Introduction to Economic Growth- I

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Lecture-13

So, how do we proceed in this particular case? Well, everything else remains unchanged. In fact, the black lines that we see here, it indicates that this is the starting situation. So, s1 is the saving rate with which we begin. And then s2 is the new saving rate. So, how do we analyze this particular situation? Well, what do we do here? What do we see here? Since we said that everything else remains unchanged. So, nothing else has changed, only this has changed.

Nothing else has changed, but what is this change going to lead to that? Is the question with a change in s. So, nothing else gets affected, as we can see because Δk remains the same, production function also remains the same, and the only curve that has been affected is the s.f(k) curve. Are we clear about that? And how is that represented here in this particular figure that we are looking at? It is represented by a red line. So, this red line indicates the new s.f(k) curve and to indicate that it is the new saving rate, it is labeled as s2.

We should also mention here that s2 is greater than s1. What is the change that is going to occur in the economy? If again numbers are given to us, numerically, we can solve them, but graphically also, we can look at what is the change that is happening in this economy. What has changed because of a new higher saving rate? This. So, this was the previous steady-state level of k. Now, because of a new, when s increases to s2, what it implies is there is a new steady state level of capital, which is labeled as k*2, and the previous one was labeled as k*1.

So, what has happened in this economy? The economy has actually, it has not attained yet by the way, the economy is still here. And what the economy has found out is that now there is a new steady state. So, there is a distinction because there is a new steady state, which does not mean that the economy is jumping from k_1 to k_2 immediately because the savings rate has gone up. So, think about it in something like this: So, when inflation occurs we know that the price of the same good that we were purchasing earlier has gone up. That is an information that does not mean that now also let us say I can afford that good.

That is a separate question altogether. So, similarly, here the first thing we need to ascertain is what is the impact of an increase in the savings rate in an economy. The impact is that the economy is now having a new steady state. And remember, corresponding to this y 2, there is also a corresponding level of steady state output. So, steady-state output has also changed.

Again, the fine distinction being it does not imply that simply by the change in the savings rate, the economy is moving to this new steady state. No, that is not the implication. The implication is that there is now a new steady state, like there is now a new price, a higher price to the same object that we were buying earlier. So, that is the impact of increase in savings rate. What impact is it going to have in the long run? The impact is as we have seen earlier.

When an economy is moving from a lower output level to a higher output level, the process of growth occurs there. So, what is going to happen in this particular context? We are already familiar with that particular story. So, how is the economy going to move from k*1 to k*2? So, now, the relevant steady state is k*2. So, maybe I should write it down here. So, the relevant k* is now k*2.

Why is now this the relevant steady state? Because why, if you still want to be very specific, since s has increased? What is going to happen then? The economy is currently at k*1. This is where the economy is located currently and it knows that the new steady state. So, k*1 is no longer the steady state that is the point. The economy is currently at k*1, but k*1 is no longer the steady state. I am just writing ss to denote steady state, why? Since s has increased, just as we said that you know you can think of a parallel that why do prices change? maybe let us say you know it was a bad year in terms of agricultural output, let us say there was a drought.

So, the earlier price now does not hold because supply has contracted. So, prices have gone up. The third point then is then k*1 is less than k*2, where k*2 is the new steady state and k*1 is the old steady state, just to be very clear about this. So, for us now, the relevant line is actually this red line that we see, the red curve that we see, which is s2 f of k. Are we clear up to here? Then, what happens in this economy? That is the question now.

First part we have determined the new steady state, but second part remember the economy now has to move towards the steady state. So, the economy has to move from k*1 to k*2 and how does this movement take place? Since k*1 is less than k*2, this implies you know go back to the mechanism mentioned earlier. And what is that mechanism? That mechanism is this. So, now this is our relevant line: this is the amount of investment, out of which this is the amount of depreciation. So, there is a Δk here; add that Δk to k*1, and you reach a new point here; call it just k_1 prime, and the process repeats itself.

So, this is what is exactly going to happen to the economy. The economy is going to move, but again, gradually, over time, it is going to move from k*1 to k*2, and in the process, what is going to happen? Our y is also going to increase in the process. So, in other words, what is the prediction? This is where you know the policy implication of Solow also comes into the picture. So, as per this benchmark Solow model that we have seen so far, it predicts that a higher s small s is saving rate. So, a higher saving rate will lead to a higher k*.

k* is the steady-state capital. And since output is a function of this k, so higher k* will lead to a higher y star. So, the Solow model predicts that countries with higher rates of saving and investment will have higher levels of capital and income per worker in the long run. So, this is the prediction that comes out from the Solow model. So now I suppose, you know what we have been talking about in the earlier weeks as well, that models are necessary so that they help us organize all these different observations that we make about the world around us, and they also make predictions like this.

