Symbolic Logic
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Lecture - 13
Using Truth Table
Testing A Set of Propositions for Consistency and Inconsistency and for Logical Equivalence

Hello, this is module 13 of our (Refer Time: 00:24) course in symbolic logic and we are going to do more of truth table and learn A something new namely to how to determine validity and invalidity of an argument.

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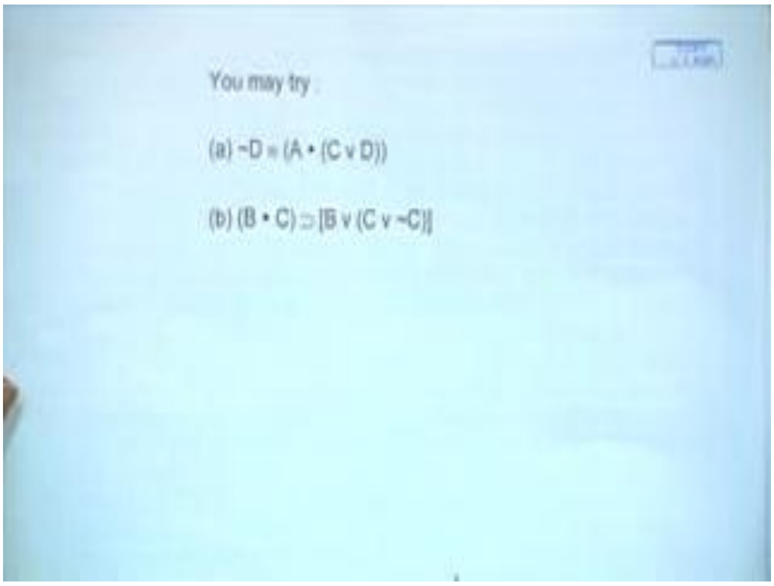

But before that, you may remember that, in our previous module we said you know why do not you try doing these problems, to see whether this is tautology, contradiction or contingent. So have you given some try to see whether this, you can do the truth table of this. So otherwise will be looking at the result for the how to do this, but the whole idea is that you will also try out and then you will look at the result at I have produced and sort of see from that, how far your work is matching mine and so on and so for.

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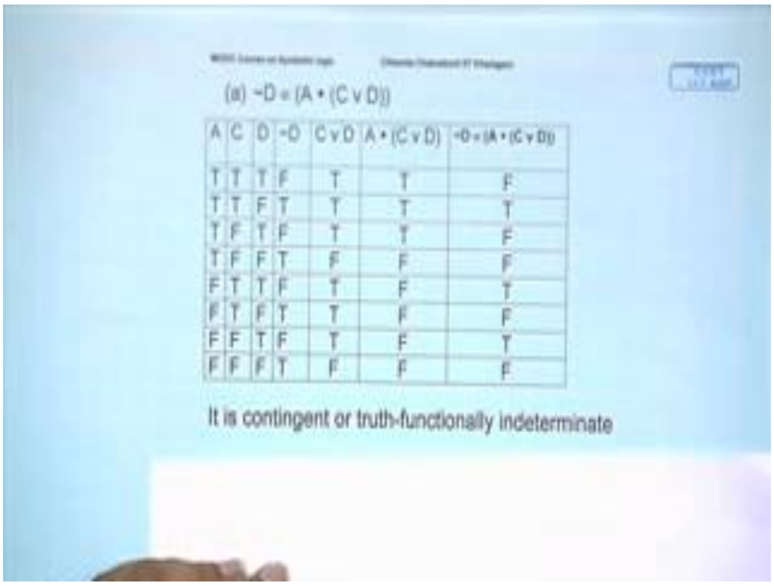

So start with the second one. Sorry the very first one this is our tilde D triple bar A dot C wedge D and this is what we are looking at we are trying to figure out whether it is A what kind of proposition it is in terms of truth values. Remember last module was about that. So the heads are going to be like this, we need columns reference columns are A C D and then there as to be A separate column for tilde D. Why? Because that is A component, and remember it is A compound, so the way we are following it there has to be A separate column assign to tilde D also. Then this is C wedge D, this is A sub connectives and here A dot C wedge D that is the sub connective. The main connective is the triple bar and that we have set for the last one.

