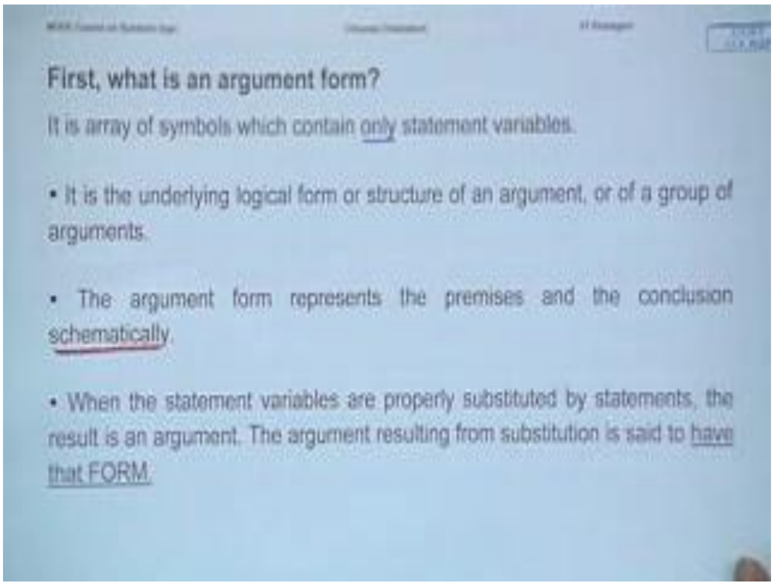

And this is what we are going to do now first of all what is this are argument form that we are talking about. Now I have to remind you that have already talk to you about what is known as statement form earlier we have talked about the statement form and where we say that the statement form is just a structure the underlying logical structure. So, similarly argument form is the underlying logical structure of arguments. So, if you want synthetically what it looks like then the answer is that it is just an array of symbols which only contain statement variables. So, array made by statement variables no constance what. So ever, you are not talking about actual statements at any point and then as I said that it is the underlying logical structure think about the skeleton. If there were something call the skeleton of an argument the logical skeleton that is then that is what this argument form is all about will see some examples very soon.

What it does is that it schematically represents the premises under conclusion and you get to see the structure of the argument clearly. So, when as is the case which statement form in argument form also if, you replace the statement variables with actual statements and properly if you substitute it what you are going to have in your hand is an actual argument. So, that argument would be the substitution instance of the argument form this is the understanding of formal logic. So, once more the argument forms visually will be an array of statement variables only and what it represents is the arguments structure the underlying structure and schematically it will represent the premises and the conclusion and any proper substitution of the formal result into an actual argument. So, you can say some arguments have these form the argument form.

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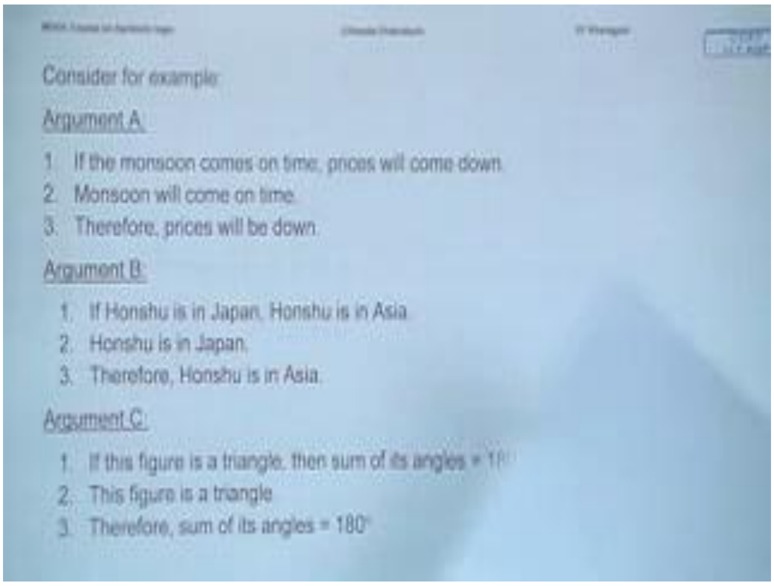

So, let us take a look actual look in to some of the samples. For example, let us consider these three arguments. So, argument a says if the monsoon comes on time prices will be come down monsoon will come on time therefore, prices will be down argument B says if horseshoe is in japan horseshoe is in Asia horseshoe is in japan therefore, horseshoe is in Asia. If this figure is a triangle that is your argument C, if this figure is a triangle sum of its angles is one eighty degrees this figure is a triangle therefore, sum of its angles one eighty degree. If I ask you what is common in that because it seems like they are talking about three different things.

