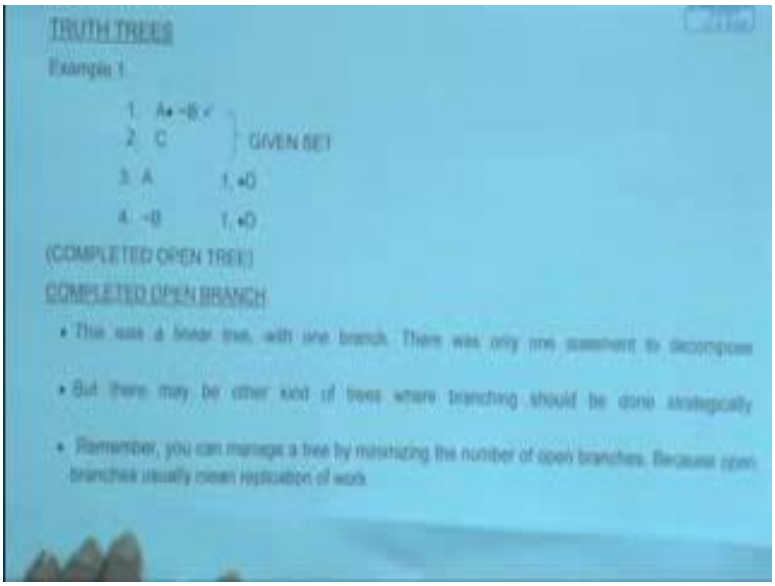
Symbolic Logic
Prof. Chhanda Chakraborti
Department of Humanities and Social Sciences
Indian Institute of Technology, Kharagpur

Lecture - 18
More on Truth-Tree
Recovery of Partial Truth-Values

Hello and we are now back into this truth tree and there is more to learn in this. So, earlier in the earlier modules, what we have done I hope you remember that and that it has given you some platform to start doing the truth tree on your own.

So, we have to learn a little bit more and how to do the trees and what are the things to see, but this is the agenda for our module 18. Today that we have gone to learn more about the truth tree how to construct how what are the ways to do it and then, there is something else that we are going to specially looking to that is called recovery of partial truth values recovery of partial truth values and this is something that we will learn how to do it from the truth trees and you will see the this is going to go along way when, we start posing it the questions that we did to truth tree. For example, this would become a major way to recover information from truth trees. So, first we are right now we are going to learn how to do the trees better and how to do the trees actually and then then the point will come how to utilize the information from the truth trees. So, first construction and then how to make it talk to us properly, this is what we are slowly going to.

(Refer Slide Time: 01:54)



In this segment we are going to looking to various worked out trees worked out trees as in I am working them out what I would request you to do it along with me as you are understanding also you try to replicate that on your own in your time, but in a piece of paper and. So, on that that it is self is a learning process you know because I am doing it and then you are doing it together and that is how you gain the way to do it into this. So, here is an example of a truth tree there are two members in the root a dot not B and there is C and this is your given set and you are asked to do the tree of this fine.

So, what you do remember the starting point is always the root and you take it good look at the root can you do any branching on C the answer is no why not because it is a literal by itself there is nothing to decompose here. So, your obvious choice for decomposition in the root will go into one when you have the root the only way the tree can be done is through decomposition on the root and in this case the only candidate for decomposition is line number one am I correct. So, this compound then needs to be broken what is the decomposition rule to be applied here the answer is look at the main connective and line number one the main connective is a dot. So, now, you know that you need to apply the dot decomposition rule on this line to get the branches growing and if you have done it I will give you just have for second to do that before we show the result. So, that we know what to compare.

So, here is your tree, if you have done it correctly then this is how it should look like I will remind you once more that it is very important that you number the lines that you are generating of course, the root has to be numbered also that is how we knew, there are two propositions in here, but this is a new line this is a new line and each of these new line also will have to be numbered what is this this is the justification panel. So, every line that you have generated I said earlier must be justified. How do you justify by referring to the line that you have decomposed in this case that is line number one and see we have mentioned one here one here then comes which rule by which rule you have decomposed this the answer is by dot decomposition rule the separator between the line number and the rule name is a comma. So, one comma dot the composition is the justification for three and you know this is how it has been line has been obtained same goes for line four fine and as soon as you have decomposed you I told you to remind yourself. So, if this is checking the compound that is your tick mark for telling yourself that I have finished the decomposition on this line.

