Applied Statistics and Econometrics
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Lecture-22
Index Numbers (Part-II)

Hello, friends. Welcome back to the lecture series on Applied Statistics and Econometrics. So today, we are going to finish our discussion on price index numbers. And before we begin, let us have a look at today's agenda items.

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So, we will start today's lecture with a brief recap of what we have learned in the last lecture, for the continuity. And then I am going to show you how we can use GDP deflator to deflate a nominal time series, then we are going to study some important composite index numbers, namely Laspeyres, Paasche, and Fisher. And finally, we are going to end today's discussion with an idea of cost of living index and its uses.

So, from the previous lecture, I hope you have understood that if we are interested to compare economic activities or business outcomes, which are expressed in monetary units over a period of time, then we should make use of price index, so that we can compare the monetary values coming from different time periods.

Why we require these price index numbers? Because they help us to take care of the inflationary pressure. So, if you remember the discussion that we had on the value index, so value index is basically a composite index of price index and a quantity index. So, with time, the value of a particular commodity being consumed or produced is changing. And there could be two sources of changes of that value change and one is coming from the price and the other one is coming from the quantity.

So, if you were interested only in the physical or the real economic outcome and growth in that, then you have to take care of the inflation factor, so that is basically how the price is going to change over time. So you have to take care of the rate of price increase. And that is what we are going to do with help of price index numbers.

So, in last lecture, we have discussed some concepts like price relatives and other type of index numbers. So, it is not a bad idea if we go back to that old discussion and have a bit of discussion on price relatives and then, we will start with new topics.

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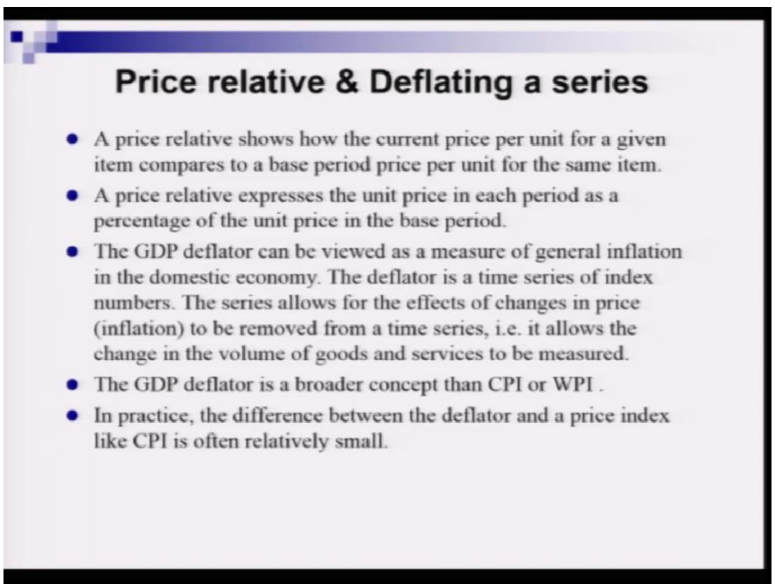

So, price relative is basically the basic concept behind price index numbers. What is it? So a price relative actually shows how the current price per unit of a commodity compares to a base period price per unit of that same commodity, okay? So a price relative actually expressing a unit price in each period as a percentage of the unit price in the base period, so that is basically the fundamental of a price index number.

Now, let us have a look at one of the kind of price index numbers and that is called GDP deflator. We have gone through several mathematical formula last time, but we have not studied any particular application of these price index numbers. So, I think, it is better if we now discuss about an example at this moment. And GDP deflator is a very well-known price index number. It is different from the other well known price index numbers like consumer price index number or CPI, or wholesale price index number or WPI.

Now, why we need a separate price index number called GDP deflator, when we already have something very no commonly used and very popular, say CPI, consumer price index number? Well, if you remember the last lecture, I have told you that when CPI is constructed, it actually takes care of a defined basket, a consumption basket and in that consumption basket, there are 100 plus items. But still, it is not inclusive of all the items being produced in the economy.