How many rows? Well as you see there are three discrete components, simple components A C d. So eight rows and we remember how the truth values are distributed. So this is the way to distribute the truth values. Out of that we built the tilde D column by looking at what we have assigned D, that how we get the tilde D and then comes slowly we build it up like so and final column if you have done it correctly looks like this - F T F F T F T F and so on and so for. That kind of mixture is what makes this statement contingent or truth functionally indeterminate alright.

So this is what the result shows us. The truth table, remember, does not tell you anything specific. It is a mindless procedure, but only when you pose a question to it can give you an answer. You need to interpret the result in a certain sort of way. So here we ask the question, can you show me in terms of truth values which category does this proposition belong to. Please note it is an actual proposition. It is not a propositional form and you asked show me truth table which category does it belongs to and it has shown you that. You look into the final column and you find out the excess mattering of Ts and Fs which make it qualify in to this category, contingent truth functionally indeterminate. Is that what you found out in your answer true if so congratulate yourself? So you have learned this task also.

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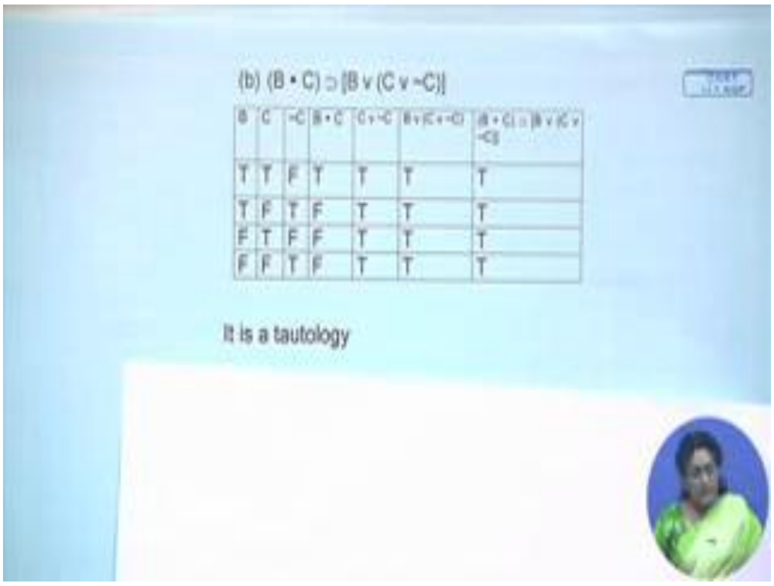

The second one, the second problem was like this, B dot C horseshoe B wedge C or not C. This was our previous from (Refer Time: 04:13) module. So again how many rows the answer is very easy, we need only four rows. But how many heads, how many columns, well that is like this, that we need this two for the reference the tilde C because it appears here so separate column. Then comes the sub connectives B dot C, C wedge not C, B wedge C dot C wedge not C and then the whole compound in itself and then again if you have done it correctly then this is what the truth table is like all right.

So in a way this is your final column, which tells you what that this proposition or statement is tautology always true. So simple, this is our first task and this is the procedure also I have shown and you found out how to classify the proposition into this three categories.

If you learnt that, then our next module, today's module is about a second task that we can put the truth table true. Here we are trying to see whether an argument is valid or invalid and we are dealing with deductive argument which a very important property that also we need to remember. So can the truth table tell us given an argument whether it is valid or invalid? That is the question we are posing through truth table.

