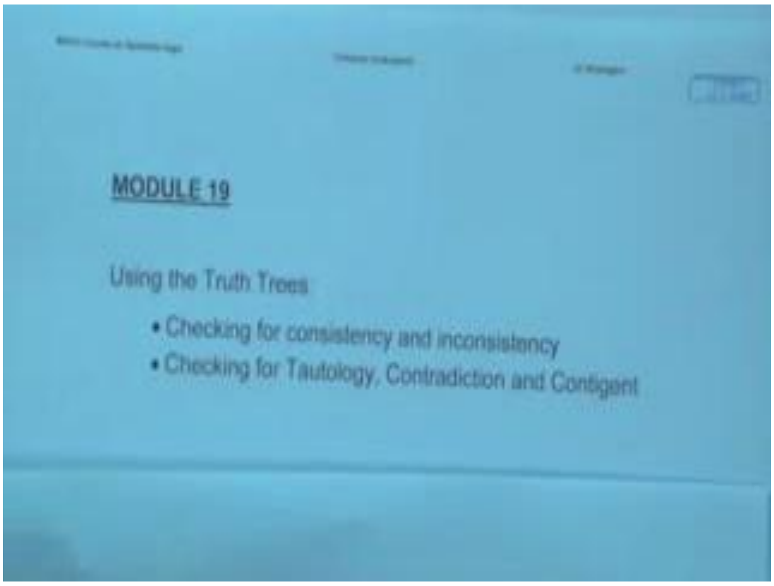
Symbolic Logic
Prof. Chhanda Chakraborti
Department of Humanities and Social Sciences
Indian Institute of Technology, Kharagpur

Lecture - 19
Using the Truth Trees
Checking for Consistency and Inconsistency
Checking for Tautology, Contradiction and Contingent

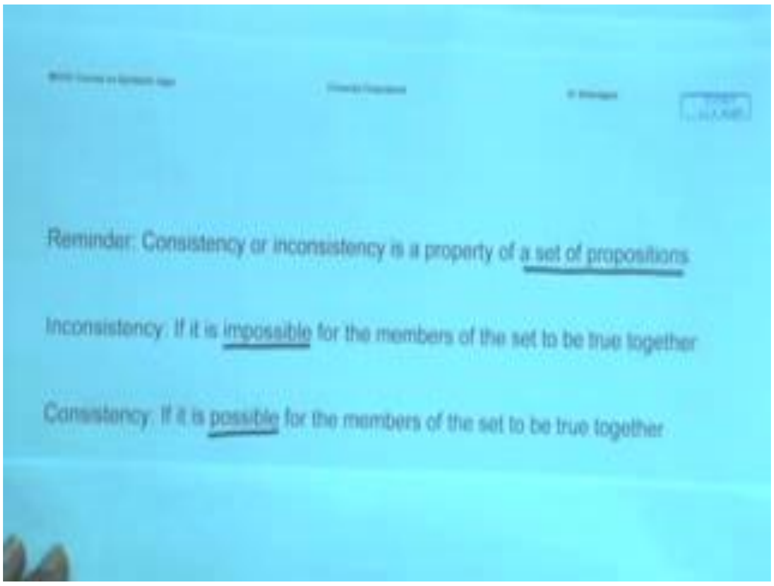
Welcome back to module nineteen of symbolic logic NOC course. So, we are learning with the truth trees and we are going to follow up on the various tasks that we want to present to it.

(Refer Slide Time: 00:36).



So, I hope by now we know how to construct the truth trees and we have been discussing that in the previous modules now it is time to pose some questions and get the procedure to do certain things for us. So, just like in the truth table remember truth table we first learnt the table and then we ask the table if it can give us certain answers. So, here with the truth trees again we are going to ask whether, it can show us for example, consistency and inconsistency of certain set of propositions and whether we can also classify given propositions or statement forms and as tautology contradiction contingent and so on so forth. So, today's task is to learn how the truth tree can help us to do these two tasks one is about consistency inconsistency and other is about classification as per their truth values.

