An Introduction to Microeconomics
Prof. Vimal Kumar
Department of Economic Sciences
Indian Institute of Technology Kanpur

Lecture - 95
Cost Minimization: Cobb-Douglas Production Function

So, here what we are doing? We are using some logic to solve these problems. But now if we take a Cobb-Douglas function and what we have? Cobb-Douglas production technology is given as K to the power L a and L to the power b.

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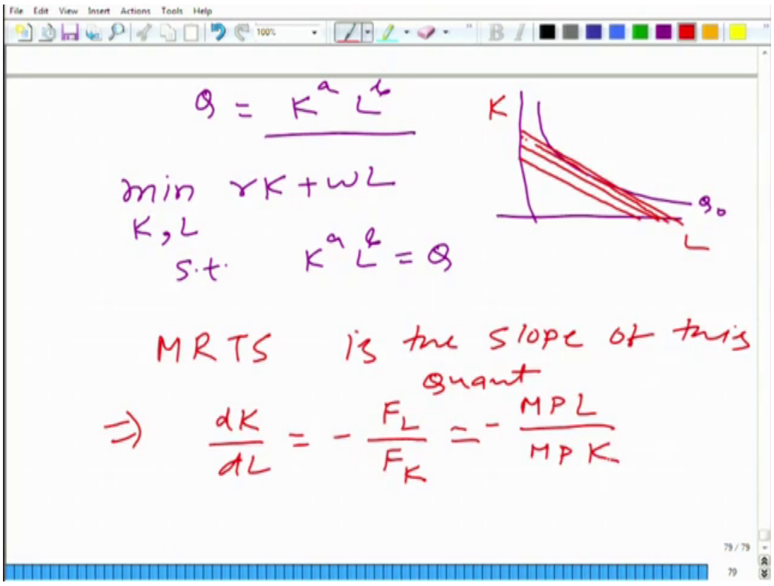

How can we solve this? Now what we have is minimize cost that is again r K plus w L and what we have to do is minimize with respect to K and L fine and what we have here is such that I will write here K to the power a L to the power b is equal to Q fine. How can we solve it?

Student: Sir here we have to use tendency relationship.

Two three different ways we can solve it. One notice that the Cobb-Douglas, in the case of Cobb-Douglas the isoquant looks like this ok. And of course, we have so we have to find an isocost, such that it is parallel to this isoquant and how can we find the how can we find the find it that the isocost line is tangent to this isoquant, how can we give the tangent to this isoquant? What is the equation of a tangent to this isoquant?

Student: Sir slope equals to.

Student: MRTS equals to.

MRTS basically MRTS is the slope of this isoquant. So, first what we can do? This is pure mathematical way, again we will come back and we will try to solve it little bit differently. What is the MRTS? MRTS is.

Student: D K.

Student: D dK by dL.

This is MRTS ok and what is FL basically marginal product of labour divided by marginal product of capital; these are the same thing ok. So, what is the marginal product of labour in this case? b K to the power a.

Student: L to power b minus 1.

B minus 1.

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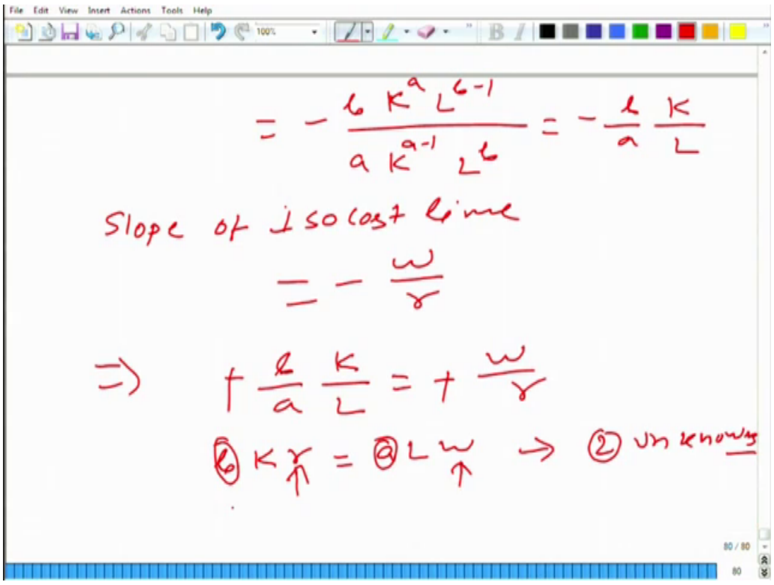

And what is marginal product of capital a?