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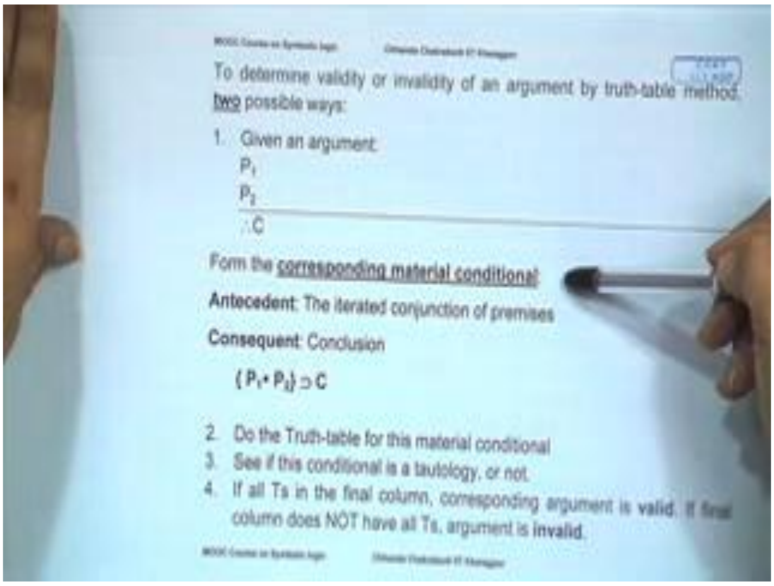

How to do this, well there are two possible ways in which you can go. One of them is that if you are given an argument like say for example, this where there are two premises P1 and P2 and C is the conclusion. Then given this kind of an argument what you can do this is one of the ways to convert it into a corresponding material conditional. So given an argument you can form the corresponding if then proposition. How were the antecedents of this conditional going to be the conjunction of the premises, the iterated conjunction of the premises?

So here you have P1 P2 you put them in conjunction and that becomes your antecedent and the consequent is the conclusion. In this case C happens to your consequent, but the difference is this is an argument and this is just a statement. So given the argument you can form the material conditional out of that. This is one of the ways. So here is as I was saying out of this argument you can form this corresponding material conditional.

Then what, well know because remember truth table apply only to propositions or statement forms, but not to argument for a sake. So directly on this argument you cannot apply the truth table. So you need to convert it into statement of some kind or statement form of some kind and then the truth table becomes applicable. So that is the whole logic behind this conversion. Once you have converted into the material conditional of this kind, then you do the truth table as usual for this conditional.

Now the result, if I do the truth table for the whole material conditional, Then what? The idea is to see whether the conditional is a tautology or not. If you have it all Ts that is, if it turns out to be a tautology you know the corresponding argument is valid. If the final column does not have all Ts that is there are some Fs there then the argument is invalid. This is our one of the methods to apply the truth table to an argument.

Now what is the inside behind this? See when you are saying this, what is that saying. You converted the argument into this material conditional. What are you saying here if P1 and if P2 then C. So if the argument is valid then whenever P1 and P2 are true C must be true correct. So in a way, what you are saying that if this is always true the argument has to be valid. This is what we said. If you find this is to be A tautology the argument has to be valid.

Now when can this be false, when you have P1 true P2 true, but C false, that is when we say it is a invalid argument right. When premises all true, but conclusion is false. If there are such situations then you are not going to have this material conditional as A tautology, there will be some Fs there. When that happens you know the argument is invalid. did you follow that? So I will repeat what I just said that one of the ways, there are two ways to do this with the truth table method. One of them is to take the argument, form the material conditional with antecedent being formed by the conjunction of the premises given and the consequent become the conclusion and then the do the regular truth table on this proposition, or proposition form. And what you see whether this term out to be A tautology or not. If it is a tautology argument is valid, if not the argument is invalid right. So that is our first method.