So, this one for example, is about weather and the connection to market economy this is about geography location of horseshoe and. So, on this is about geometry, but what is common in this and you might be able to see that first of all not only there are three statement in each premises under conclusion, but there is certain pattern that you can recognize namely that the first statement is a conditional statement or an if then statement and the second in every case is the assertion of the antecedent of the conditional right the first cause in that conditional and then the conclusion is the consequent of that condition.

Now, what is that that you capturing your bare eyes can capture that there is some sort of a pattern though it is not overly stated? So, that is that is what we call the underlying logical structure which we may represents in terms of statement variables in certain base. So, keeping that in mind, this is what you going to gain like this.

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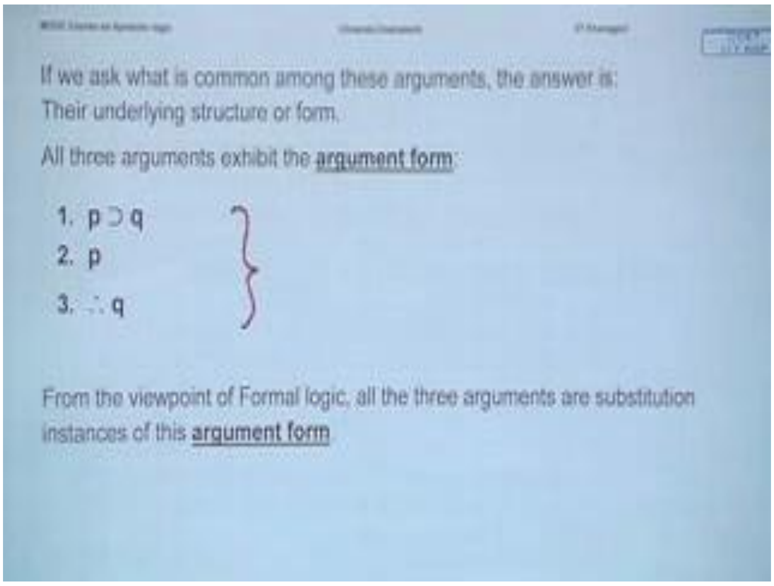

If you try to capture what is that argument form that we just saw then probably you will pick up something like this let us remind ourselves that the p, q these are statement variables they are like holds moulds were we can actually plug in substitute actual statements. So, these may though looks like statement, but they are not they are propositional variables right stock statement variables. So, together what they represent is the structure that all this three argument exhibit and that is what we are going to call an argument form now, why we are looking into that the answer will come clear as we go along, but at the outset please note that formal logics approach to this would be that what you saw earlier as three arguments are actually substitution instances of this form once more, we are not really try trying to encapsulate the form from the examples rather the form precedes the substitution instances.

So, the form is first what you do is you replace this with any choice of your statement actual statement and you are going to get as an actually argument which is substitution instances of this form. So, if you want it speak about horseshoe and its location fine if you want to talk about geometry that is fine this is the form will whichever, way you want to go that you will exhibit in the substitution instances, if you understood what argument forms are then let me further this point by saying that argument forms themselves can be valid or invalid we am remember what validity invalidity is.