(Refer Slide Time: 01:52)

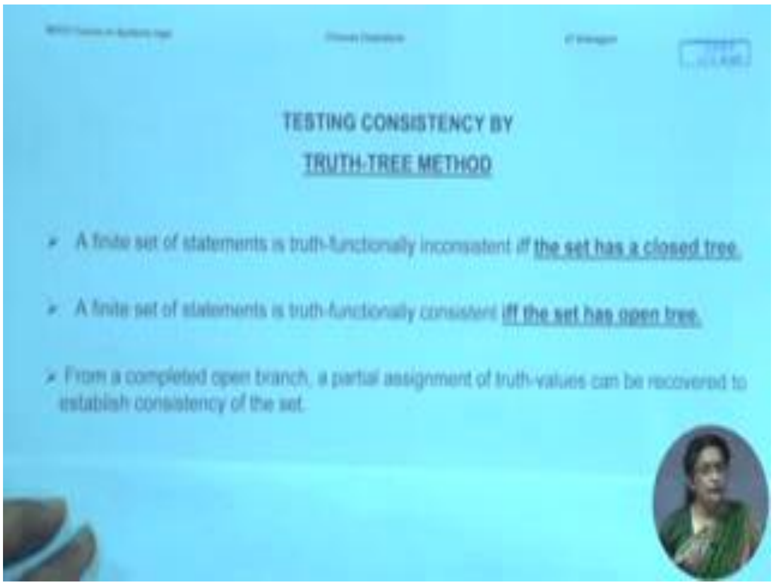


So, that is what is on the agenda for today on module nineteen just to remind ourselves what consistency inconsistency tells you may remember that it is actually a property of set of propositions. So, given a set of propositions we can ask whether the set is consistent or inconsistent and if you recall then, the set is inconsistent, if it is impossible for all the members of that set to be true at the same time in a in simultaneous manner or in a compatible manner whichever way you understand. But there much not be even a single possibility when all of all the members have going to come out is true that is when we call the set inconsistent remember that and then consistency is just opposite of that namely it should be possible for the set members to be true together how many such possibilities the answer is always at least one. So, minimally speaking the set is going to be consistent if there is even one chance when all the members of the set is coming out to be truth.

Now, if this is the situation in in what we say is the property of a set of propositions then how can the truth tree show that to us. So, in that I will again remind you that when you are doing the truth remember this what I have said again and again. And I will repeat it again is that whenever we are doing the tree as you are listing a proposition in the tree whether in the root or whether in the branch you are claiming or assuming it to be truth. So, given that if the set is inconstant you know there may must not be there cannot be even a single branch when, you are going to have all the members of the set to be true alright because if the state is inconsistent then the members cannot be true at the same time. Therefore, in if you do the tree of this kind of a set there is not going to be even a single branch when every member of that set is going to be true.

So, we will youth utilize this insight to fill the definition of inconsistency in terms of truth tree similarly when you are looking into consistency we are going to look into the completed open branches whether there is even one. And if there is one completed open branch it tells you that completed open branch when you follow that you are going to find the truth conditions under which the members of the set of the propositions are going to be true. So, if the set is consistent you are going to see in the tree at least one completed open branch of this kind keeping these together now we can define what consistency and inconsistency the true truth tree method is going to be like.

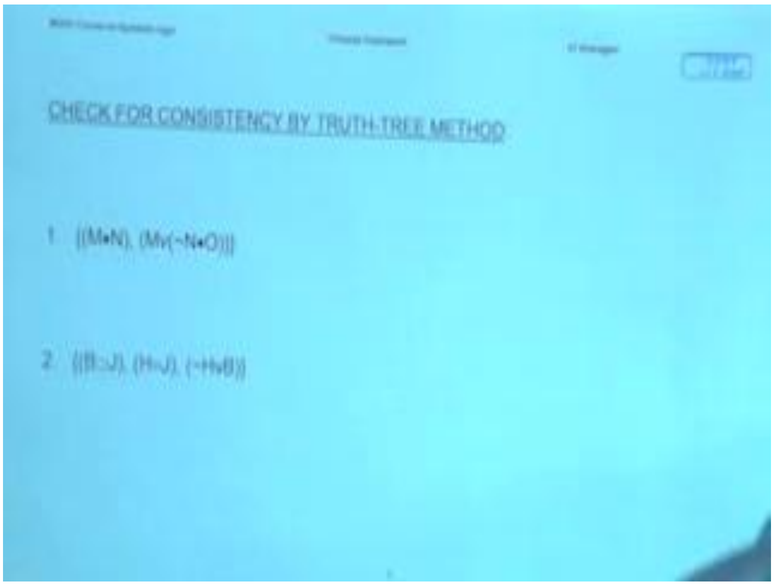
(Refer Slide Time: 04:54)