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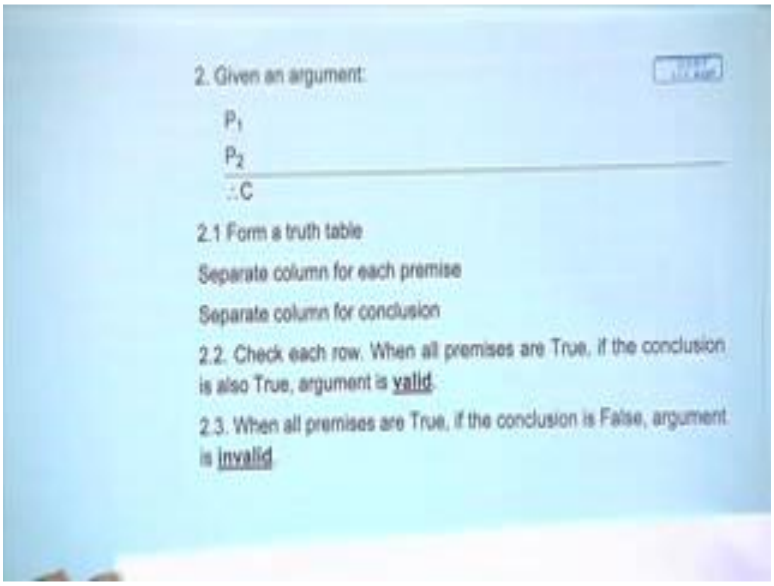

What is the other one to do? Well similar something similar, but not the logic remains same, but just few steps we do not have to do. For example, given this kind of argument that your P1 P2 and you have therefore, C. What we do is we do the truth table; we do not convert it into a material conditional. We do the form a truth table on what, on just take the premises, and give them separate column. One separate column for the conclusion we do not form any material conditional, but we have the reference columns and we directly bring in each premise and we assign it a column, each and then one column for the conclusion.

So, then still the logic remains the same. We are going to look into every row and check what is happening when all premises are true. Remember if the argument is valid, whenever all the premises are true, the conclusion is also going to be true and when all premises are true, but conclusion can still false you have invalidity at end. So I will repeat the second method also, whichever suits you. You do not have to do both the things, but either of this will do. The second method simply says that given an argument like this do not convert it into a material conditional instead you form a truth table.

As is usual for truth table you have the reference columns for the components, then one column for each premise and the separate column for conclusion. Will show you example and then we check, we do the truth table as usual and then we check every row and check specifically when every premises true, what is happening in the conclusion.

When if all the premises are true then also the conclusion is coming out to be true because you can see, you have the rows right. So that rows represent possibilities. So you check what is happening when the premises are true and if you find the conclusion also true every case then arguments is valid, but if you few find even one case where the premises are all true, but the conclusion is false, call the argument invalid. Did you understand so that these are the two methods?

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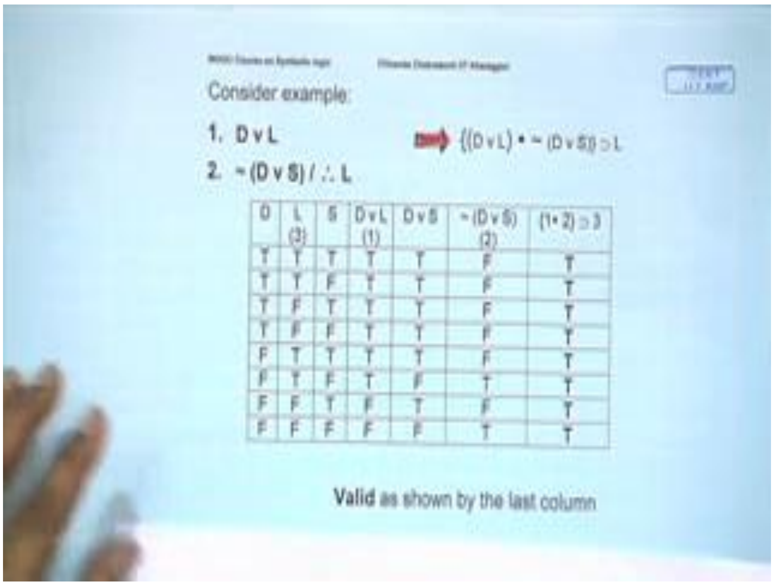

And now the time comes to see some actual examples and I suggest that you also try to do this. As I explained it to you, as I show you the example you try to do this. So here is a argument. The first premises D wedge L. The second premises it is not the case the D wedge S or tilde D wedge S. This slant line is a separator line between the premises and the conclusion. The conclusion is L presided by the triple dot, the triple dot stands for therefore, correct. This slant line is A separator line, that is separates the conclusion from the premises.