Student: (Refer Time: 02:58) K power a minus 1 1 L power b.

And what we get?

Student: K by L.

Student: k

K by this should be same as.

Student: Minus.

The slope of.

Student: Isocost.

Slope of isocost line and what is the slope of isocost line?

Student: Minus w by L.

Minus w by L.

Student: R.

Or minus w by r sorry thank you. So, what we get basically is that one condition is that minus b a multiplied by K by L should be equal to minus w by r fine and what we get basically is if you look at here, bk r should be equal to a L w and this is an equation in 2 unknowns ok. What are the unknowns? b is given in the equation.

Student: A.

A is also given in the equation.

Student: K and L.

This is r and w are market determined what does this form need to decide?

Student: K and L.

K and L. So, these two are the variable for this problem fine. Second equation, how do we get the second equation?

Student: We would calculate the mode the mode of output.

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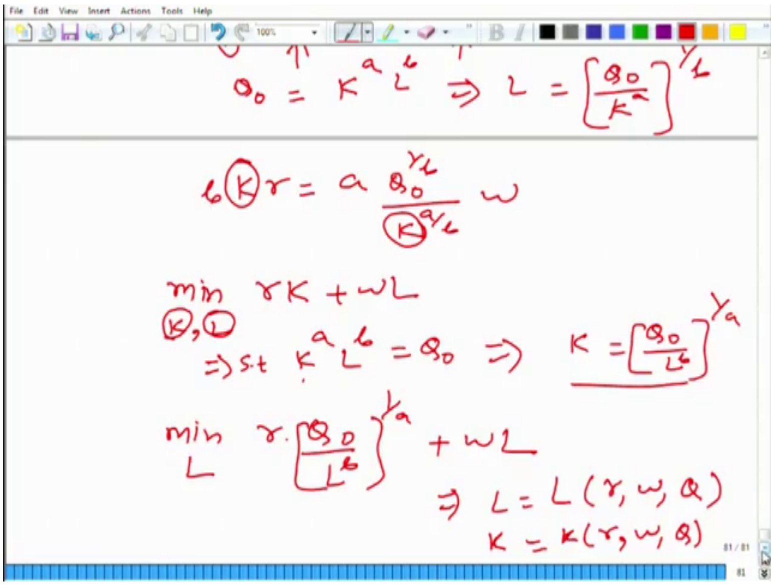

Second equation is from here; Q is given, Q naught is given and what we have is K a

Student: L.

L w. So, what we can do from here, I am not going to solve it completely just giving you hint. What we can do? We can figure out L in terms of.

Student: W.

K and K and Q naught ok. So, what we get? Q naught by K a to the power 1 by.

Student: 1 by b (Refer Time: 04:57).

This is power fine. So, now, if we put it back b K r a, L is nothing but Q naught to the power 1 by b, K to the power a by b w. In this we have only 1 unknown that is K and we can solve it. What did we use? We use the tangency criteria. So, somewhat we have used mathematical and little bit of graphical understanding of the problem.

Another pure mathematical way is just to look at the problem again minimum r K plus wL, we have to minimize with respect to K and L and what we have here is K to the power a such that K to the power a and L to the power b is equal to Q naught. We can get rid off one of these two variable right here fine. What we can get? Let us say let us get rid of K in this case.

So, K is Q naught by L to the power b whole to the power 1 by a fine. So, what we are saying basically here that K and L are not independent of each other, they vary in one particular way and this equation gives that variation. So, what we are saying basically we are making it as a function that K is an implicit function of L basically and we plug it back. So, now instead of minimizing with respect to K and L what we have is we minimize only with respect to L and r K we can get from here and this is one equation, where we have to which we have to minimize with respect to L and we minimize it and we get the what we can get we can get.

Student: L.

L as a function of.

Student: Q naught.

L as a function of.

Student: Q.

R w.

Student: R w and Q naught.

And Q naught ok or let us write here just Q and K is also a function of r w and Q. Is it clear?