Here comes the question. So, at lined for are you done is the tree complete or not and remember the way we have defined the completed tree. So, have you reduced the tree into the level of literals only all the decompositions that were it to be done, is it completed done finished what you think and the answer is that everything has been finished on this because there was only one candidate for decomposition here and that has been taken care of an what you see here is a branch it does not look like a branch to you, but this is a single branch and on it there are only literal is now available this when is all ready checked compound fine.

So, what you have in front of you can be called a completed open tree completed because all decompositions here have been finished and the tree is now at the level of literals open tree completed open tree, because everything has been finish and still the tree branch has not closed down why not because, if you follow this branch there is no literal and it is negation both appearing on the branch am I clear. So, this is what you have and this is what you have finished doing a small tree with me fine this is all right.

So, this is now how to read it is rebel come there just now we are learning the procedure how to do this on your own. So, will do more of this that before that few points for me to mention notice that it does not look like a tree, but it is a tree and you have generated a branch it is only one branch and it is a linear branch. So, it does not look like a by forget

it very rather showy kind of tree, but it is a tree none the less there is only one statement to decompose and we finished it. So, that tree is completely done.

In here you had very little choice right in here means when you had the root you had very little choice how to do the tree the only answer is only line number one is the point to start or decomposition on, but note that there can be other situations other trees where it is not very clear which one to start with and why. So, in those cases what I am trying to point out is that the tree maybe needed to be done strategically, what is the strategy the strategy is to see that the tree remains manageable one of the ways to manage the trees is to see how quickly.

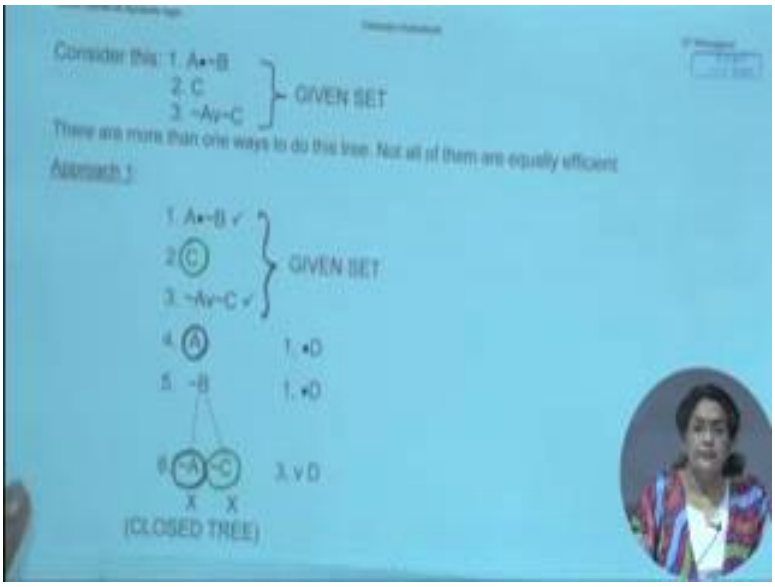
Say for example, you can generate close branches or how quickly you can have completed open branches remember these are the indications that the tree is about to about to be over the doing of the tree has to be over how to minimize the replication of work in a re branch these are the perceptions these are the insights that is behind the tree doing strategy. When I started out I said the truth tables are very mechanical, but trees truth trees and not that mechanical you can manage them you can you can sought of make it go in the way you want and so on. So, one of the reason is that it is a sought of strategized operation I will show you some examples of that.

So, one of the ways in which you can minimize your work or manage the tree is to keep a track of the number of open branches why because the number of open branches if it is large then remember we have a rule that we every open branch that runs through a compound gets the result of the decomposition replicated. So, which means in every branch that it is opened under a certain compound you have to repeat the decomposition results right. So, if you try to minimize that is you the number of open branches that is you can you try to see how quickly can you generate some close branches; that means, that on those branches you do not have to work on any more.

So, minimizing the number of open branches is one kind of a strategy get advise that I can give you how do you know which branches are going to open, remain open and which ones is going to soon close down will that is where your grasp of the decomposition rules come you need to understand this decomposition rules and sought of approach the tree with an open eye look at the root and you know what the decomposition rules can give you. So, you can soon see it is like a chess game, 2, 5 steps

down you can see that, if I do this the branches going to closed down first right. So, that is the kind of work that is the kind of perception that you going to gain the more you do trees the more you will have better grasp on this, but right now I will just mention this that some trees will give you no choices some trees will give you choices and if you use this choices intelligently the truth tree really can be an efficient tree to show you the results. So, let us see other examples as we go around.

