Introduction to Economic Growth- I

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Lecture-12

So, what we are talking about here is how to determine the steady state capital stock okay. So, again a quick recap what is the steady state capital stock and how do we find that. So, for that we need to make use of this equation that we see here right inside this box. So, I will repeat that once again because this is one of the most important equations that is required in the Solow growth model. So, here Δk, this delta is for change okay. So, this Δk should be equal to 0 at steady state.

Why is that so? As we had said that you know if we are thinking of steady state as an equilibrium, just like in the equilibrium there is no change in that variable. Of course, unless and until there are you know some exogenous change that is happening, but by itself if the equilibrium is stable. Then that is not going to change. So, very similar concept we use here as well.

And although if you are curious as to why we do not use the term equilibrium but rather we use the term steady state, in some time we will see why we call it a steady state. But I can give you a hint here right away that in the term itself steady state, something is remaining steady. So you can start thinking about it while we proceed. So, at steady state this Δk that is this change in k is equal to 0. In other words what it means is if the steady state has been reached then there is no requirement for k to change.

So, physical capital stock will neither go up, it will neither go down. It is going to remain unchanged. Now, if this Δk is equal to 0, as we can see from this equation, then how can we solve for k* or k^*_{ss}? Sometimes, we also call it or denote it by k_{ss}.

Yes, we can now solve for it because now the left-hand side is equal to 0, right. So, what we have is $s \times f(k) = \delta \times k$.

How are we going to solve that? So, δ that is the depreciation rate that is exogenous that value is given to us. Small s that is the saving rate that is also exogenous that value is also given to us. $f(k)$ that is the production function that has been converted to per capita terms and we have k, right. So, now there is one unknown and one equation. So, we should be able to solve for k*.

So, I hope if a problem is given based on this, then as long as we know this particular equation, we should be able to solve for it. How does this appear like graphically? This is important because in most of the analysis that we will be doing, this graph is going to help us a lot. So, what do we see in this particular graph? Again, on the x-axis, we measure per worker capital or small k. On the y-axis, we have investment and depreciation. This $s \times f(k)$, we have seen this before, and there is one $f(k)$ which is up here somewhere.

This is not drawn here. But it is up there somewhere, right? $\delta \times k$ we have seen this before as well. So, as we noted from the previous slide that when $\Delta k = 0$, what it implies is this, right? And what is that particular point at least in this particular graph? That is right here, okay, and this is denoted as this k*, or as I said, sometimes you know, we also call it the k_{ss} just to be very clear that this is the steady state. Now, as we can see from here, if we have found out the value of k* — steady state capital stock, we just have to extend this up here, and what we find the corresponding value here is going to be our y^*_{ss}. So, very neat and simple figure.

So, graphically that is why we rely a lot for the Solow growth model because very quickly we can see a few things, right. So, this is how we represent this equation that we saw here and at steady state as we said this $\Delta k = 0$. So, $s \times f(k) = \delta \times k$. Now, this is how it looks graphically. Let us also try to understand what is the intuitive meaning of $\Delta k = s \times f(k)$.

What it signifies is that the economy has reached a certain level of per capita or per worker capital stock, where the investment is just enough to cover depreciation because remember $s \times f(k)$ is an investment and what is $\delta \times k$ — that is the total depreciation. So, in other words if the only investment that is occurring in an economy at k* is to cover depreciation. It means that nothing is being added to new physical capital stock. So, if an economy reaches a stage like this, then it is said that the economy has reached the steady state capital and correspondingly, as we saw from here, if we know the value or if we have found out the value of this k*, then correspondingly we can also find out the value of y*. Are we good to go? So, maybe now we would be asking ourselves a question that I understand how this model has been set up and a quick recap as we had mentioned earlier.

How did we set up the model? There is a supply side of the model represented by the production function. There is a demand side of the model that comes from the national income identity. And then we brought them together and this is the representation that we have. In the meantime, we also defined what is investment, right? And when we brought them together—that is, demand and supply together—this is where we have arrived at. So, this is what we have done so far, and what we have managed to find out is that we have also solved this k* here, right? So, up to here, we should be all good.

But naturally, now we will be asking ourselves a question. We said that the Solow growth model is a growth model. So, where is this growth coming from? Because right now, what we see in front of us is the k* that we have solved for. So, we can solve for that either graphically or we can use this particular equation here as we mentioned earlier.