So, we will define it like. So, that a finite certain set of statements is truth functionally inconsistent if and only if the set for the statements has a closed tree will remind ourselves closed tree means a tree which has no open branches all it is branches are going to be closed. So, there is not even one possibility where you will find the truth conditions for the members of that set of propositions there will not be any situation when you are going to have the members of the propositions are set to be true at the same time. So, this is what we are saying when we set the set is going to have a close tree on the other hand as we said if the set happens to be consistent then, the test for that in truth trees going to show what that the set has an open tree remind yourselves that having an open tree means that the tree there is going to be at least one completed open branch in that tree right and that should settle the question whether the state is consistent or not.

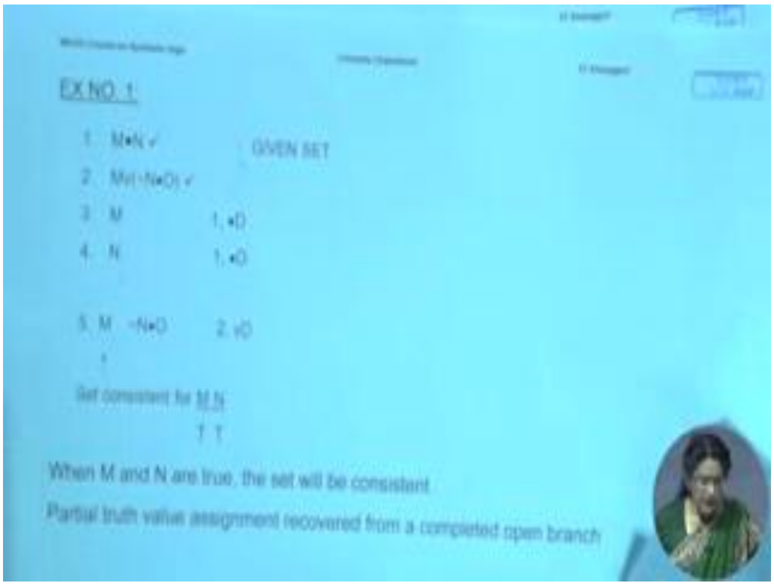
So, if you have understood the basic concepts and then if you understood how the tree works then it is obvious that this is how you are, you see it in the truth tree making and remember that, if you have an open tree there is going to be at least one completed open branch and I remind you that from a completed open branch you can recover partial truth value assignments for all the components in that tree which makes the set all the members coming out to be true. So, whenever you see a completed open branch try to recover the partial truth value assignments and that will tell you what conditions under what conditions the propositions are all going to come out to be true let us take a look into an example. So, that we can we can see this.

(Refer Slide Time: 07:08)



Now, I would request you that you keep your pen pencil and paper ready and. So, that you can do this along with me as we do this here is the first problem. So, this is a set of two propositions one of them is M dot N the other one is M wedge tilde M dot O except that there is the around tilde N dot o. So, two members set is it consistent or inconsistent we are going to find out and the other set has three members. So, B horseshoe J H triple bar J not H wedge P these are the three members and our question remains the same, whether this is a consistent set or an inconsistent set. And just do the tree the tree should speak to you. So, I will ask you I will give you a lead time to start at least on the first problem why do not you try this while we try to set it up and then we will actually match our results. So, that it is easier for it to grasp where you may have gone wrong or where you are correct am sure you will be able to handle this by now.