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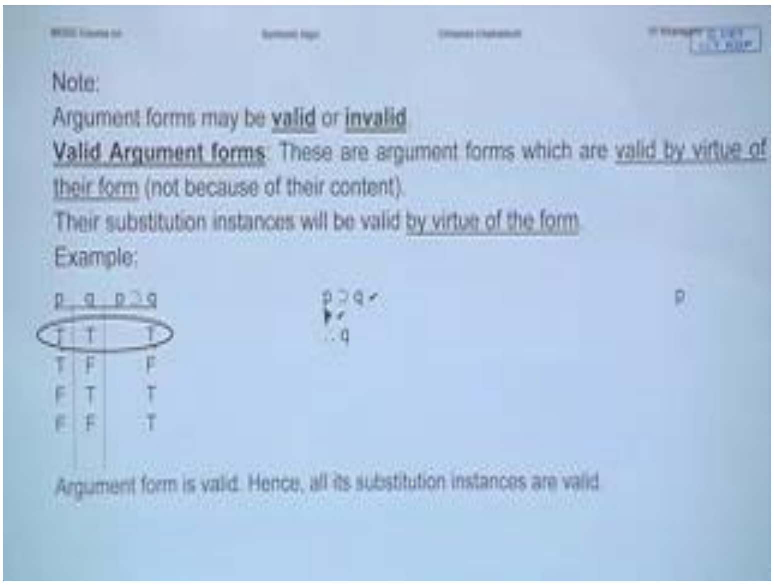

Now, I am taking it to the formal plaint to claim that argument forms themselves can be valid or invalid valid argument forms would be those argument forms which are valid by virtue of its form. So, not because of the content, but there is something structural property is there which makes them valid this is how formal logic could understand it and therefore, no matter what you substitute then with no matter which proposition you substitute and them with if the substitution is proper the substitution instance is going to be valid just by having the form the form will ensure that you have valid argument in your hand.

For example let us take the argument form that we learned from that earlier three arguments A B C this was your fourth. So, this is p horseshoe q p therefore, q if you do a quick truth table on this then you will see that this is we are laid down the truth table now we see where is, when the premises is all true what is happening with the conclusion this is your first premise this is your second premises. So, we look under p horseshoe q and we find this is when its true and we look under p and this is when it is true. So, together when the truth is c that q is then true in other cases you have one of them false here and here. So, we do not be worry about that the only time we need to worry is when the premise is all true what is happening under the conclusion and the answer is then conclusion got be true.

So, this is a valid argument form if this is the valid argument form please note what you have gained is that just by calling or proving the argument form as valid what we have gained is the say over the entire class of arguments that would be the substitution instance of this form. So, all the three arguments that you saw remember A B C and so on. They are you do not have to do separate truth table for any of them anymore all three of them now you can call them valid how by virtue of the form that they represent this is our answers. So, you do not have to do separate truth table to for each case to establish validity you can now refer to the very fact that they have a separate form and if you know that the form argument form is valid then by virtue of the form you can invoke that they must be valid also.

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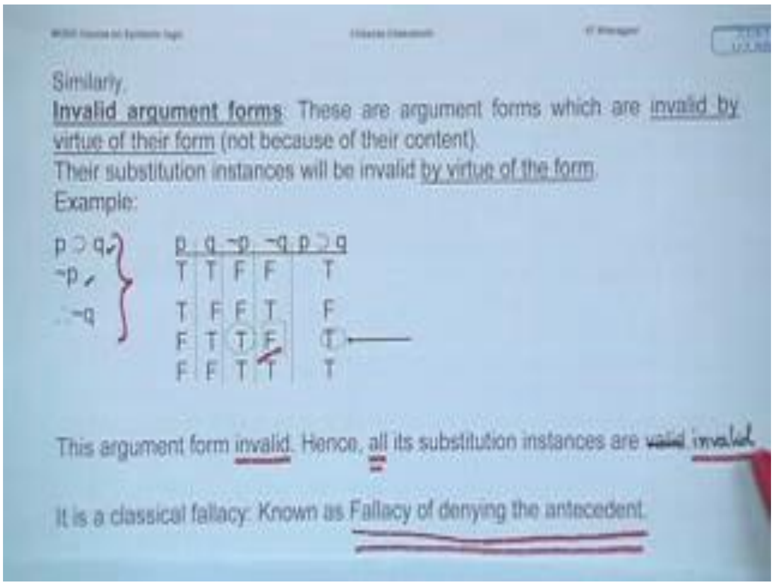

So, this is valid argument form and similarly you should know that there can be invalid argument form also argument forms which are invalid because again of their because of their form. So, certain property structural property is there in them which makes them invalid and we will try to show that also, but the point to note is that if you found such invalid argument forms all their substitution instances would be also invalid by virtue of having that form that is the classic point. So, just by having the same form they would be automatically invalid it does not matter what their content is what you want to substitute then with does not matter the property of invalidity will stick to them by virtue of having this form and that is what we have to understand from the formal logic point of view.