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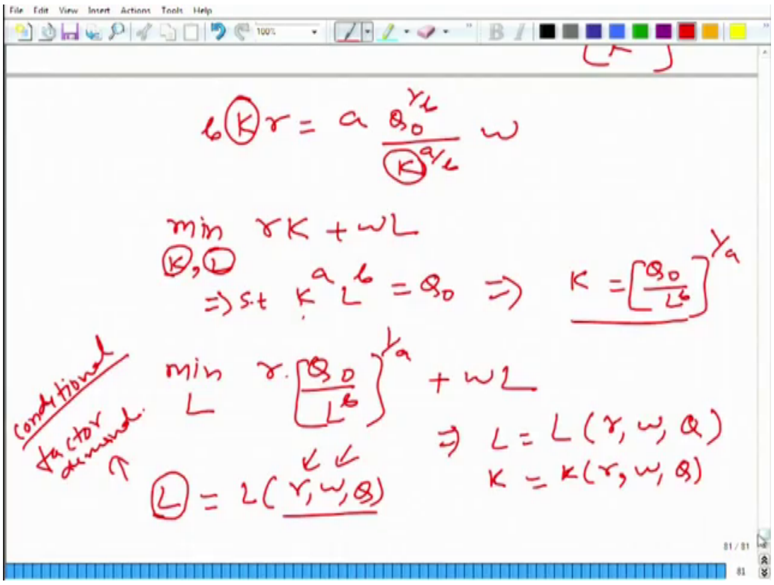

So, what does this give let us focus on this equation only now, what does this give?

Student: Minimum amount of labour required to.

Not minimum amount of labour, the amount of labour required to minimize the cost of production of Q; minimize the production of output, when the market price of labour is w and market price of capital is r not the minimum this is the amount of labour you require.

So, in other word what is L? L is the amount of labour that this firm demands from the market. So, can I say this is kind of a factor demand? Fine and we also put a word here conditional factor demand not just factor demand conditional factor demand. What is it conditioned on that it has you know L is not just a function of r and w, it is also a function of Q; the amount of output required, that is why we are putting a word there conditional. This is conditional factor demand. If you want to correlate it with a concept that we have learned during consumer theory this is very much like Hicksian demand.

What did we do in the hicksian demand? We kept the indifference curve fixed, we kept the utility level fixed and we said what would happen to our demand of a particular consumption good when the market price of this good or the other good change. So, that is what we are talking about. Here, what we have done? We have kept the isoquant fixed.

We have kept the isoquant fixed, we are not changing the isoquant and what we are talking about?

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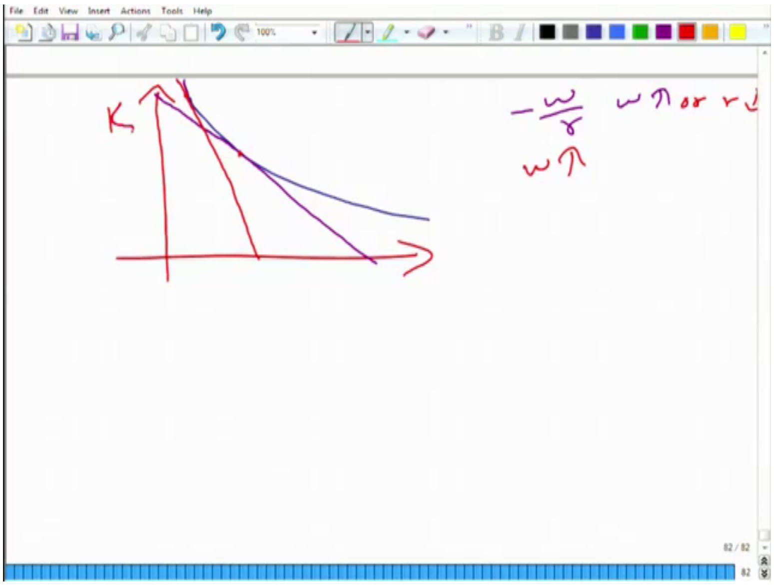

So, what is happening basically let us see; graphically, let us see what is happening here is this is isoquant let us say depending on a particular value of r and w our isocost line is this. If value let us say what happens if the slope is minus w by r. If w goes up what will happen to the slope? If w goes up.

Student: It will more steeper.

It will become steeper, it will become steeper. So, how would it look? Probably of course not exactly, it will look like this and we are moving here. That is why remember it does not matter, whether w has changed, w has increased or r has decreased. We will move from the factor demand would move from here to here and see what is happening basically, if w has gone up labour is becoming labour has become costlier. So what would this firm do?

Student: It will distract firing labour. We have to reduce the firing capital instead of.

It will it will produce the in the production it will use more of the cheaper input

Student: Yes sir.

And which one is the

Student: Cheaper.

Cheaper in comparison to other capital.

Student: Capital.

So, the amount of capital would increase to produce the same amount of output. Fine, is it clear, ok. So, that is what happening here.