An Introduction to Microeconomics
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Lecture - 96
More on Cost Minimization

So, we saw these three examples and what we try to do, we try to calculate the minimum cost to produce Q amount of output, given the price of 1 unit of capital is r and price of 1 unit of labour is w and of course not just 1 unit, it is 1 unit per unit of time.

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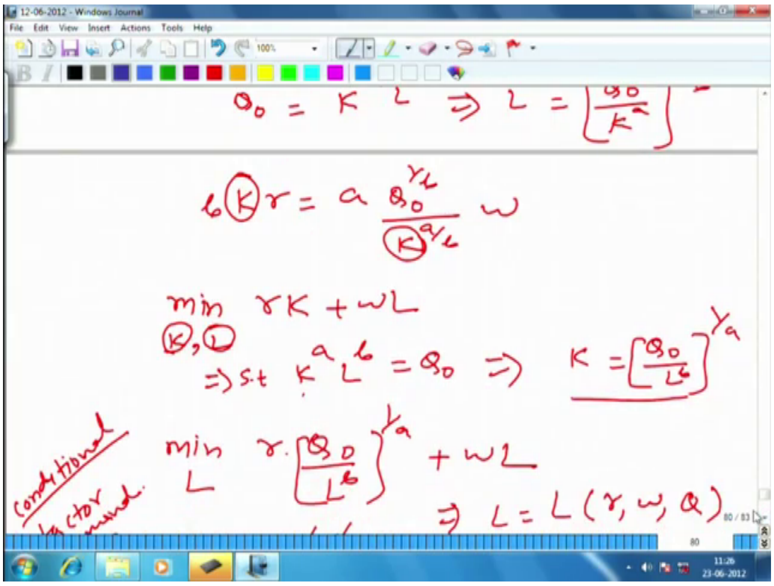

It can be per day per hour, just we have to be consistent about it and we have learn what we have derived that the cost function, let us say this is cost; cost function is a function of w r and output.

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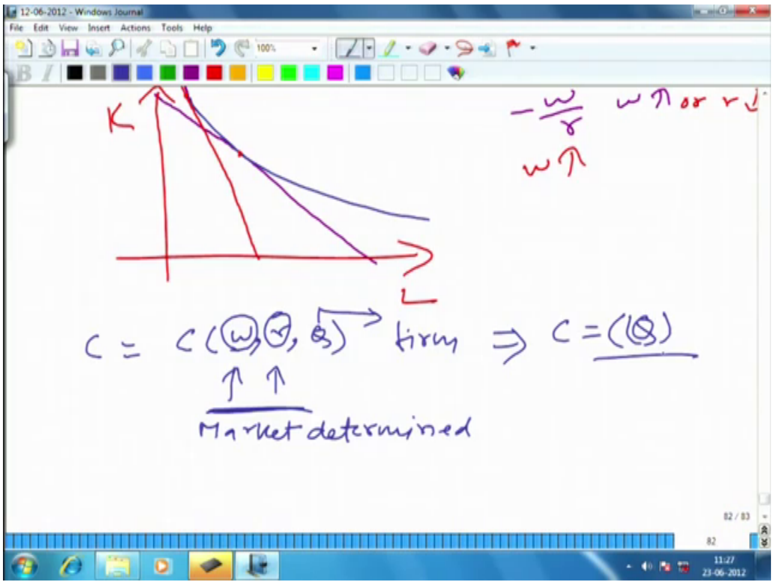

So, notice this w and r; these two are given by market typically, it is a market determined and Q, Q is decided by firm.

So, for a firm which is not able to influence w and r, cost function is function of output ok fine, I just want to show you to more things here.

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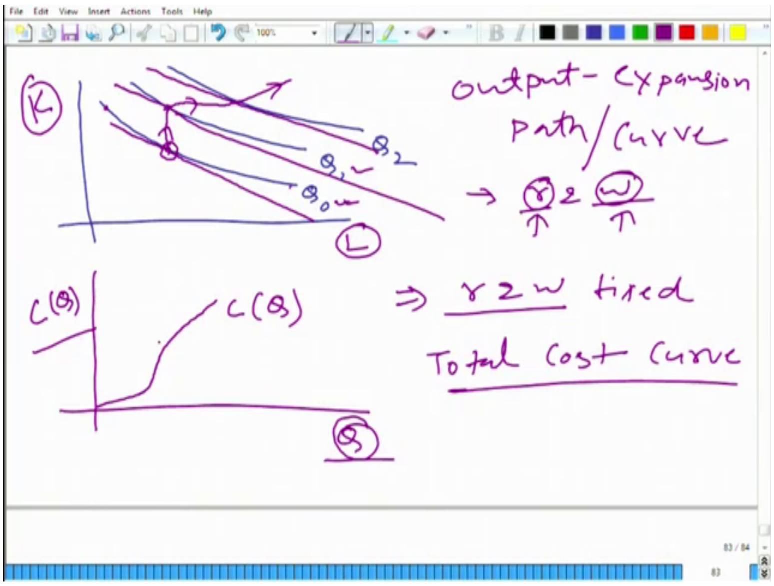

First of course, we can take this not necessarily a Cobb-Douglas example, but isoquant we have different isoquant Q naught, Q 1, Q 2; In the consumer theory we talked about income expansion path. Similarly here we can talk about output expansion path.