And there are many items like your real estate, they are actually probably not part of the CPI calculation because CPI actually talks about direct consumption items like your cereals, beverages, say milk, water, etc., right? So, when you want to have an overall picture of inflationary pressure or price rise in an economy, CPI may not be able to cover the entire economy, because it is constructed from a selected group of items. That is why we need something more general and GDP deflator actually does it?

So, a deflator is again a time series of index numbers. And that time series allows for the effects of changes in price which is basically inflation to be removed from a time series on some economic variable expressed in monetary units. So basically, it allows for the change in the volume of goods and services to be measured after taking care of the inflationary pressure. Although we are differentiating between CPI WPI and GDP deflator but in practice, if you ask me, I would say that there are very little difference between these price index numbers.

So, for your practical purposes, if you do not have access to GDP deflator, but if you want to deflate a country’s national income or GDP, then you can also make use of consumer price index numbers, the results are not going to vary that much. So now, we are going to look at a numerical example through which I am going to show you how one can make use of this concept deflator, it can be a GDP deflator or it can be a consumer price index number or a wholesale price index number. How we can convert time series given on a nominal economic variable to convert that economic variable in real terms.

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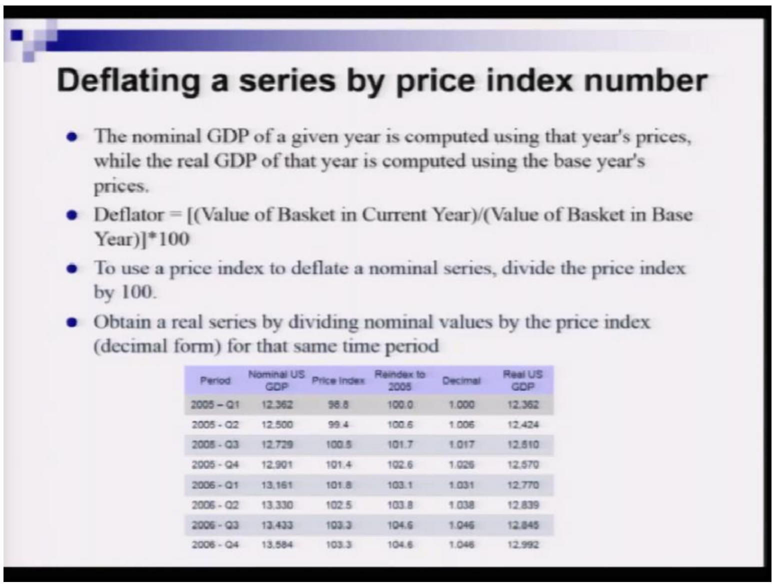

So, for that purpose, we are going to concentrate on quarterly GDP estimates from U. S. economy. So here, the U. S. GDP in a quarter is represented in billion U. S. dollars and we have data on total eight quarters from years 2005 and 2006. And let us see how we are going to make use of these deflator concept to express these nominal U. S. GDP to real U. S. GDP time series.

So here, please concentrate on the table that you see at the bottom of the slide. So here the first column gives you those eight quarters. So, these are my time periods. And second column gives me the nominal U. S. GDP, expressed in billion U. S. dollars. And the third column gives me the price index number value.

And note that, although we are interested to express the nominal U. S. GDP in the real U. S. GDP so to be more specific, I want to express my nominal U. S. GDP figures in 2005, quarter one price level of the U. S. economy. That is basically what I mean by real U. S. GDP. I have to make 2005, quarter one, the base period.

But I have already told you in the last lecture, if you remember that for the base period index number should take the value 100. But here you see it is not 100, it is somewhere close to 98.6 or so. So, it seems that the base here actually is somewhere else. So, what to do in this context? How to proceed? So here first, you have to change the base period, so you have to re-index the index number. So here, I am going to show you how you can re-index this price index numbers such that the base period is 2005 quarter one year, so for that time period you get the index value 100. It is very simple.

So, what you have to do? You have to divide this number 98.8 that you see in the first cell under this third column by the same number, so you re-index that number and you get the number 100. So now what you have to do? You have to divide all other numbers that you are seeing under the same column. So, you are basically now going to divide 99.4 by 98.8, 100.5 by 98.8 and you are going to generate new number. So, that new series is basically the new price index number where the base now has become 2005 quarter one.