(Refer Slide Time: 11:30)



Consider this example for example, that we have the branch right here sorry we have the root as this three members in the root a dot not be C not a wedge not C now understandably given this then the job is to decomposed, but where do I start and more importantly is why that is the question to ask clearly 2 is all ready a literal, you there is nothing to be done on to, but one is a choice three is a choice where do I start and why? Now let us see I will show you parallel ways of doing this trees and maybe more than one ways to do this tree. So, that you can understand what I was talking about the strategy and sought of becoming with that I hope the proficiency will increase also.

So, more than one ways to do a tree and my point is not all of them are equally efficient here. So, getting that is which one works for you is going to be practice matter of practice, but let us see right now. So, I will say I will start with approach one supposes somebody is doing the tree likes of. So, this is your given set that is your given set and the person said whose ever it is that let me start with line number one for no apparent

reason, but the person probably thinks that is the first which has coming in order. So, I will do that one fine that is. So, here is 4 and 5 obtained from line 1 by dot d composition fine so far.

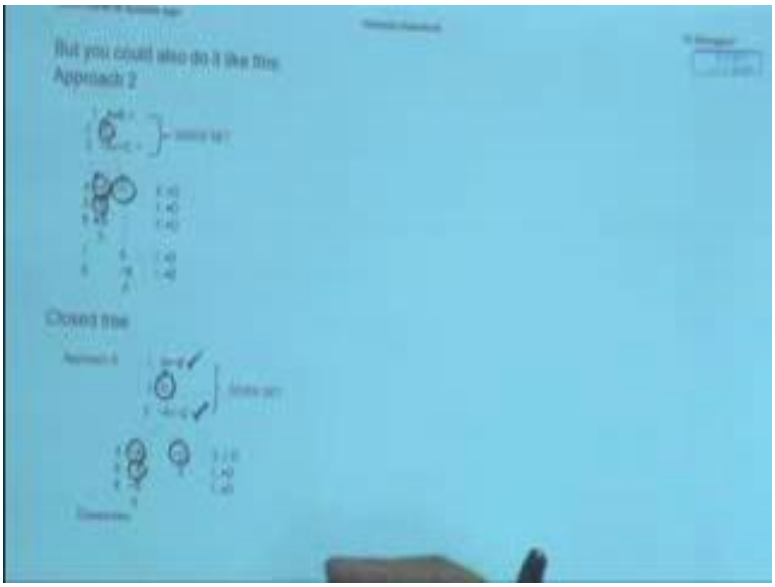
Now, what and then immediately this gets a tick which means that line one has been decomposed, but remembers line three is still there. So, you have open, but not a completed open branch how you write the result of the 3 under the open branch. So, this gets by furcated and here is the result of your writing this this is not a or not C obtained from line three wedge decomposition fine and then you put tick here got me now is this complete is there anything else to be done take a look and you will see that all the branches are at the level of literals. When you are at this kind of stage you need to now pay closer attention to every branch take a look into this branch is it open or closed and you follow it through and here when you reach here you find oh here is still the a and this is a which means what that this branches going to closed down fine let me encircle that for you. So, you encircle it like. So, that is a clear marker why this branches to closed down let us take a look at this other branch tilde C tilde B a, but here is c. So, you have tilde C and C notice how far you have to go this is all one branch. So, go until up to the root to C whether it is open or closed and in here you found that here is a contender.

So, here you have tilde C and your have C what happens here the branch closes down. So, in under each of this branch you are going to put the cross signs remember the cross sign is there to indicate that this branch is closed this branch is closed what is that make this tree it is a tree where every branch is closed down. So, it is a close tree gets it. So, this is how to do the tree it is again a fairly simple tree, but my idea as to show you how you can approach the tree on your own and hope you followed the procedure as I have explained to you.