So, we are following suppose you say I am going to follow the first method right. To check by truth table will follow the first method. What is first method say; convert it into the corresponding materiel conditional and then the sentence become like this. This is an argument this is A compound proposition. What have you done, you made you took the two premises form the conjunction and the consequent is the conclusion got it and then we do truth table for this one. So how many rows we have one two three, three discrete constants there is going to 8 rows and we will do the truth table as we know it is. So here is A our our table heads or the column heads. These three are your reference columns. Please note these are alphabetically ordered.

Then we take the sub connectives. D wedge L, then D wedge S and this is a tilde D wedge S. So first we need to compute this and then we have tilde D wedge S and then we have the whole proposition namely the whole material conditional here, but we are using this numbers and numerical, Why? Well into for abbreviation; remember this is our premise number 1. So we call it 1. This is our premise number 2. So we gave it a number 2 and this L which happens to be in our part of the reference column is our 3. So these 3 columns are what we need to check. When this is true, this is true, what is happening with L, that is what we going to check but the one way to do this is the first method says we are going to put the whole conditional statement as a my last column and see whether it comes out to be a tautology or not.

Many of you have done the truth table correctly and assign the truth values properly and computed the sub connectives properly, then this is what you are going to find. What we are looking for is, when this is true and this is true, what is happening. Please check that this is not the case where you have both the true, but this is true any way.

So you have T. You do not have a situation where the premises are all true, but the conclusion is false. you do not have any situation like that. Please look through into the rows. This is T, this F and this is T. The combination that you are looking for rule out is that when this is true and this is true, but this is false you will not find any of any possibility like that and this is all going to be true. So what you found out. You found out that the given argument is valid.

By looking at the truth table of this statement you are commenting on the argument and argument is valid as shown by the last column. Let us take the same problem, but we are now using the second method. So I will remind you, this is the argument. If we do it by second method, this is our first method, the result is not going to be change, but look at the way the second method is applied.

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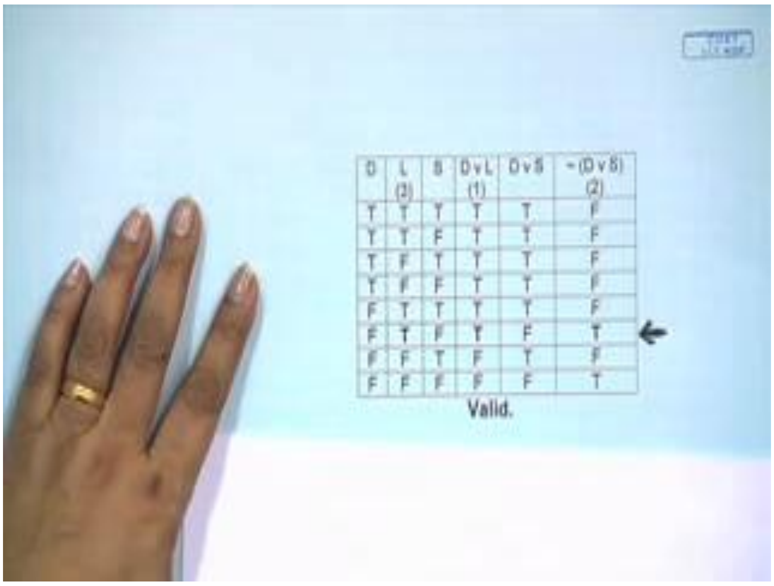

So here we do not convert into material conditional, instead what we have done we have taken the discrete constants and these are your part of the reference column. Here is first premise one column given, this is D wedge S, but you can directly also go into this column and call it the second column. What are we comparing this column, this column and this column as before and what are we checking when this is true, this is true, what is happening with the conclusion or the consequent and again you will find that there is not a single case where you will find that when this is true and this is true, this is false.