(Refer Slide Time: 08:31)



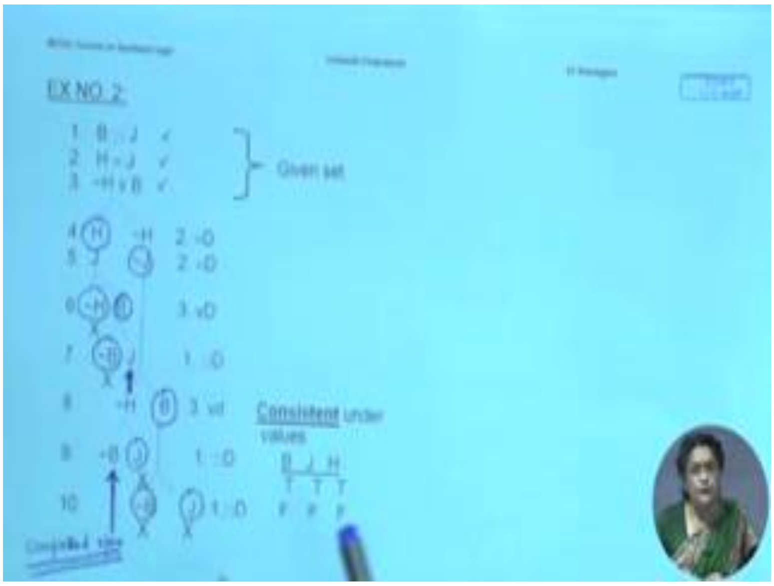
So, the first tree, we are going to set it upwards remember the given set has only two members M dot N M which tilde N dot O and I will remind you that whichever one you are trying to decompose you should know why and remember to put a tick mark against that just to tell yourself that this one is decomposed already. So, you probably thought that this one is and this one is. So, let us do number one first. So, you have done it then this is the result of your decomposition on line number one and this one gets a tick like. So, after that comes this and then we have the composed it like and the answer is the justification is given here.

Please remember to put tick against the composed propositions now you ask yourself whether we have found a completed open branch at this stage or not and I had asked to look both in the left branch as well as the right hand branch and the answer is that on the left hand branch that is a completed open branch how do we know that every decomposition that was supposed to have been done on this is done already and all that is left on this branch are nothing, but. So, on this branch there is nothing more to be done fine if that is the case you have found the answer here you have found an answer because all you needed is at least one completed open branch in order to know what kind of set you delivered. So, this is the completed open branch on that basis you can call the set consistent.

Now, your question is do I decompose this further the answer is it is not necessary because the answer that you wanted to get is made obvious by this branch already you have found this completed open branch and that should settle the question. So, on top of that if you want to do that that is a separate, but that would be redundant even if it is open you know the answer even if it is closed, you know the answer. So, at this point that is we will stop the tree now what comes is this that from this completed open branch not only we can claim that the set is consistent, but we could also tell under which truth conditions this set is going to come out of truth.

So, let us take a look you are falling through and them literals are M and N which tells us when M is true and N is true this is going to come out as true and this also is going to come out as true use your knowledge of the connectives to see why that must be true. So, we will just write this small truth able right from the completed open branch this is nothing, but recover truth values from the completed open branch. So, this is our answer that whenever M and N are true the set will be consistent and this set is shown by truth method to be a completed open sorry a consistent set. So, this was our first problem c I mean it is not difficult.

(Refer Slide Time: 11:56)



Let us try the next one. So, now, we are on the example number two which has three members and we have B horseshoe J H triple bar J not H wedge B now this is slightly different situation. Where it is there is no clear choice it seems to you it is may seem to you that there is no clear choice where to start because each of this if you recall the composition rules then each of this is going to generate a branching. So, before you start on it is better to ask yourself which one should I start with and why by now you should know the results of the decomposition. If you do on any of this you should know what you are going to yield as the branching of result. So, overall I would say that the line number two is the most probably time taking one in the sense that it is going to generate two lines.