So, let us take a look at what would be an example of an invalid argument form for example, see this one this may look like similar to what we have earlier say, but suppose there is a form which says that the premise form is like this. If p than q and then not p you are saying if p then q and then you are are also see saying not p and from that you concluding therefore, not q this is an argument form let us see whether its valid or invalid. So, we can lay out the truth table likes. So, remember us separating the tilde p tilde q because there are all appearing in the argument form. So, we place them like this again what are we looking for is when the premises are all true what is happening with the conclusion. So, this is your first premise and this is your second premise. So, we look under each of them here is p horseshoe q true and here is not p one of them is false. So, we do not need to see that row this is where it is already false this is also false we do not need to see this row, but this row has p horseshoe q is true and not p also as true this is when you have all the premises true at the same time what is happening in it conclusion here and you find that it is false right.

So, when the premises are true it is possible for the conclusion to be false that makes the argument form invalid. So, this is an invalid argument form and what you have claimed. Therefore, is that every substitution instance even the once that you not aware of that are not in front of you eyes that you have not yet thought of every single substitution instance of this form is going to be invalid that is the point, and just because you are meeting this there are many such argument forms by the way which are invalid and this one the one that you just saw is a classical invalid argument form it is called the fallacy of denying the antecedent the fallacy of denying the antecedent fallacy means they are logical errors logical mistakes, but people keep on doing them. So, one needs to identify them and the medieval logicians have done an excellent work of classifying many such fallacies this is one of them which is known and routinely people do this, but its non the less a mistake an error and it is called fallacy of denying the antecedent, but our points sticks that this is an example of an invalid argument form and if you encounter this in any argument you can safely call that argument invalid even without just by referring to the argument form you can call it invalid.

Now, why are we discussing this argument form valid invalid etcetera the answer is here is that you need to understand that valid argument forms has a very important property for formal validity, what is that property is that valid argument forms are truth preserving truth preserving they have quality that they preserve truth as in, if you start with truth.

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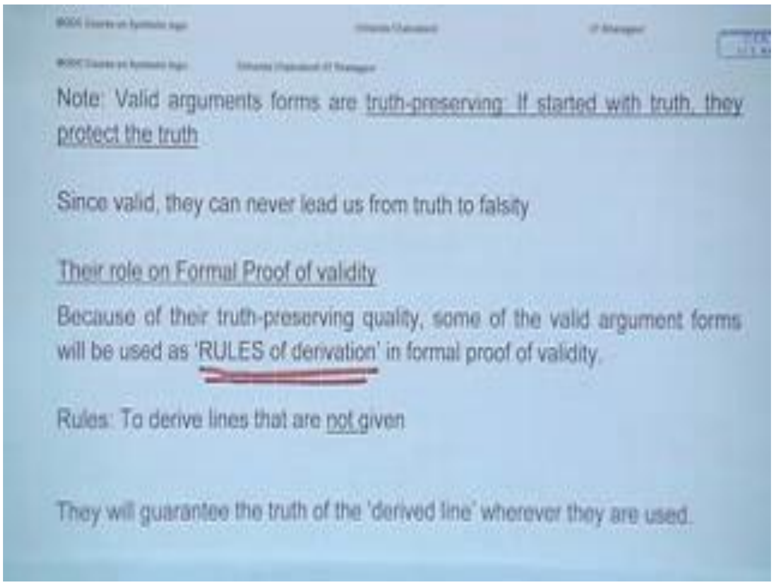

And if you use this valid argument forms they will protect the truth they will preserve it. So, there is no way that you can start with truth and use the valid argument forms and still land into falsity get it. So, if you remember in formal proof of validity you have step by step progress. So, if you start with a step that is that is true and then apply valid argument forms to that line there is no way you can land into falsity now there is no way you can derive falsity out of that. So, that is what they do therefore, now you know that the role of valid argument forms in formal proof of validity is important.