So, you see now, in the fourth column, I am showing you all these numbers that are basically the newly construct index number and then, you have to remember that here these index numbers are all in 100 plus figures. So, when you want to use the price index number to defeat a nominal series, you always remember to divide the price index number by 100. So that you have to convert that large number in the decimal form; and the fifth column here in this table actually showing you that, so you divide these re-index numbers by 100 and you get the decimal values.

Once you have the decimal values, then you obtain a real series by dividing the nominal values of the economic variable by the price index number in decimal form. And that is basically is shown as the last column of this table. So here, you see I have actually divided the nominal U. S. GDP value say 12,500 by this decimal number 1.006 to get the real value of the U. S. GDP expressed in the price level of 2005 quarter one; and that the, and the value is 12,424.

So, if you now are interested to explore the connection between price index number, deflation and time series data, then, I request you to do one small thing at this moment, I am not doing because in this classroom environment, I have limited time, but you can do it very easily. So, I now request you to plot this nominal U. S. GDP figures, against the quarter, the time periods and then, you also plot similarly, the real GDP values that you obtained by the deflation measure. You will see that the real GDP values will always lie below the nominal U. S. GDP figure. So, only for the first time period they will be equal, but the real numbers are going to be actually less than the nominal U. S. GDP figures because there is positive inflation.

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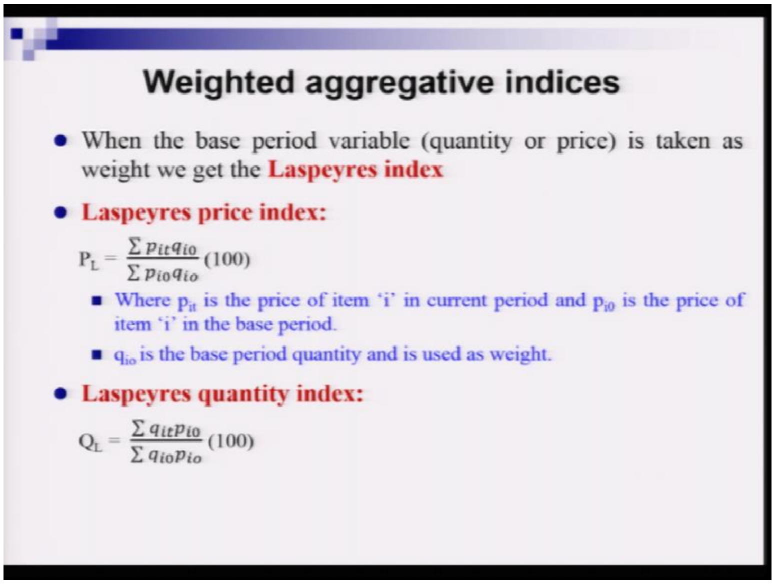

So, now, we are going to discuss a little bit on the weighted aggregative indices. Last time, I said that this is a kind of composite index, but last time in the class, we did not have enough time to now go through these indices. So, now we are going to have a look at them.

So here, there are two major types of weighted aggregative indices and they are developed by statisticians named Laspeyres and Paasche; and later on, we will see that there was another statistician Irving Fisher, who actually made use of these two developments by Laspeyres and Paasche to come up with another formula for index number and we are going to actually look at them individually one by one. So, first I will start with Laspeyres index in this slide. So, you remember you are always comparing the current value of an economic variable in the current period and you are comparing that number with the value of the economic variable in the base period.

Now, the variable could be either price or quantity. So, when you make use of the base period variable as the weight then you get the Laspeyres index. So, of course, Laspeyres index could be of two types: one for the price variable and one for the quantity variable, because ultimately, we are interested to see the change in the value and value actually changes from two different sources of change and that are, and they are price and quantity. So, we have to have separate indices for both the economic variables price and quantity of the commodity.

So here, we have the Laspeyres price index formula. Here you see, I have n number of commodities in my consumption basket and for all these n commodities, I know what is the price level for different time periods and the quantity consumed or purchased in different time periods and when I say that, I know the values of p and q in all time periods, I mean that from the base year to the current year, I have data.