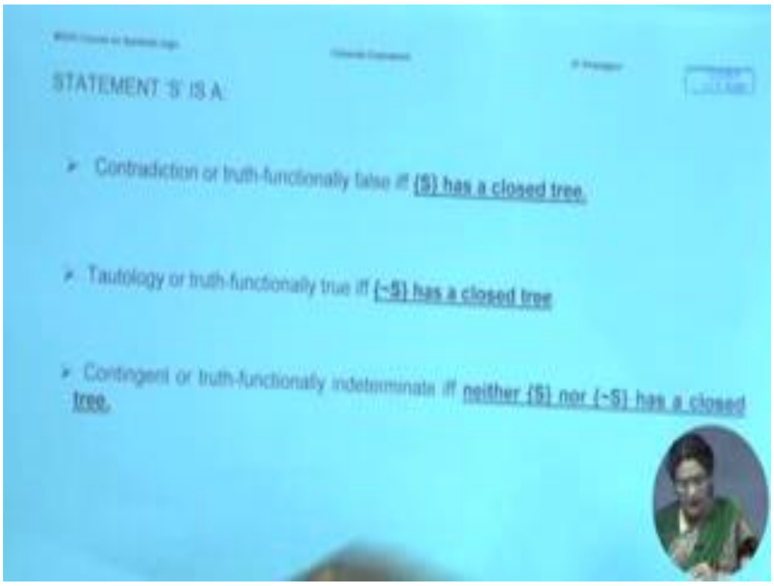
So, if you leave it postponed for any subsequence steps then you have to repeat the result on every open branch. So, instead of that let us get it over right. So, we are going to do it like. So, so line four and five both are by triple by decomposition and we need to write the rule also twice remember you now you have generate two branches and please remember to put a tick on the decomposed line number two. Now what let me remind you that I have asked you to hold one branch while you are doing the operation on the other? So, for example, if you are going to now tackle the left hand branch keep the right hand branch on hold here is the result of line three this is the by protect branching result. So, under these we have now put till the H B why we chose it because apparently it is imply we can generate a closed branch because we knew that if we if we branch it then we are going to have not H and here is H. So, we can close it down and, but this one is still open, but this is line number three wedge d we put a tick on the decomposed line what is still list now number one. So, now, under these we are going to repeat line number one and it is result.

So, here is a result of line number one de composition again you find that here is tilde B J when you find the tilde B you knows that this branch is going to close. So, this is closed this is closed, but please note this one is open at this juncture you can ask yourself is this a completed open branch or not these are closed, but is this completed open the answer is yes. Every decomposition that, you had to do is over at this stage right and the branch only contains now literals. So, you have found at least one completed open branch which makes the set consistent and you can recover the truth value the partial truth values from this completed open branch it is the further question is I have found the completed open branch by just by doing the left hand side do I need to do the right hand side the answer is not necessary because the question that you had in mind can be settled by just this result.

So, here you can say that when whenever J is true B is true H is true the set is going to come out as consistent and that will be perfect answer to you, but if you want to you can always do the right hand side and here the answer is going to be like this that we have to we have to repeat the decompositions one by one on here as branches happen and you may find that this would closed down this would closed down, but here is the tilde B tilde H tilde J and that branch is completed open everything that needed to be done as for as decomposition is concern is finished and everything is now reduced to literal and it would give you another set of truth values under which the set would come out as consistent that would have to be whenever B is false, J is false, H is false.

So, either of this values would settle our question that the set is consistent I have to remind you now that you know it is very important as you can see as you are advancing in your free making that it is important that you circle and circle the literals which is giving you the closed branch. So, that you remember. So, that your search is complete and you can see follow through every branch to sort of get the answer quick. So, that is one thing the second is that search for not just the closed branch with also the completed open in this cases completed open will give you a lot of answers logically important information. So, we have found that this set is consistent under this kind of values either this or this would make the set consistent.

(Refer Slide Time: 17:33)