I also said that this is only one way to do this tree there may be other ways to do it. So, we will take a look into that the result is not going to vary result may not change, but the tree may look different the number of steps may decrease increase and. So, on and then you decide how to proceed with this see here. If you review this tree then this approach shows that the person preferred that first to keep it linear as far as possible when, there is a choice between 2 decompositions and 1 of them is linear no branching only single branch sought of comes out no bifurcation that is branching, but no bifurcation and this one gives you the bifurcation option then this approach shows that the person is decided

to keep it linear as far as possible and then only when, it comes to a compulsion then that bifurcation comes just to keep the matter of replication to the minimum how many steps here 6 let us take a look into other ways of doing it.

(Refer Slide Time: 17:33)



See this is approach 2. So, you can also do it like this. So, here is that set does not change this is your given and then suppose somebody said that I want to start with three line number three nothing wrong with it as per say because you will have the same result, but the trees going to look completely different. So, you need to put a tick and line three, but your line number one decomposition is still left. So, how to do that while you have to do it here on these branch ones and again on the right hand branch again why because this belongs to both the branches remember the parental property analogy there I gave you.

So, both the branch deserve their result of the decomposition of that and I said when you working on one branch you please keep the other branch on hold do not even if it is the same rule application do not try to do not attempt to do decomposition on all though open branches at the same time because, then you are more prompt to error and some of the important information's might be overlooked and so on. So, one branch at a time when you are doing decomposition even if it is the same decomposition rule that you are applying.

So, will after line four then this person is doing on this side line number one decomposition and result is a and B and therefore, you if the person says we have this

sorry there is not tilde v this is just B and then, you get that this is where you need to stop it because you have a and not a and line 7 and 8 you have a tilde B and here you are you need to see that here is tilde C and here is C. So, branch close. So, in a way you have generated a closed tree this is also close tree, but look at look at the number of steps. So, earlier it had 6 steps and here you have 8 steps, point is that between these 2 approaches you might prefer doing the first way though it is this is approach two is not essentially wrong, but it is taking your little bit more time little bit more time and it is done rather in a sought of random way without any specific logic, but just a person is rather impulsively choosing one option over the other, but there is a third way to do it also.

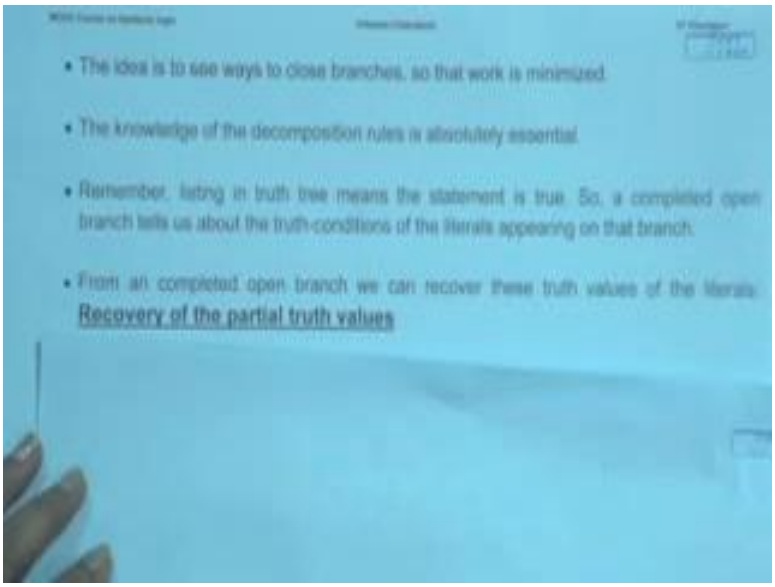
So, take a let us take a look into that this is approach 3. So, we have the same set this person said look I am doing this one line number three line number 3 is here and the results are here, but the immediately the person as spotted that there is this chance to close the branch down right and the branch is closed which means that we do not need to applicant the result of decomposition of line number 1 into 2 both branches see this is line number 3. We need to put a tick here and this is line number one, but when you have done line number 3 and one side has close down because there is C tilde C then, you write the result only on the left hand side and left hand side again gives you the same reason to close it down like. So, and it is a close tree the result does not change, but what change is your strategy the number of steps that you going to take and the rule and so on so forth.

So, this is one way to get into what we call the doing the tree doing the tree is not a blind process which is, what I am trying to explain to you that the whenever because you knew and because you are approaching at with the newly acquired knowledge. So, let me tell you that first of all you need to really understand the decomposition rules your understanding of the decomposition rules will depend on how well you understood the truth tables of the connectives. So, each one is built upon the previous one if you remember the connectives. How they function what is the truth table you will have no problem in understanding the decomposition rules and those are going to be the stipples for doing the tree.