So now, you can compare one particular time period against the base period. So here, you are using the base period variable as the weight. So, what we have to do here as we are calculating the price index number, we use the base period quantity consumption as the weight. So, you see in the formula pl that stands for Laspeyres price index number or some people also call it lpi. So here in the formula, in the numerator, you see there is a sum and in the denominator also there is a sum.

So, what does the numerator say? So here, you see this tells us the value of the consumption basket in the base period when it is evaluated at the price level that exists in the current period t. And how can we interpret the denominator? Well, this is also a value of a basket. Now, value of the basket is calculated by multiplying the quantity consumed of the i-th commodity in the base period with the price of that i-th commodity in the base period.

So here, the value that you get that actually talks about the value of the consumption basket of the base period when evaluated using the price of the base period itself. So, this ratio is now multiplied by 100 and you get your Laspeyres price index number. Now, let us move on to the concept of Laspeyres quantity index. So, what we are going to do? Here we are going to actually compare the quantity consumed or quantity produced in time period t with the quantity consumed or purchased or produced in the base period 0.

So here, we are going to make use of the base period price as the weighting factor. So, now, let us look at the Laspeyres quantity index formula in detail. So, that is denoted by ql or as this is for the quantity, some can also express Laspeyres quantity index with the abbreviation lqi.

So now, let us focus on the numerator of this formula. So here, you see, I am expressing the value of the quantity that is there in my basket and I am evaluating the value by using the price of the base period. So, I have and number of commodities, qit's are basically the quantities of i-th commodity that is there in the data set for the current time period t and I am multiplying each individual commodities quantity with the price of that particular commodity, but that exists in the base period, okay?

Now, let us concentrate on the denominator. So here, you see that I am going to talk about the value of the basket of items and here, I am actually evaluating the value of the basket. So here, I am going to multiply qi naught so, that is the quantity of i-th commodity that is there in the 0th time period by the price of that commodity in the 0th period.

So basically, I am now calculating the value of the basket for the commodities that were there in the base period and for calculating the value I am using the base period prices, okay. So once this ratio is computed you multiply the ratio with 100 and you get your Laspeyres quantity index.

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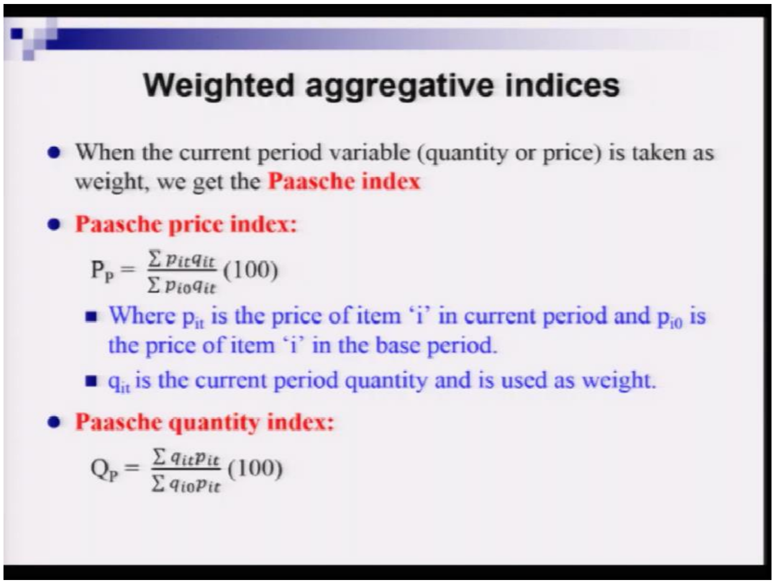

Now, we move on to the case of Paasche index and again, there could be two types of Paasche index; one for the price variable and one for the quantity variable. Let us have a look at them one by one. So, in the previous slide, if you remember we used the base period variable values as the weights. But here, if we use the current period variable values as the weight, then actually we get the Paasche index.

Now, at this moment, I should tell that there is no a priori preference whether you should go with base period variable values as the weight over the current period variable value. So, there could be possibilities. So, at this moment, I should tell you that the choice of the weighting factor is quite arbitrary in nature and that is why you see that Laspeyres and Paasche, these two statisticians they have chosen different variables or different time periods as the base. So, keeping that in mind we should say that, here we cannot compare which one is better whether it is Laspeyres or whether it is Paasche.

