Introduction to Economic Growth- I

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Lecture-11

So, after talking about the supply side of the economy, which is represented by the production function, well, that is all about the supply side. You see how simple it is? It is not complicated at all. And for the supply side, we just have the production function. In case you have forgotten about the nitty-gritty of the production function, like marginal productivity, and what is diminishing marginal productivity, then please take a look at the textbook. So, we are done with the supply side, now we move on to the demand side of our economy. So, remember, on the demand side, we are asking ourselves a question: where, well, all the output that has been produced has to go somewhere, right, and where is that somewhere? Then there are two sectors where it is going, and this again comes from basic macroeconomics that we are already familiar with.

The output either it goes for consumption demand or investment demand. Now, generally, we also add G and net exports NX in the regular macro course, but the Solow growth model is at least a benchmark model. It is a model that does not have any government. So that is why $G = 0$ here, no G.

Also, this is a closed economy model, the benchmark model. Now, one can keep on adding all these features to the Solow growth model and make it more interesting, and answer more questions regarding growth, etc. But in the benchmark Solow growth model, It is what we call a closed economy model, that is there is no trade involved here. Well, that makes our life simpler, is not it? Because $Y=C+I$. But just like we had converted the supply side into per-worker terms.

Here also similarly, we have to convert Y, C, and I into per-worker terms ($y=c+i$)in a similar manner. And next, what we are going to do is we are going to talk about each of these terms separately, that is, consumption and investment. We begin with the consumption function, and here we define another notation here which is small s, which is the saving rate. This is the fraction of income that is saved and in the Solow growth model, this is a very important assumption; this small s is an exogenous variable. What it means is that? We are not going to determine any optimal S or S star from within the model.

So, this is like saying that the savings rate of an economy is say 30 percent. So, S is 0.3. This value is already given to us. That is why it is exogenous in this particular model and this is a very important assumption because there are growth models.

Where this S itself might be endogenous, that is, S is a function of something else, and the model determines the optimal value of S, but in the benchmark Solow model, S is exogenous. And the next thing that we define here is the consumption function and how do we define that? Remember, everything is in per worker terms. That is why we see small c, small y, etc. So, c is equal to actually income minus the portion of income that has been saved. So,

$$c=(1-s)\cdot y$$

So, I suppose we are also familiar with the consumption function because we have encountered this before in macroeconomics as well okay. The next thing that we are doing, so we are done with consumption, and now we have to define investment and then the next step is very simple: we just have to bring them together. So, saving per worker that is defined as y-c. c we have from the consumption function, we just plug it in there from the consumption function, what we are left with is s·Y, S is this small s here. Now from national income identity, we know that y = c+i because it is a closed economy model.

So, there is no G and no imports and exports. We can arrange these terms here and while I am saying this here and while we are seeing this on the screen here, I would very strongly urge you that keep writing. So, we have to write down these equations so that we know where exactly they come from. So we just have to y = c+i. So we end up with what? We end up with i = y-c.

And because saving is equal to investment, remember it is a closed economy. So what we end up having is this last one. $i=s\cdot f(k)$. This is a very important equation in the Solow growth model. So, we are kind of all set now.

So, these are the elements that we have covered so far. We have covered about the supply side of the Solow growth model came from the production function, demand side of the Solow growth model it came from the national income identity, only consumption and investment. And then what we end up having here is $i=s\cdot f(k)$. Now, how does it look like in terms of a graph? Now, this graph is very important. I would encourage you to please draw this graph every time you are studying this, so that you know we can then play around with it.

Now, let us see whatever we have studied thus far, how can we represent it graphically? The production function is something we have seen before; this is represented by this, you know, the curve, which has been labeled as f(k). So, that is the production function. And from here, because s is exogenous, remember, the value of s is given to us like, say, S is 30 percent, so 0.3. So, we have f of k.

If we can plot that, then we can also plot the curve $s \cdot f(k)$; is that right? That is exactly what has been done here, the second line that we see here. So, maybe I will mark it. So, this is our production function. This is the $s \cdot f(k)$ curve that we have drawn. What can we infer from here? This is very important.

Now, suppose this is the starting level of k because on the x-axis, what is it that we are measuring? We are measuring capital per worker. So, let us say an economy currently has this level of capital per worker. Now, if this much information is given to us, and we know its production function, and if we know its s, then this is given. and production function. We have drawn the production function, and s is given to us.

So, we can also draw $s \cdot f(k)$ and this is let us say you know this is also given to us, some value, some starting value. Now, given all these variables, let us say the values of all these variables; now, the next question is, can we determine the question we are asking ourselves here? Can we determine y, c, and i corresponding to k1? That is the question. And we can do that because we will solve this also in form of an equation, but we can also do this graphically. How? I think it is simple enough. So, we start from here because this k1 is already given to us.