And if you would like to see if such a prediction is, you know, empirically, does it hold yes or no? So, here in order to check that on the x-axis, we have investment as a percentage of output; average investment has been taken for over 40 years on the x-axis, and on the y-axis is the income per person. Remember, the prediction of the Solow model is that with higher rates of saving and investment, those countries will also have higher per capita income in the long run. This long run is important that is why the time period between 1960 and 2003 has been considered. What do we see here? So, this is why we had looked at similar graphs earlier. What the points here from what we can see, more or less a positively slope line can be fitted.

What does that mean? It means that there is a positive relationship between investment as a percentage of output and per capita income in the long run. And is this what the Solow model had predicted? Yes, this is what the Solow model had also predicted. So, now, I suppose we can see why the Solow model was, you know, so popular in the 1950s, 1960s, and even 1970s in many cases because, you know, it is like a neat recipe, right? It is like telling us or telling the governments of different economies that you want higher per capita income. Now, in the long run of course, not in the short run, then what should you do? You should be investing more, you should be saving more, and if you do that, then your income per capita income will be higher in the long run. So, it is like a very neat and clean, you know, prescription that has been given, an economic prescription, so as to say, and naturally, this has become very popular, especially during that time.

Now, if we are clear about this, let us move to another variant of the same thing, and that is what happens when there is population growth. I should mention it here: breakeven investment in the presence of population growth. What does this mean? This means that remember the way we are defining all our terms, they are in per worker terms. Now, in this analysis that we have done earlier, where we had seen this. One assumption was that the population was not growing; we said ceteris paribus, only the saving rate was growing.

But if you want a more realistic representation of an economy, in an economy, there is growth in population, is not it? So, now the next question is, what happens when there is growth in population? When there is growth in population, let us say the population is growing by a rate of 2 percent. So, in that case, what is going to happen in the Solow growth model? The break-even investment that equation that we have seen earlier, now that is going to change slightly. Now, let us first try to understand it intuitively, and then we will get back to the equation that we see here. So, earlier, we said that investment had two components.

One is depreciation. Why depreciation again? Because physical capital is subject to wear and tear. So, to maintain its level of performance, etc, there is depreciation, and depreciation is also part of investment. The second component that we had seen earlier was the addition of new capital stock. This is what we had seen earlier. So, I will just write it down earlier.

Now, think about it. Now population has gone up and worker population we are talking about because everything that we have done so far is in terms of per worker. So, let us now suppose that worker population has gone up. Now is there going to be any change here? Yes, there is going to be one change here that we will see, which is showing up in this particular equation, and what is that change? New workers have to be equipped with capital. Otherwise, they are going to be unproductive.

So, think about it like this. Let us say that earlier there were 5 workers in the economy and all 5 of them required a computer each in order to be productive, in order to produce output. Now, let us suppose that the worker population has gone up. So, now instead of 5 workers, 2 new workers have come. So, now there are 7 workers. Now these two new workers will not be productive unless and until they are given to computers.

So, we now add one more step to what we had seen earlier, we have to also equip new workers with capital. And this is the new addition, a new additional feature. So, what are we then talking about? which earlier had two components; now, investment has actually three components. So, think about it: suppose you are an entrepreneur; this is how you might be thinking about it. First I have to maintain my existing capital, it is you know important, it is mandatory to do that, otherwise my existing capital you know will be rotting away very soon.

So, that is why I have to spend on the upkeep of that and that is known as depreciation. In the process, new workers have also joined. Now, I cannot make them sit idle; in that case, they are completely not productive, and they are not helping in the production process at all. So, next what I am going to do after taking care of depreciation, I am going to equip these new workers with capital, otherwise they are not going to be productive. Now, after taking care of these two things, whatever is left that is going to be addition to new capital stock.

So, I have taken care of my old capital stock, I have bought may be two new computers for the two new workers. Now, I am going to add to my existing capital stock and expand, right? So, if we combine all these three things together that is what investment is all about now. Earlier we did not have growth in population or growth in worker population. That is why this part was not there in the earlier version.

What does that lead to? That leads to the change in the break-even investment equation that we had seen earlier. So, earlier recall, there was only $\delta \cdot k$. Now, there is $(\delta + n)\cdot k$. where n is the population growth; this is small n, call it population or worker population growth that is the meaning of small n. So, now the break-even investment is delta plus n k, no longer Δk itself. How does our equation of motion for capital, how does that change? So, earlier, we had Δk equal to s.f(k).

So, in the earlier version, this is what we had. We had $\Delta k = s\cdot f(k) - \delta k$, earlier version as in when there was no growth in worker population. Now, because there is growth in worker population, this is the new change that we have. This term has now come in, and in the previous slide, we explained why this new term has come in. So, this is known as break even investment and as before then what is going to be our steady state capital stock.