But two different formula exists. So here, in the case of Paasche index, we use the current period variable values as the weight. So, if we now focus on the Paasche price index, you see that here weights are qit in denominator and in the numerator and the explanation is going to be the same as I did in the last slide for Laspeyres. So, you can follow that same argument from the previous slide and you can actually get the interpretation of the numerator and the denominator.

Now, let us move to the case of Paasche quantity index and here, you see, I am going to make use of price as the weighting variable and I am going to use the current period values for that variable as weight because that is the assumption made by Paasche. So, here, you see, in the denominator and in the numerator you see pit is used as the weight. And explanation is similar, if you go back to the previous slide, then whatever I said that way you can explain the interpretation or the meaning of the numerator and the denominator.

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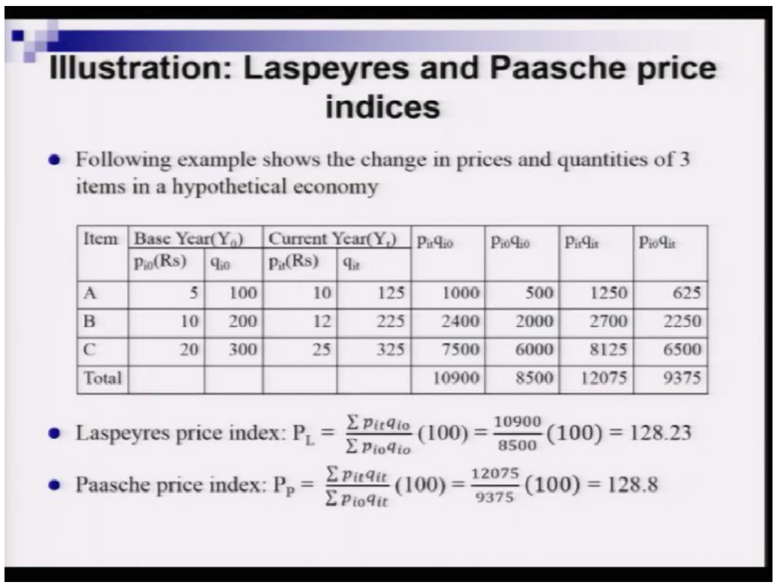

So, now, let us have a look at a numerical illustration how to compute Laspeyres and Paasche. And there could be four different cases because within Laspeyres and Paasche we have both indices for price and quantity. But, as we are mostly interested in deflation, how to convert a nominal economic time series to a real economic time series, we are going to ignore the cases of quantity indices. So, in the numerical example, we are only going to illustrate the cases of price indices.

So here, we take an example and needless to say that these are all arbitrary numbers. So, this is basically data on price and quantities of three goods in a hypothetical economy and these goods are A, B, C, they are given a no under the heading of item in column one. And you see the numbers are shown. Here, the base year current year numbers, the prices and the quantities and then, here I am showing you how you can multiply different sales with each other and then you sum them to get the values of Laspeyres price index and the Paasche price index.

So, I have already explained you the formula in previous slides, here again, I am showing you the formula in the bullet points. And now, if you plug these numbers to these formulas, then you can get two different values for Laspeyres and Paasche price index number. Note that, they are quite similar in nature. So, the first one, Laspeyres price index number gives you a value of 128.2 and the Paasche price index number gives you a value of 128.8. So, there is not much difference. So, you can say that for all practical purpose. It does not matter whether you choose a Laspeyres and a Paasche. And this is going to be more clear when I am going to talk about the case of cost of living index number.