So, say for example, when you see this root here then unless you know how the dot moves how the wedge moves and so on. You are stuck at some point, but on top of that what I am saying is that knowing just memorizing the decomposition rules is never the

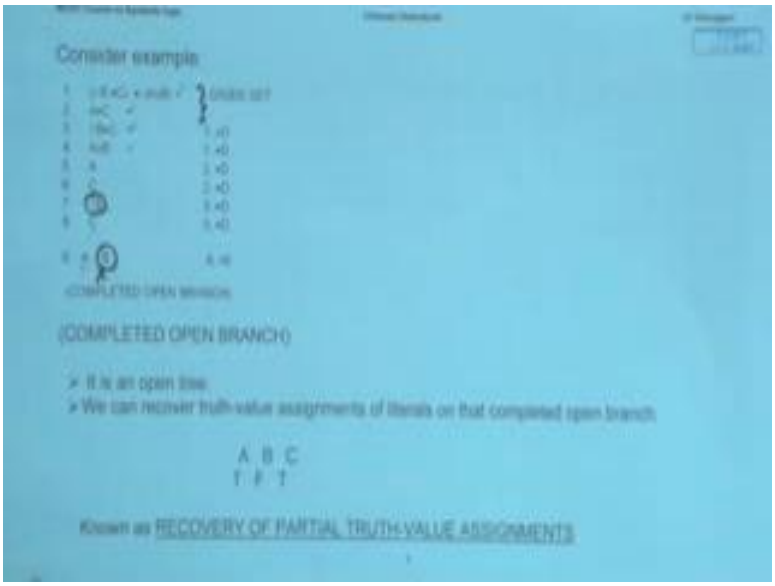
solution you need to have a grasp over them you are need to understand. Why they work in the way they do by paying attention to what the root is talking to you about and it is important to know the rules, but it also important to understand the rules.

(Refer Slide Time: 23:04).



Now, remember that I have said that listing in the truth tree means that the statement is true right. So, whenever you have a completed open branch I mentioned this earlier it is telling you something namely about the truth conditions of all the literals appearing in that branch right because listing means that the statement is true. So, when you have a completed open branch it means that it is also showing you where the literals are going to be true. When that happens this is the reason from a completed open branch you can recover the truth values of the literals I will show you and that process is known as recovery of the partial truth value assignment to the literals.

(Refer Slide Time: 24:11)



So, let us take a look let us take a look and then maybe we can understand this better suppose we have this as your given set. So, you have tilde B dot C and average B as the first proposition A dot C tilde B dot C as your next sorry A dot C up to here is your given set this is your given set two premises, two statements in your given set and then you are doing the tree as usual. So, you may start see both are dots. So, you do not have much of a choice and probably you took the first proposition and here other results of that dot decomposition and remember this is why I said you need to check the compounds.

So, this is very important that you do this has you go along. So, this is the first one is done then you have. So, many contenders here, which one do you start you say I will start at A dot C. So, your result of A dot C the moment you do that you tick that you check it and then there is this tilde B dot C. So, check it and write the results then you please see that they are still line number 4 and this is the reason we need to have this checking in as reminders and here is the result of the decomposition of this.

When you are at this stage line number nine where you have decomposed line number four which decomposition your tree is at the level to stop, but this is when as I said you need to check which branches are open which branches are closed for example, if you follow this through then you will find that you have a situation here on this right hand branch where you have B appearing here and tilde B appearing write here. So, that branch closes down part look at what is happening on the left and side the left and side if

you follow it through we have a C tilde B C a and so on and so forth. So, all literals and none of them are like A and not A or B and not B. So, this remains open not only open it is a completed open branch because every decomposition that was supposed to be done on this branch is finished and everything is reduced to the level of literals fine.

This provides us the right opportunity to recover partial truth values. So, from this I can see that A is true, C is true not B is true not B is true means B is false and these are the only literals that I have you get the repetition of this information here C is true a is true which is all ready given here. So, from this we can now form a small sought of a truth table out of this see from this branch we can say what, we can now say is that a is true C is true and B is false and that would keep this given set the truth conditions this process is known as the recovery of truth value assignments. So, this is something to know from a completed open branch you can do this process to take care of the partial truth value assignments we are going to use it in our next modules and so on.