And if all values are given to us and if we know the production function, then we should be able to solve for k* numerically as well. So, either way, we have solved for k*. And since we know the production function, we should also be able to solve for y*. But then comes the question: where is growth occurring in this process? We do not see any growth occurring here, at least in this particular figure. So, to answer that question, let us look at it from a slightly different direction.

Now, just as an equilibrium analysis, just think of any market that we have covered in microeconomics. So, we just have a demand curve, we have a supply curve, we know where the equilibrium is—but what if the economy is currently not at equilibrium? So, the same question we ask ourselves here. Let us suppose to begin with, the economy is not here. We know that this is our k*ss, just to be double sure that this is the steady state. But let us suppose, and this is perhaps the most common case, that the economy is here at this point. If the economy is at, so what does this imply? This implies that the economy is at a certain level of k that is less than k*ss.

So, we are below the steady state. Let us suppose this is the situation, and this is taken to be a very common situation. Then the question is: what happens to this particular economy? In other words, we ask ourselves the question that the economy is currently here, and we know that this is the steady state for the economy—how does the economy move towards this particular steady state? Are we clear about that? In the process, I will also give you the answer right away—it is in the process we will see how growth occurs. So, this is the explanation that was given by Solow. Let us see that when an economy is to begin with at a point like k1, where k1 < k*, then how does an economy move from k1 to k*? Is this clear to all of us? Then we can proceed from here.

So, let us take a look at this point k1, which is below k*. Now, what happens at this particular point? When an economy is at k1, we can extend this up to here. And what is this measuring? This is measuring the total investment in the economy. How do we know that? Because this is touching this particular curve which is represented by s · f(k), which is nothing but our investment.

So, in other words, we know that at the level k1, what is the total investment that is being made in an economy? Now, out of this investment amount that we see here, a part of this, this part actually is for depreciation. How do we know that? Because remember, investment is depreciation plus change in capital stock, both are components of investment. So, we know this, we have this, we have this one already, we also know the amount of how do we know the amount of depreciation? That is because this is the line, it is touching the line here. So, this is Δk, this is our depreciation. So, then what are we left with? Then this is the only thing.

Then what are we left with? Then this is the only thing that we do not know. But right now, we have the values of investment, and we know how much the depreciation is. So, we know this entire thing, out of which we know this amount is going for depreciation. So, what are we left with? We are left with this. So maybe it helps to label. So, this segment AB is what remains, and this AB is representing the change in capital stock.

Now, in the next step let us ask ourselves a question. So, in other words the capital stock has changed, right. So, it has increased typically in this particular situation. Why? Because we can see that this Δk. This is greater than zero, right? We see this is positive.

So, what it means is that more capital stock has been added to the economy. So, what is the new capital stock? So, a new capital stock should be the existing capital stock, right? And what is the existing capital stock? That is this amount because we started from here, plus this Δk. So, in this process there has been more capital that has been added to the economy. So, in other words, we have this k1 + Δk now. So, what we have is equal to I will just write k1 because here we are representing by k + Δk.

So, where are we moving then in terms of this particular figure? So, to this k1 if we add, suppose we take this Δk, we bring it down here, we reach a point k2 now. Because this new point is the previous capital stock, and to that, new capital stock has been added. So, in the process then what has happened? In the process the economy has moved from k1 to k2. And if you remember, it is not here in the figure here, but if you want, we can add that here. We also have a production function, remember? Maybe it is just not here.

Otherwise, this particular diagram becomes too clumsy, and too many things will be here. But when we are drawing it ourselves, we can always draw that. So, here there was earlier a level of output. y1. Now, we have k2 and that corresponds to a higher level of output here.

But still, see, we have not reached a steady state. So, these are not steady state numbers. This is just the dynamics in the system. What we mean by that? By that we mean that if the economy has started from a point that is below the steady state, then how is it that the economy will move towards the steady state? So, we are looking at a particular process that is happening here. What is going to happen at the next step after the economy has reached k2? Well, still k2 is less than k*.

So, it is still at a point which is below the steady state. So, what we had said or written about k here and Δk, the same thing applies here as well. Is moving to a neater diagram here. So, this is our k2 now. What happens after this? Now, we can do that exercise once again.

From this k2, we can find out what is the total investment. This is the total investment. We can maybe just label it. So, Ak2 is the total investment, out of which how much is the depreciation? This much is the depreciation. So, we know the total investment, we know the depreciation amount.