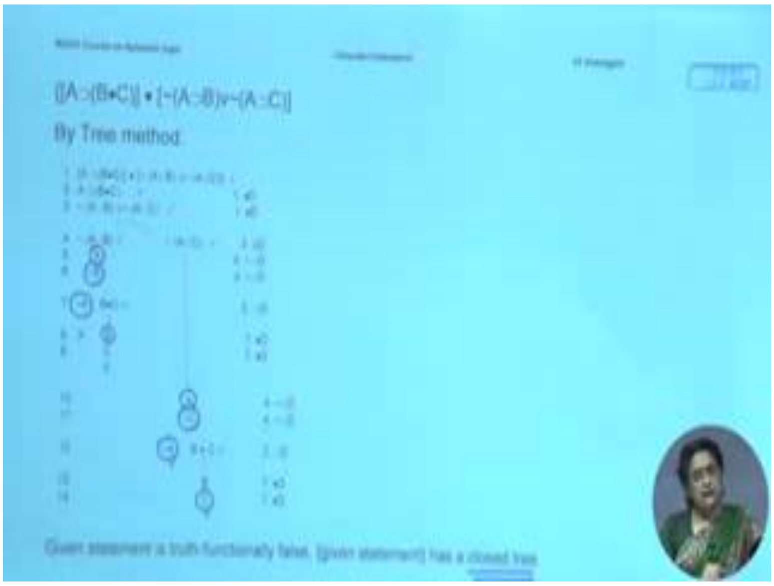
Let us move on to the next task which is whether we can categorize given a statement whether we can categorize it as tautology contradiction or contingent. Now in truth table what you did is to looking to the final truth table and see what kind of truth values are shown there that option is not here in terms of truth trees we have to understand whether the statement of is this kind or that kind. So, I will again I will I we have to fall back upon our understanding of the truth trees and I will remind you that the truth tree lists only truth conditions. So, given that now you try to think about a contradiction a contradiction is that kind of a statement or statement from which only has false substitution instances.

So, given that kind of a proposition what will happen is that there is never going to be a situation when this statement is going to come out as true. So, if you are doing a tree of that kind of a statement which is always false what will happen that branches will close down because whenever you have an a completed open or open branch what you what you can see is when the statement is going to be true, but if the statement is of that kind where you do not have such recommendations you are going to have a closed tree. So, the on that basis using that insight we can now define it like this that you can if you have statement s and if you do the tree on the statement s if it is a contradiction what will happen the set of s is going to have a closed tree alright. So, we can put it as necessary and sufficient condition also if our proposition or a statement has a closed tree it is contradiction if it is a contradiction it is going to have a closed tree both is true. So, this is our result for contradiction it is and we will we will take an example to see that also.

On the other hand tautology proof is not simple and you may you may think or read up on this, but there is no direct proof of tautology as such rather in terms of truth trees what we can do is to see whether the negation of a given statement has a closed tree or not why remember if the proposition s is a tautology it is negation is going to be a contradiction and just now you have found out how to determine a contradiction. So, our modus operandi will be like this that takes a random proposition if it is negation results in a closed tree the original proposition must be a tautology.

So, that is indirect proof that is indirect proof why we cannot establish directly there is a lot of theoretical discussion in that, but I will not get into that, but try to understand the logic behind what we are doing we have already established how to show a contradiction and we know that the negation of a contradiction is our tautology. So, that is following that we will say that if the negated statement results in a closed tree original statement must be tautology on the other hand the third category contingent or indeterminate what will happen is that neither the tree for the proposition will be a closed tree nor the negated proposition is going to have a closed tree. So, this is how we will find out our classification of proposition.

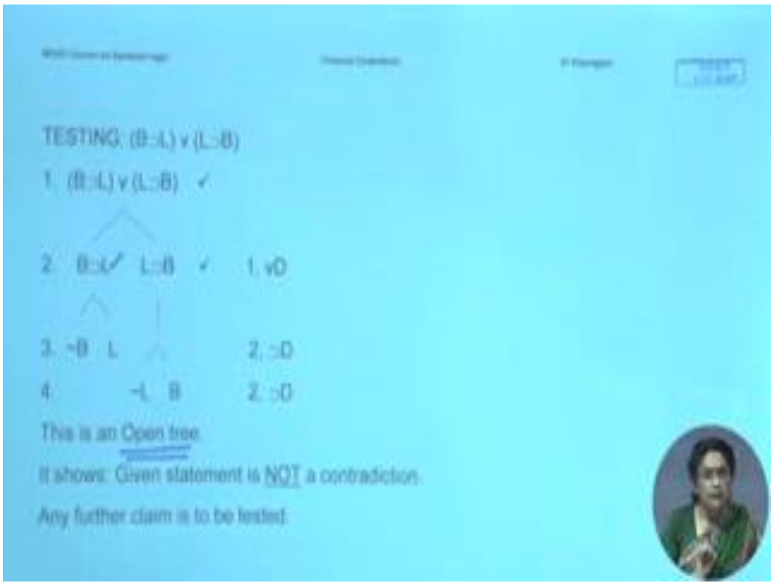
(Refer Slide Time: 21:42)