Now you please go through the rows by yourself to see when is 1 and 2 both true and you will find that this is the only row, this is the only time when that has happen. First premises is true here, second premises is true here. So true and true and this is the crucial row, which will tell you whether the argument is, what fate argument has and you find that is when the conclusion is also true.

Which makes it valid, but you have to check every single row, validity is something where you need to eliminate all possibilities. So every case there is, first of all there is in here, there is not even one situation when you have both the premises true. This is the only time both the premises are true and luckily fortunately that is when your conclusion is also true which makes the argument valid.

The result as in changed obviously the method changes, but the result does not change. So this was our first way of doing it, converting in to A material conditional looking for tautology or just assigns columns under premises and the conclusion check, what is happening when both of the premises are true what is happening with the conclusion and the result or the verdict remains the same right.

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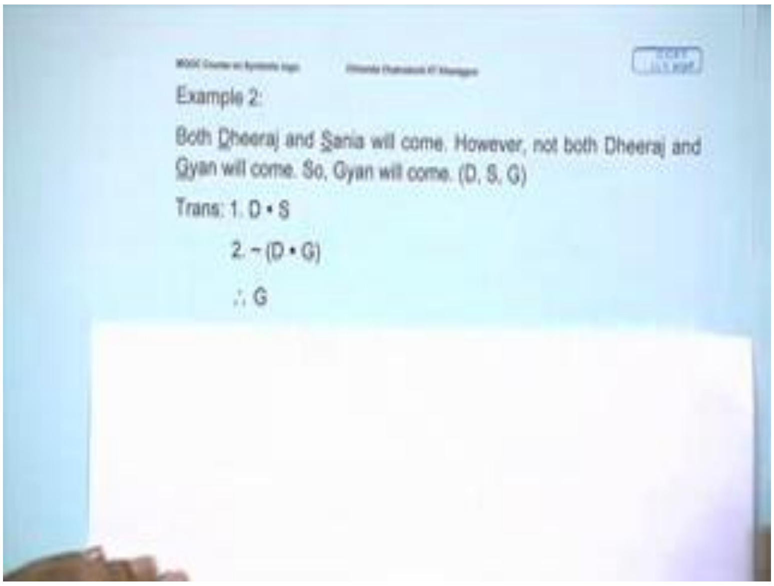

So this is how we go. If you gain the little bit of confidence in this procedure then let us try this kind of word problem. See so far we have gone into the symbolic level, but here is an argument, both Dheeraj and Sania will come; however, not both Dheeraj and Gyan will come therefore or so Gyan will come. This is A problem the argument is given in word. So, the translation skill also shows up and then you do the truth table technique to find out whether this is valid or invalid argument.

So, first thing is to do is to translate it. We have done translations already. So that should not be A problem, if you still remember the translation skills have to be practice and have to be learn. So this is first sentence premise number 1, this is premise number 2 and this is your conclusion. And these are the given keys D.S.G stands for Dheeraj, Sania, Gyan sentences involving them. What is the translation of this argument, let us try.

Both Dheeraj and Sania will come, that is clearly D dot S. Next sentence is however, not both - Dheeraj and Gyan will come not both. So the translation will be like this not both. It is not the case both Dheeraj and Gyan will come and then the conclusion is very simple it is just G.

Now you have a choice. So translation is done. Now you have a choice, how to apply the truth table. You can either form a material conditional or you can do the straightway column assignment to each of the premise and conclusion and do this. The question is what is the result of this. So if you can you quickly do the truth table and will compare the result right now, how many rows you know already there is going to be 8 rows and it is a matter of just lining up. One thing I must tell you that at the alphabetical order. Try to follow the alphabetical order. So it is going to be D.G.S not D.S.G; so not by the appearance order, but by alphabetical order.