And then what are we left with? We are left with this. So, let us label this, let us call this a b, this is r mu Δk, right? And what happens to this new Δk? So, if we add this new Δk, in other words what it implies is in the economy more physical capital stock gets added. And how do we represent that graphically here? So, this new Δk, if you just bring it down here and add it to the existing k. So, now we have k2 plus Δk, we will call it nu because for the earlier one also, we had called it Δk. It is still Δk because it is a change, but another one and that helps us to arrive at another point here, which we are calling it.

k3. What has happened in the process? The economy has moved from k1 to k2, then from to k3, and like this again at k3, you can repeat this exercise; you can find out the Δk; you can add that here, and you will find the k4. So, gradually, if we keep on doing this exercise repeatedly, there will come a point where we will reach this, and here we see that Δk is equal to zero. So, no new addition to capital stock is going to take place. The only investment here, do we see the only investment that takes place here is this amount? And this whole amount is also equal to depreciation. So, once the economy has reached the steady state level of capital stock, the entire investment that occurs is just to maintain the depreciation of capital; no new capital stock is added.

So, this is a process that occurs in an economy over a very long period of time. These changes do not occur overnight, not even if it is a matter of years, but as we can see, it is a cumulative process. And at every step, what has happened, again, this is not drawn here, but we can always do this. If we would like to convince ourselves, there is already a production function up there because, without that production function, we cannot arrive at this $s \times f(k)$. So, if we want for k1, there will be a corresponding y1, and for k2, there will be another corresponding For k3, there will be another corresponding y3 such that y3 is greater than y2 is greater than y1.

And this process continues until one reaches k*for which there is a corresponding y*. So, what has happened in the process? I hope now the story is playing out. You see here y1 to y2. y2 to y3. So, this is how growth happens or growth occurs in an economy as an economy moves towards the steady state.

So, this is the summary of what we have seen thus far. So, as long as K is less than k*, investment will exceed depreciation as a result of which Δk will always be positive. And as long as Δk is positive, so we are adding new capital stock. So, think about it very intuitively. Since production function, production is a function of capital stock, the more capital we add, we are going to have higher output, right.

So, Δk greater than 0 implies we are adding to our capital stock. And what is happening in the process? Because we are adding to our capital stock, output is also growing. Does this continue infinitely? No, it does not continue infinitely, it continues until this point has been reached. Why? Because at the steady state Δk has become equal to 0, right. Now, let me ask you a question here and let us see whether we have an answer to that.

So, my question here is then If an economy has reached this point k*, which is the steady-state level of capital, then is it implying that the economy has stopped growing in terms of GDP? So, just listen to this question carefully and ponder about it or think about it in the context that we have set up here. So, the question is once an economy has reached k*, so by now we understand the process right, we should be very clear about the process right. So, suppose now an economy has moved from k1 to k2, k2 to k3, k3 to k4, it has done all that you know movement and it has reached its steady state capital stock k*okay. The question being once an economy has reached this point k*, does it imply that output is not growing in the economy? Why are we asking ourselves the question? Because we said at a point like k*, Δk is equal to 0, right? So, in other words, if there is some, again, the production function. If we were to draw it here if we were to extend it up to here, then there is going to be some y* here, right? And as we said that once the economy has reached this steady state point since Δk is equal to 0, so the economy is not going to move away from that point unless and until there are exogenous factors.

So, does this mean that when an economy has reached the point k^*, which has a corresponding y^* here, and we said this k^* is not changing. Then that implies y^* is also not changing. Then does that imply that this economy is not growing in terms of income? That is the question here. Think about it. If we can answer that question, we will also know why this is known as the steady state.

If we are absolutely clear about what we have covered thus far, then comes a very interesting exercise that we can now start doing with the Solow growth model. This is also a part of the policy analysis. So, you know maybe we will just repeat it once during the policy analysis, but one can also think of it as a policy analysis. Now, let us suppose that in this economy, so now we are looking at a scenario. Remember here, we said that if an economy has reached this point of $k^*$and there is a corresponding y^*, then the economy is pretty much going to stay there.

But can there be new steady states? That is the question. Well, there can be new steady states. How? So, let us suppose that in this particular economy that we are talking about, the saving rate has gone up, it is quite possible, right? So, let us say s_1 earlier was, let us say, 0.3, which is 30 percent. Now, maybe s_2 new saving rate people have started saving more as a result of which the savings rate has gone up.

Everything else remains unchanged. That is, the production function is not changing, our delta value, and nothing else is changing. This, in our terms, we generally call it what? We call it Ceteris Paribus. What it implies is except for this change in saving rate nothing else is changing. Now, the question is, what is the impact of this increase in saving rate in this particular economy? What changes? So, this is the question that we are asking ourselves now.