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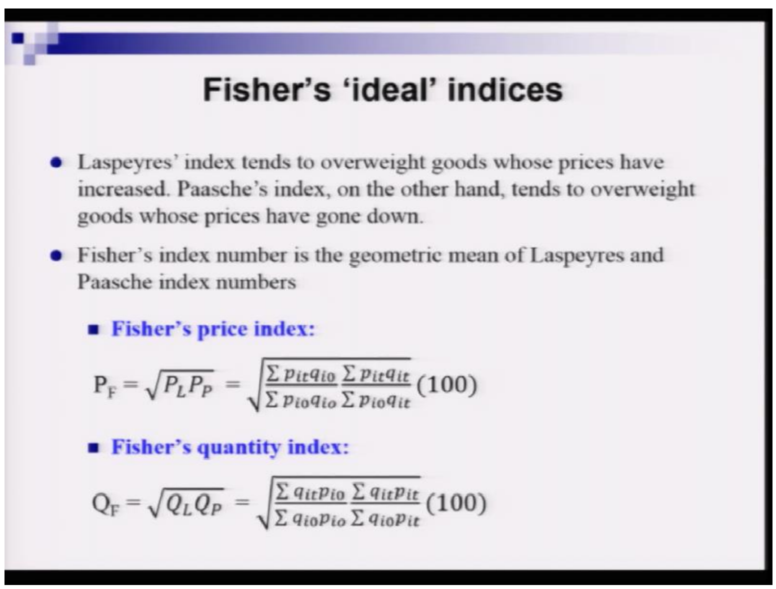

So, now, we are going to discuss the case of Fisher's ideal index number, which is developed by a statistician Irving Fisher note that he is different from Ronald Fisher, who invented or developed these ANOVA that we discussed last time. So, why did Irving Fisher think that there should be another index number?

Because Laspeyres and Paasche's index number were already developed, right? So, Irving Fisher, statistically proved that the Laspeyres index number tends to overweight the goods whose prices have increased, whereas the Paasche's index number tends to overweight the goods whose prices have gone down. So, there is some mathematical logic why Fisher thought that Laspeyres and Paasche are not ideal index numbers.

There is another point that we can talk that is in addition to what Fisher found mathematically or statistically and that is the choice of a base period and the current period weights, they are quite arbitrary. So, I told you that there are lots of subjectivities involved with Laspeyres and Paasche price index and quantity index numbers.

So, how you can minimize the impact of your subjective choice that you are going to make use of Laspeyre number or you are going to make use of Paasche number. Well, they could be very close at the end of the day, but they could be very different also, it all depends on the nature of the data set you are handling.

So, Fischer introduced this concept of ideal index numbers and he said that as these two are different numbers, why do not we take area, why do not we take mean of these two numbers. But he preferred geometric mean over the arithmetic mean. So, he defined transformed a new price index number, which is nowadays called Fisher price index number, that is basically the square root of Laspeyres price index number and the Paasche price index number.

And similarly, he also proposed Fisher's quantity index, which is denoted by qf and that is again the square root of product of the Laspeyres quantity index number and the Paasche quantity index number. There are many other very good uses in real life world and for that you have to learn about one more concept and that is called cost of living index number and that is what we know we are going to discuss next to conclude our class.

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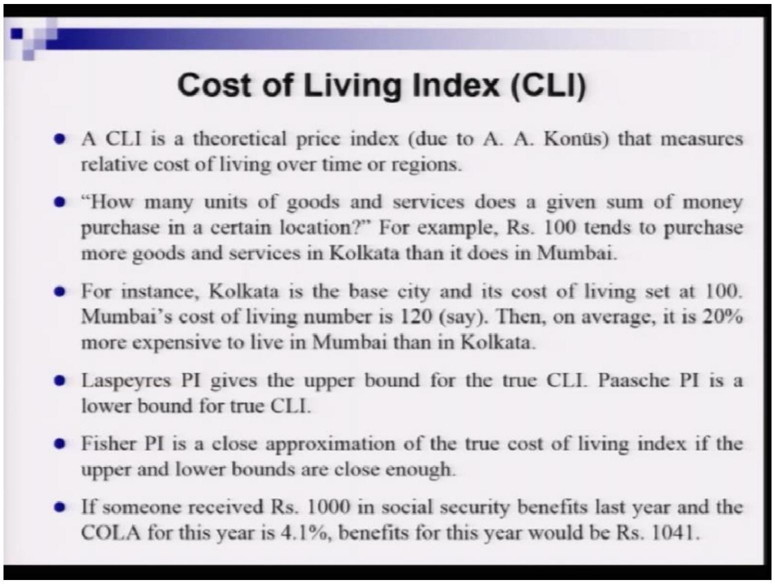

So, the idea of cost-of-living index number is developed by a Russian Economist A. A. Konüs and it measures the relative cost of living over two time periods or two different regions, okay. But what do I mean by this cost-of-living index number? So, it is best to understand this concept through a simple story. So, suppose someone wants to compare the cost of living in two different cities in India, Kolkata and Mumbai two metropolitan cities. And of course, if you have rupees 100 or rupees 500 in pocket you can buy more staff goods and services in Kolkata rather than you can buy in Mumbai, why? Because the general price level is higher in Mumbai compared to Kolkata, okay.