We have drawn this line of f (k) already. We have also drawn $s \cdot f(k)$. So, the next thing that we have to do is we project this k1 onto the production function. And what do we get? We get output. Why? Because output is a function of k and this implies if we know the value of k, which is k1, therefore we get y1 that corresponds to k1.

Are we good so far? So, graphically, you see, if we know what is the k for an economy, that is, the per worker capital in an economy, and if we know the production function how it looks like, and if we know what is the savings rate actually even without the savings rate actually we can always know what is the output. So, the main thing to understand here is that this output is a function of per-worker capital. So, one thing we have already solved for. Now, this output that has been produced, as we know, gets distributed as consumption and as investment. So, next thing we should also be able to determine that.

How can we determine that? Well, we have already also drawn this particular curve s of f(k). Now, the same k1 if we project it here, remember this will give us the amount of investment. Why? Because this k1, when plugged in here this becomes, this will give us the amount of investment. And then how do we know how much is the amount of consumption? This remaining part, why? Because this is y1-i1 which is c1. So, the first step is to find out what the output corresponds to the given level of k1.

After finding that, the next thing that we found out is this. This is step 2 and since we now have y1 value and i1 value, the remaining is this which is c1. So, then, the question that we asked here is just written right here: Can we determine if the values of y, c, and i correspond to a starting level or any given level of k? The answer is well graphically we have found out the values for all of them. So, just as I had drawn at least partially the figure was already there. But you know I at least try to draw a little bit of it.

I would again very strongly encourage you to please keep drawing this diagram so that we are very clear in our head as to how we are arriving at these different values. So, that was the first very basic representation of the supply side and the demand side of the Solow growth model. But remember, we said there is one final step that is to be done. We said there is a demand side, and there is a supply side. But how do we know it is a demand and the supply of the same entity that we are talking about? We have to bring them together.

Only when we bring it together that is when we have entirely defined our economy. So, that one step now remains. In order to do that we need another extra element and we introduce that here which is this notion of depreciation. Why is this notion of depreciation important? The notion of depreciation is important because we are talking of physical capital here. Physical capital is subject to wear and tear and in order to maintain one can say the quality or the performance of that physical capital, what one has to do is one has to like servicing.

Now, the car is not really a physical capital per se; it is not an asset in that sense, but I would just give an example so that we know the context in which we are using the terms depreciation, etc. So, think about it like this: suppose you have a car. Now, when the car is newly purchased, we know that its performance is fantastic, everything is pick and span, performing very well. But generally, no matter which car we have, once every 6 months, servicing is required. We generally say that I have sent the car to the garage for servicing.

What is this servicing? This servicing is because if a car is being used on a regular basis, there will be some wear and tear. So, if we want the performance of the car to remain intact or not to suffer too much, we also have to make sure that we go in for regular servicing. Now think about it like this also in an accounting sense: the money that we spend on servicing as per this framework that we are talking about is actually a part of investment. So, it is in this context that we are bringing in depreciation here. Because as we saw in the last slide that a lot depends on this capital term, capital per worker, and this is physical capital, so like huge machinery, etc.

Now, things like that are subject to wear and tear, which is known as depreciation. So, delta is represented as the rate of depreciation, and how does that look like? If we are graphing that then it is a straight line passing through the origin. Why is it a straight line passing through the origin? Because if there is no capital, there is no depreciation of that.

If there is no car, we are not sending anything for servicing. So, that is why it passes through the origin.

And it is a positively sloped line that we have here because higher the amount of k, depreciation is also higher. So, the amount that you spend for servicing one car, imagine you have a fleet of cars, and each car requires servicing. So, naturally, your delta k is going to be higher. So, that explains why this line is positively sloped and it also passes through the origin. So, the basic idea being, remember, we are trying to now bring the demand and the supply sides together, the basic idea being that investment when we invest when an economy invests or even at the individual level when we invest, we increase the capital stock.

But at the same time there is depreciation of capital as well and that reduces it. And that is represented by this equation that we see here, which is the change in capital stock. Now the change term is denoted by delta here. I would also quickly like to remind you that I am following the terms and all the notations that is from Mankiw. So, just to keep parity with the chapter, the same notations have been used here.

So, delta generally indicates change. So, here delta K is change in capital stock which is equal to investment minus depreciation. So, this is how or the other way round, now think about it, I or investment is a change in capital stock, that is the amount that we have added as new capital plus depreciation, why? you know that servicing example once again, the amount that we spent to send the car for servicing that is not really consumption, but that is also investment because that is required to maintain the quality or the performance of the car. So, looking at it the other way around, investment is equal to a change in capital stock plus depreciation. So, we can write this differently also. So, I will just write it here because this is how we also define investment.