Now, what we can do here is we can take see how much is the minimum cost to produce this Q naught amount of output and then we can say that we can try to figure out the cost and this is this and then what we have here something like this ok. So, optimal point is here, here and here. So of course, I have picked only 3 different level of outputs Q naught, Q 1 and Q 2.

One can put many more levels of output and try to figure out the minimum cost to produce these respective outputs and here we have L and here we have K and if we try to draw a curve passing through all these optimal points, what we get is basically output expansion path or output expansion curve. And what we can do? We can again here what we have done? We have taken r as well as w; r is the rental for per unit of capital per unit of time and w is wage for per unit of labour per unit of time.

So, we have kept r and w fixed and what we are doing? We are changing Q naught to Q 1 to Q 2 and we are seeing that ok. Here do the optimal bundles lie and we are drawing a curve passing through all those optimal bundles and that is how we are getting the optimal expansion path.

But remember here this optimal expansion path that we are getting is on graph K L or L K to be precise ok. So, what we can do? We can take these optimal paths and we can draw here, we can say here, here we have output and here we have total cost C of Q and it is not exactly to the scale what we have here is and this is what we get here is C of Q; this is again here we are taking r and w fixed and what we are varying?

We are varying Q and we are seeing it is impact on total cost and what here we have? This is total cost curve. Remember this total cost curve, this gives cost to produce respective amount of output, remember this has already been minimized, this is not any cost like Q naught can be of course, produced at higher cost also if you take this combination and this combination.

Why we are selecting this particular combination? Because this is the list cost required to produce Q naught amount of output. So, here we all the list costs are represented not just any cost ok fine.

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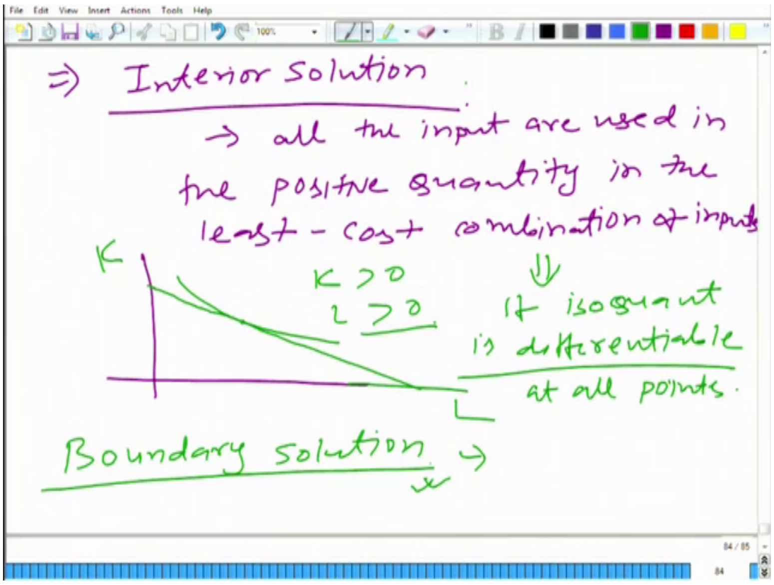

So, in the three examples that we saw we can put we got two types of solution. One is interior solution and what do we mean by interior solution? That all the inputs are used in the positive quantity, in the least-cost combination of input.

For example, we talked about Cobb-Douglas production function ok. What did we get here the solution we had at this point; here we had L here we had K. At this one, this is the least-cost combination, here K is K is greater than 0 and L is greater than 0. So, what we have is interior solution.

As opposed to interior solution what we have is something called boundary solution and what do we mean by boundary solution that all the at least there is one input which is not used in the which is not used in the least-cost combination of input to produce Q naught amount of output. So, at least we are not using one input, then we get boundary solution ok.

For example, when we say we took a production technology where both the inputs were perfect substitute of one another and in that case we got the boundary solution. What happened that at the least cost combination only one of these two inputs was used.

So, one thing if here if you pay attention to this interior solution, if isoquant is differentiable if isoquant is differentiable at all points then the tangency criteria that we had learned is applicable. We can use tangency criteria to get the least-cost combination of inputs or the optimal solution of the cost minimization problem.

Remember even when the inputs were perfect complement we got interior solution, but there we cannot apply the tangency criteria. Why? Because isoquant is not differentiable at all the points ok. So, if we want to if we want to use tangency criteria we have to be certain about two things, one that isoquant is differentiable at all the points that we can check in advance.

But the second is that all the inputs are being used in the positive quantity in the least-cost combination. So, this is bit contradictory if you think of it. Why it is contradictory? We want to know about solution that is why we are using this technique, but what we are saying that we can use this technique only if solution is of this kind. So, how would we know that solution is of this particular the interior solution kind ok so how can we proceed in such cases?

Student: Sir first we can differentiate.

That is fine, that there is no problem with that.

Student: We assume that.

We assume, we assume and we proceed and then we check our result with our assumption.

Student: Contradiction yes sir.

If there is contradiction then there is problem ok. So, we do not have to worry about it.