So, you can basically also say that this story can be expressed or retold in terms of time. So, if you have rupees 100 in pocket in 2010 then you could have bought more units of goods and services back in the past compared to what you can purchase with the same amount of money today, because the price levels have gone up.

So, for instance, if Kolkata is my base city and the cost-of-living index number is set at 100 and I found that Mumbai's cost of living index number is 120, then I can say that on an average it is 20 percent more expensive to live in Mumbai than in Kolkata city. So, what use one can make of the CLI? So, here when you are choosing for your staying location, suppose you are currently doing a job in Kolkata and you want to leave your job and you want to relocate to a better job. And suppose you are you got an offer from a company in Mumbai and you just want to say whether you will be equally well off or not if you move to Mumbai, because you know that cost of living is much higher in Mumbai.

So, then you can add just your offered salary that is offered by the company based in Mumbai and then, you can compare your current salary that you are getting in Kolkata and then, take a call whether you are actually going to be better off if you relocate to Mumbai. So, how can we compute the cost-of-living index number? We are not going to know details of construction of cost-of-living index number, it is very complicated. But we can talk about the bounds of the cost-of-living index number from what we have learned in this lecture.

So, a Laspeyres price index number gives the upper bound for the true cost of living index number, whereas the Paasche price index number provides the lower bound for the true cost of living index number and it is a theoretical result. I am not going to show you the derivation or proof for this. But this is a good result that you must remember, okay.

So, if I now know, theoretically that my cost of living in index number is going to fall between Laspeyres price index number and Paasche price index number then how I can find one particular value of cost of living index number. Actually, you cannot find the true value of the cost of living index number, you have to get a proxy. And Fisher's price index number can provide you the proxy.

So, if you look at the Laspeyres and the price index numbers and difference between these two is not huge. So, then you can apply these geometric mean formula. So, you multiply this Laspeyres and Paasche price index number take square root and that will give you the Fisher price index number.

So, as it is a geometric mean of two numbers. So, the geometric mean always will lie in between these two original numbers, which are Laspeyres and Paasche. And then, you can say that well for that logic, I can use Fisher price index number as a proxy or a guess for the true cost of living index number.

And to end discussion on cost-of-living index number, I would also say that, these cost-of-living index numbers also find applications or use in the social security schemes. So, we all know that Government's in many countries provide a lump sum amount of money to the needy people of the community bet it senior citizen, bet it widow or bet it poor students.

So, there are many social security monetary benefits that are going to different sections of the community and Government wants to provide money in such a way that they are equally well off, when there is inflationary pressure in the economy. So, of course, if you get rupees 1,000 two years before and if you continue to get rupees 1,000 in 2021 then the quality of living will fall or the welfare or the utility of the person will fall, why? Because you can purchase less quantities of goods and services from the same amount of money.

So, the monetary payment by the Government under a social security scheme should be adjusted by cost-of-living index number. And there is a concept called cost of living adjustment or COLA in the short form, so this is very useful to recalculate the monetary benefits under a social benefit scheme, when there is inflation in the economy.

Suppose, there is a hypothetical social security benefit scheme where rupees 1,000 is being provided to a person and that was paid last year and Government knows that there is inflation and cost of living adjustment for this year is 4.1 percent, suppose statisticians have computed this number and government knows about it. So, this year's benefit will now be 1,041 rupees, because these extra 41 rupees will take care of the inflationary adjustments.

So, we finish our discussion on price index numbers. And now, we know how to express nominal economic variables into the real economic variables, so that we can talk about the physical or the real change in economic variable. And we are all set now to start our discussion on the basic time series modeling. And in the next lecture, we are going to start discussion on the classical time series analysis. See you then. Thank you.