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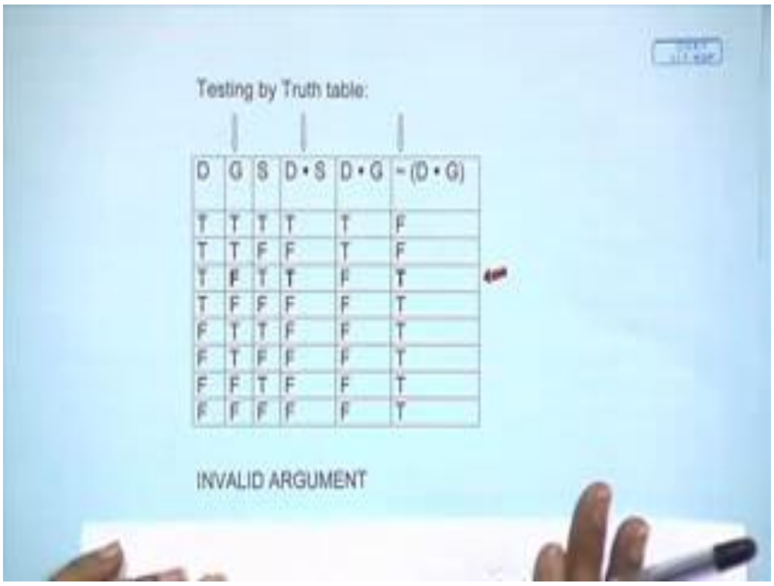

So if you are fast you can do the truth table as I said right away, but let me at least show you the column heads. If you are following the second method that is not converting into material conditional, then these are your reference columns. This is the first premise this is the second premise. You can skip this one also. If you want you can skip this column and go directly here, but since we are beginners. So I thought about putting that in and we are comparing these two columns with G, which happens to be conclusion.

Once more be, very careful to see each row, when this is true, when this is true, what is happening with G. If you find that one of them is false, obviously that is not a point of worry, but it must not happened when this is true and this is true, this is false. That must not happen. If that happens then the argument is invalid, right that is our mode is operating.

So we check how we have done it. If you done it correctly, then this is the kind of table that you are going to generate, 8 rows this is the way to distribute the values and these are the results for each column. We are checking now what is happening here, this is true this is false and this is true. So we are safe. This is false, this is false and this is true, no problem. This is true, this is true and then here we find conclusion is false. Once more this one is true, premise number 1 is true, premise 2 is true and the conclusion is false and that row is crucial to settle what, that argument is invalid all right.

So whether you look into this, whether there is a second row or third rows. This playing this is not the issue. Having even one such row can demonstrate that the argument is invalid. That is what we earlier established. Validity is an exhaustive notion all or none, either always valid or it is invalid. So it does not matter, how many times it happens that your premises true and the conclusion false.

Here we have found one clear decisive situation when you have the premises all true, but the conclusion is false. That is what is sufficient to make the argument invalid. So this was our module on testing arguments with truth table and determining whether they are valid or invalid, you have learn earlier that validity and invalidity are very important notions. They differentiate between what we call good deductive argument from bad deductive arguments. So you just saw that the truth table technique is quite adequate and competent to perform this task.

Earlier we have learnt how to disclassify say propositions into pigeon's holes like tautologies, contradiction, and contingent. This was with the arguments. Which are not statements right we have learn that, but it is important to know that even with this truth table technique we can compute values in a such a way, that the arguments can be classified into valid and invalid. The invalid arguments are not desirable ones, the valid ones are right. So this was our module, on that in the modules to come we will put the truth table to other tasks, but right now this is where we were and let me remind you that we have learn two techniques two ways to demonstrate validity or invalidity namely the conversion into two material conditional or not.

So with that I am going to end this module here.

Thank you very much. Hope you learnt something.