What they serve as some of them are going to be used as the rules of derivation remember when I defined the formal proof of validity which said that this would be sequential process of deriving lines from previous line according to certain rules and I did not explain what those roles are the time has come now to talk about it, but understand that this rules are nothing, but what we would called valid argument forms. So, some valid argument forms would be identified as rules of derivation to serve this purpose this rules what they do is to allow us to derive new lines in a proof that are not given some lines with are given and then, you need to derive new line what would protect the truth of those new lines the answer is this rules of derivation or valid argument forms will guarantee the truth of the derived line wherever they are used this is our introduction to your the whole mechanism of the proof.

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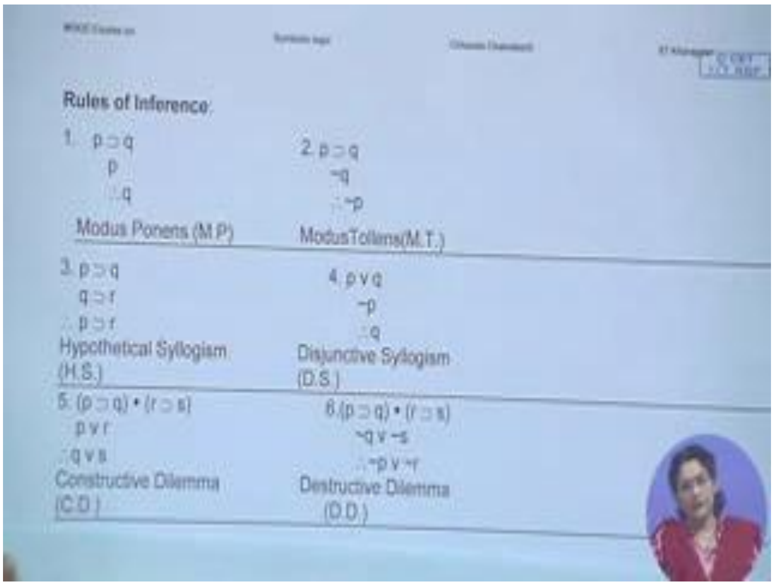

Let us now look into something very important. So, we are going to now talk about what is known as the rules of inference that you are going to use in the formal proof of validity these are all valid argument forms. So, let us get acquainted with this is what we have seen earlier if p than q, if p and p therefore, q once more. So, you have if p than q and p. So, there is an assertion these are 2 separate lines are you need each of these lines before you can prove the consequent out just from p alone you cannot pull q out just as if, you have only if p then, q you cannot pull q out. So, you need two lines to pull this q out this is what we uncounted earlier as the common structure argument form in all the three arguments and that we saw there is name for it and we are going to call it modus ponens that is a medieval name, we will shorten it and call it the MP rule. MP stands for modus ponens if you want to know then this is by affirming we are firm the slating.

So, what you are affirming is that if p than q you are further affirming that p has happened therefore, q has happened lets come to its corollary this is the second role which says that you have to have the premise if p than q and not q. So, together from these two lines you can say therefore, p is not the case if you are stumped by this then let me remind you that q is the necessary condition right. So, if we and if it is if it is true that if p than q and if it is also true that q has not happened then you safely infer p also has not happened and that is the valid argument form it is called modus ponens shortened form is m t and we are going to refer to it as m t.

Few more this line number this rule number three is an ancient rule starting from the Greek times we have had it is colloquially called the chain rule, but the full name is hypothetical syllogism hypothetical means if then and syllogism is an Aristotelian term. So, the name is hypothetical syllogism we are going to use the abbreviation hs what is it say that if you have two premises like this if p than q if q than r see the chain then, if p then r please note that you not saying p has happened you are not also saying r has happened the entire argument form places in to a hypothetical or conditional mode that is the name. If p than q if q than, r if that is the case then p then r nothing more is being stated, but that helps us to get rid of the q if you if you need it. So, in a proof you will see that it rule has some advantage this one is simply saying this is call disjunctive syllogisms because it uses the wedge this disjunction and in short its call d s what am I doing the first premises p wedge q and then says not p all right. So, it follows therefore, q now I have to remind you or certain emphasis on this that this rules of inferences is very rather formal and rigid.

So, when you have such a situation please note the sequence what the rule allows is that you have a wedge and then you have the negation of the first disjunct the position of the disjunct also matters if you have. So, the first disjunct is negated. Then you can pull the second disjunct out what you cannot do is to change the swap the situation just by this role you need a different rule to do that and will see that in the rule of equivalence, but right now please note the disjunctive syllogism only allow you in this sequence.