So, we can see that even from the diagram, we can also solve it from here as before we said $\Delta k = 0$ and if we do that. So, at a steady state, what happens? Since $\Delta k = 0$, this implies that the left-hand side (LHS) of the equation is equal to 0. Therefore, we have: $s\cdot f(k) = (\delta + n)\cdot k$. So, pretty much same as before, but earlier it was $s\cdot f(k) = \delta\cdot k$, now it is $s\cdot f(k) = (\delta + n)\cdot k$ that is the new feature that we have here now. And we have the diagram looks fairly similar except that now there is a new component which is this small n and what does small n represent? Once again, it represents the percentage increase in population. So, there is population or worker population growth that is taking place here.

Every other dynamics remains the same, we can convince ourselves again about that how let us suppose we start at a point k_1 where $k_1 < k^*$. If we are starting from a point like that then we are here. So, this is our Δk, then we add this Δk to k_1, and we arrive at k_2. Again at k_2, we have a certain $\Delta k'$, and add that to k_2, and we arrive at k_3, and so on and so forth.

We reach this point. So, the same procedure happens in an economy. So, one important thing that we can observe here, although at every step we are adding that Δk, but I hope it is already visible to us. This magnitude of Δk, you see this much is a magnitude here or may be it will be clearer here. Suppose we are starting from a point like, let us say, k_0, you see this Δk is this much. I am talking of the magnitude here, the difference that we can see here.

You see how this Δk is reducing. So, the closer we get to the steady state, this Δk is reducing, and that makes sense because, ultimately, here at this point, Δk is equal to 0. There is also another, you know, subtle, you can say you know the thing that we can draw from here. If a country is very, very far away from the steady state, way below the steady state say something like k_0. Then its initial increase in capital stock, that is Δk is going to be larger compared to a country, let us say, which is at k_3, assuming that the countries have all other parameters the same, we will talk more about this later, not right now. But let us assume that we can have two countries on the same scale.

Then what we observe is that the country which is far below the steady state, it will initially its Δk is going to be very high. So, imagine if you add this much of Δk to k_0, just take this portion here and plot it here. It reaches this point, whereas the Δk here is quite small compared to the Δk here, you see that right. So, if a smaller portion of Δk is added here naturally, yes, everything is inching forward towards k^*, but the pace the rate is going to be different. So, these are some of the finer points about the Solow growth model, and whenever we are studying these points, we always put it in the context of an economy.

So, think of an economy that is way below its steady state versus an economy that is very close to its steady state; both will have Δk greater than 0. But that magnitude of Δk is going to be different, and that will have an implication on how fast or how slow an economy reaches the steady state. So, you can just play around with these things. So, one more exercise for you. I hope that after all these explanations, you should be able to figure it out yourself.

So, what is the impact of population growth? Again, we are using population growth and worker population growth interchangeably, but it is n that is changing here. So, what happens here, if let us say that there is an increase in it, worker population has gone up, then what it does, this leads to a figure like this, where the green line that we see is the new line. Please try and draw this, because as I have mentioned earlier also for to understand the Solow growth model, it helps a lot if we keep drawing these figures repeatedly. So, if population growth has occurred here, so from n1, it is now n2, and n2 is greater than n1, so more of population growth. What has happened is now, the economy has now a new steady state like before that we had seen when there was you know increase in savings rate.

But now the new steady state is lower compared to the previous steady state. I hope you can figure it out and again the dynamics and everything of this model is going to remain unchanged. What is important is this prediction of the Solow model. Higher n is going to lower k^*, lower k^*.

So, that will lead to lower y star. So, the Solow model predicts that countries with higher population growth rates will have lower levels of capital and income per worker in the long run. So, in countries where the population keeps on rising very fast, their long-run income per worker terms is going to be lower. Is that what we see? Well, this is the empirical evidence like before. We have population growth on the x-axis, and we can see that this is over a good 43 years, to be precise, and on the y-axis, we have per capita income. These are the plots, the scatter plots of different countries, and what do we see? More or less.

If you have the data, you can run a neat regression and see for yourself, but it is going to be a negatively sloped line. And you can again see why the Solow growth model was so popular at that time. And we can also perhaps relate this with what we see around us, countries with a higher rate of population growth they have lower levels of per capita income compared to countries where population growth is lower. So, very quick recap of what we have done so far, this is pretty much the benchmark Solow model. So, a very quick recap: as we had you know, we took a look at that excerpt from what Robert Solow had said that is contrary to the Harrod-Domar model.

The long-run growth here does not stem from capital accumulation or investment, and this movement towards the steady state that we saw, where Δk kept on increasing in the sense Δk is greater than 0. k kept on increasing because we were adding Δk to k. It kept moving towards the steady state. Now, that converges to a steady state rather than growing forever because of diminishing returns to production.

This is a very important point. Why? Because this whole procedure could take place because of the very shape of the production function, which we had discussed earlier. That is why we said the production function for the Solow growth model is very very important because if we change the production function, the implications are also going to change. So, this dynamics that we saw in this model in the process, you know, where y kept on increasing, that is entirely due to the property of the production function that exhibits diminishing returns. So, this is a very important feature, and if we are up to this point, we will now move on to what is known as the augmented Solow growth model.