Let us take this example for example, you have a horse shoe B dot c and tilde a horse shoe B wedge till the a horse shoe c it is a single statement it is a compound statements it is single statement let us see let us directly do the tree on this and let us find out what will happen. So, I will let you work on this, but if you do it properly and without getting nervous or anything or without getting into it is mistake no then the tree is going to result like this what you will find no matter how you do it is that every branch is going to closed down, you are going to have a literal ns negation in every single branch can I stop after doing this on the left hand side the answer is no, why? Because we do not know just by showing the left hand branch whether the tree is going it be a closed tree closed tree means every single branch is closed not just some branches are closed not just at least one branch is closed right.

So, we need to exhaust the possibility and if you do this is the result that you get to see remind again remind reminding you that you need to encircle the literals which causes the closed tree. So, your result is going to be like this that is a truth functionally false or contradiction because the given statement directly when it is done it has resulted into a closed tree.

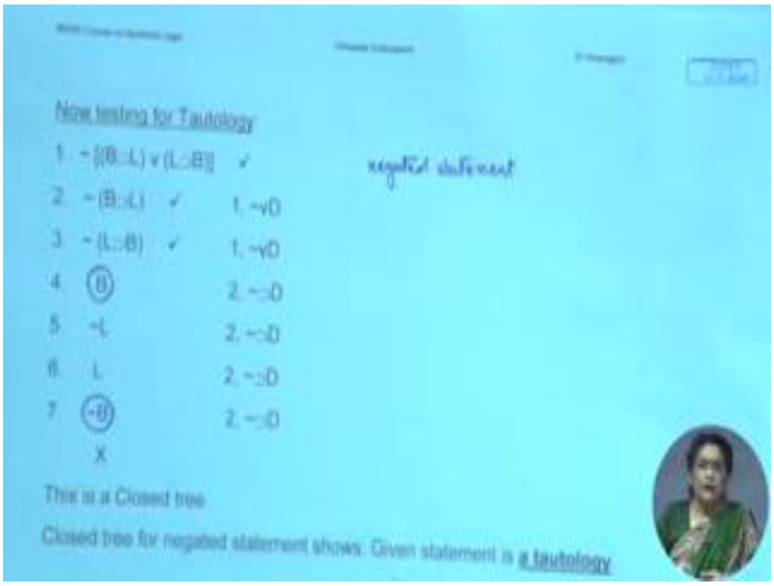
(Refer Slide Time: 23:24)



Let us try this other one this seems like a simple problem this is B horseshoe l or l horseshoe B now given this you have a choice you can do a tree directly on this one or you can do the negation of the you can take a negation of the proposition and do the tree on this either way you will have to find out there is no set algorithm to tell you which one for what kind of proposition. So, let us try this. So, here is if we do the tree directly on this and this is simple tree. So, very quickly you can do it also you are going to generate two branches like.

So, when you have done that let us work on the left hand side keeping the right hand side on hold. So, because there is still this to be decomposed on this side, not be l and this one gets not l B take a look into this tree what you have is every branch is open, but at this stage what do you know the simplest and the most logically correct answer is that it is not a contradiction can I quickly say that this is a tautology no for that you have to do the other test after this namely you have to negate the propositions see whether it is tautology or not at this stage you are only entitled to say that it is not a contradiction.

(Refer Slide Time: 25:04)



So, let us do this negated tree this is your given statement that here is a negation and here comes the tree if you are doing it correctly, please remember to tick a check the compound statements as you are decomposing them and here is the result. Now after this all the decompositions are done, and please follow the branches true and you will find there is this tilde B as well as B the result so, l and tilde l, either way the branch closes down right when that happens on the negated statement you are in a position to say this has to be our tautology. So, that would end our lesson for today, that we have just learnt some tasks by truth tree one is how whether the given set is consistent inconsistent we also saw whether the given proposition is tautology, contingent or contradictions.

With that we will finish this module and will continue with more on truth trees in the next module thank you very much.