So, change in capital stock plus delta k, this is important. Now, if there is no depreciation, let us suppose if for some reason there is no depreciation. then delta is equal to 0. Let us say you have a supercar; it does not depreciate, it does not require servicing at all. Well, then there is no depreciation at all, then your investment is just this change in capital stock.

But since most of the physical capital is subject to wear and tear, there is depreciation this is what we or this is how we actually define investment. Now recall we have already seen before that $i = s.f(k)$; remember from the previous slides, we have seen that this is nothing but i or investment, is equal to savings, small s is the saving rate, and f(k) is y. So, that is the total savings. So, if we bring that back in here, then this equation. We plug this in here, and what we find here

$$\Delta k = s \cdot f(k) - \delta k$$

So, this is what we finally arrive at, and this is a very important equation we are looking at because this is what we call central to the Solow growth model. If we have this much information, now we are fairly good to go both in terms of graphs and also if we are given numbers, we can also now solve for a lot of variables. So, this is known as the equation of motion for capital. And what is that? As we said, $\Delta k = s \cdot f(k) - \delta k$. So, in very simple terms, the change in capital stock is equal to your investment minus the depreciation.

This is very central to the Solow growth model. What it does is that it determines the behavior of capital over time, and as we had seen in an earlier slide that this capital, in turn, is going to determine the values of y, value of c, and value of i. So, that is why understanding this motion of capital is very important because that is very crucial to our model. Once we understand how k functions, from there itself, we can determine the other endogenous variables. Why? Because, as we have seen, $y=f(k)$.

So, you see how many times k comes back here. So, that is why it is crucial. Also, c or consumption, this is also a function of ky because s is exogenous to the model. So, if the value of s is given to us, all that we require to determine c is again the value of k. All that we require to determine the value of y is, again, just the value of k. So, that is why it is written here that it is very crucial then to understand how k moves.

That is why it is written that this is central to the Solow growth model because k in turn, it determines the behavior of all other endogenous variables. So, before we move further, this thing we have to remember. It is important you remember this equation of motion for capital because this is the equation on which the entire Solow growth model stands. So before we move on to the notion of steady state, I think before we do that, let us just quickly see how we can represent whatever we have discussed thus far.

So we have already discussed this before which is s.f(k). Remember there is some f(k) out there which is not drawn here repeatedly because for this motion of capital all we need is s.f(k). But remember somewhere up there f(k) is also there and we had also talked about this depreciation. So, delta k is the depreciation of capital. Now, these are the two elements that are very crucial as we mentioned that they are central.

So, we have plotted this so you can plot this graphically. We also know how to plot this graphically. So, two elements we already know how to plot them, and the next thing that remains is to bring them together. Now, how do we bring them together and what is the interpretation of bringing them together? Now, as we have seen while we were doing IS-LM or AD-AS or even in case of a simple demand-supply, what do we do? We bring the demand side, and we bring the supply side together, and we define it as an equilibrium. Now, we said that we do the same thing here, but instead of using the term equilibrium, we use this term steady state that you see here. In some time, we will look into this: why is it known as a steady state? Right now let us just stick to this term and move ahead a little bit.

what happens at steady state. So, we can think of a steady state, as we said, like an equilibrium. What happens in an equilibrium? So, recall any demand-supply figure that we had drawn in microeconomics; demand and supply cross each other; that point is the equilibrium. Similarly, here, we define the steady state as that level of investment that is just enough to cover depreciation. That is a point where $\Delta k=0$. That is there is no need for k to move either to the left or to the right.

So, delta k is what? It is the change in capital stock. So, a point where there is no change in capital stock, delta k is changed. So, if that $\Delta k=0$, it implies that no change is being made. No change means nothing is being added, nothing is being taken away. If we have reached a point like that where $\Delta k=0$, then that is defined as the steady state. And the k that corresponds to this steady state is generally denoted by k^*.

Sometimes, we also, or I at least, write as k_{ss} to denote a steady state. So, I think before we proceed any further, we can at least go through the previous slides and we can just take stock of what we have done. We should make sure that we have understood up to this point because this is all that we have in the benchmark Solow model. But if we have understood up to this point, then we can actually start exploring the model even more from here.

So, I would request you to pause for some time. If required, just go back to the previous slides. See how we reached here. This is very important. We had to reach up to this equation. So, just see what all ingredients that we had used or required so far to reach up to this point.

And finally, where we have reached is we have brought the demand and the supply side of the economy together. And we have now arrived at the steady state capital stock and this is where I think we can just pause for a while here before we move on to the graphical representation.