Few more to go along with this is called constructive dilemma let us take the note of what is what is it saying we are going to call it the c d abbreviated if you have a premise that says if p than q and if r than s. So, it is a conjunction of two horseshoe statements on top of that if you have either p or r notice the position of p and r p is the antecedent of this one horseshoe and r is the antecedent of the second horseshoe statement. So, if you have together this sentence and p wedge r than you can derive q or s or q wedge s notice the position of q wedge q and s q is the consequent of the first conditional s is the consequent of the second condition if you have a situation like this along with this premises together you can derived q wedge s and this is construct dilemma its corollary is the destructive dilemma which you are going to refer to as d, d what is it say it is out of a contribution of this earlier rule and the modus ponens if you have a conjunctions of if p than q if r than s you also have not q wedge not s.

So, you have negation of the consequent of the each of this conditional as a disjunction then you can pull out not p or not r gets it. So, if you have the negation of the consequent then you can derive along with the both the premises will be necessarily than you can derived either not p or not r and that is your destructive dilemma rule.

This six rules are all valid argument forms and they are also rules of interest there is more we are going to have total nine rules of inference to start with this is simple simplification as we can see.

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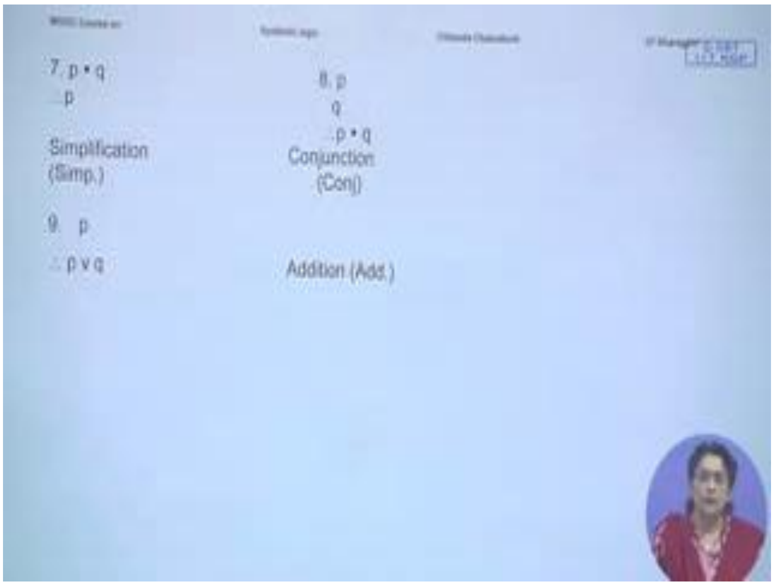

if you are told p dot q is true you can infer p must be true remember the dot truth dilemma you will see why this has to be valid always the rule is called simplification, but we are going to refer to it as again I have to remind you or insist upon the fact that look at the position what does it allow it says that if you have a conjunction you can chop the second conjunct out and pull the first conjunct out what you cannot do is to change the position of p dot q you need a separate rule for that simplification does not allow you to simply change the position of p dot q all right. So, it is exactly follow it exactly that if you what it allows you is to if you have a conjunction statement you can simply take the first conjunct out by simplification rule this is conjunction rule.

If you have p as true and q as true then this rule allows you to join them together and that is the job of conjunction and it is called conjunction which we will refer to as co conj this is a different and should be institutively obvious to you if it is known that p is true then this rule called addition which we are going to refer to as add allows you to add a statement with disjunction. So, if p is known to be true therefore, p wedge q where q can be anything that you need or anything that you want to add why is that true because we know that with disjunction if one of the disjunct is true the disjunction must be true the whole disjunction must be true. So, this is what it is we are going to use this rules in our next modules to get into the proof properly, but it is very important at this stage that you go through the rules of inference by yourself I have tried to explain and will see how they apply, but it will take some time, but you need to give yourself that time to get the understanding properly we were going to refer back again and again to the rules.

But in this module, this is where I will stop and in the next module show you how to utilize this rules, but before that gain knowledge of the rules the more confident you are about the rules the better you will be performing in the proof. So, that will be our end of our module number 22.

Thank you very much.