An Introduction to Microeconomics
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Lecture – 86
Elasticity of Scale

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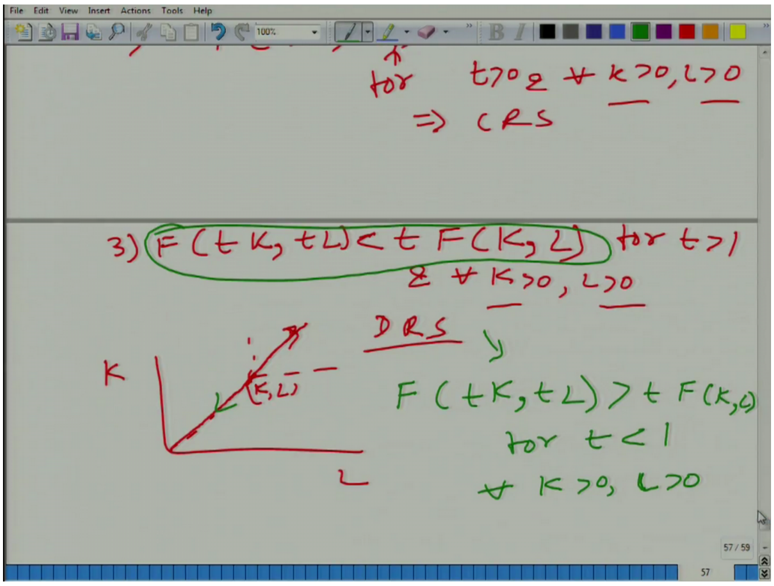

So, we have learned return to scale and what we have seen that; it is defined for we can talk about a production function exhibits, constant returns to scale or increasing return to scale or decreasing return to scale. If it satisfies some condition over the complete range of all the input variables ok, but what if; you know in a sense we are talking about a global the global phenomena for in the context of this production function ok.

But, some time it happens that one of these phenomena is exhibited over a small range of input values and some other over some other range of input values. So, what we are going to do rather than defining it globally, we will define it locally, and for that we will talk the term is elasticity of scale.

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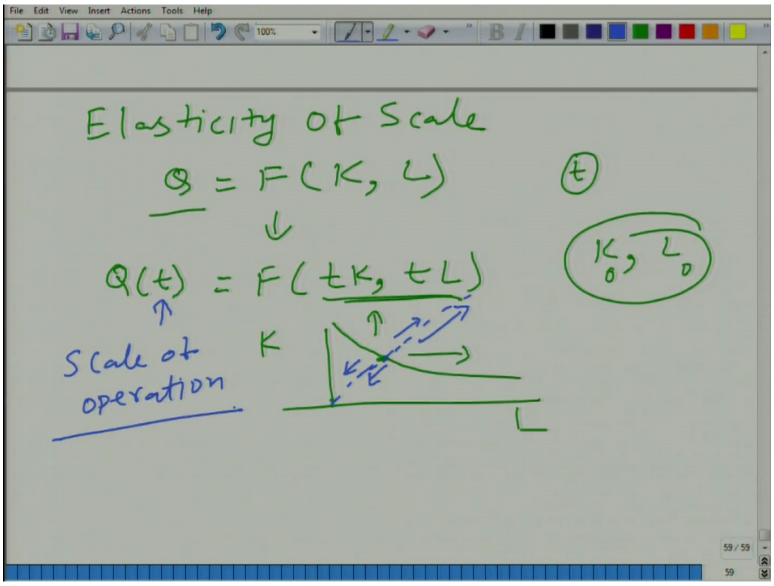

So, what is happening; let us look at it we started with Q, that is output it is a function of capital and labor. And what we said that; we will scale up the operation and how can we scale up the operation? Let us say if we take a t variable, that is a scalar t greater than 1, that is what we will stick to t greater than or let us say just t right now t. So, what we are talking about it? Scaling up operation means that instead of using K comma L we are using t K comma t L not remember not t 1, t 2 but.

Student: T.

T K and t, because remember what did we say; that all the factors of production or all the inputs should be increased in the same proportion. So, it means as soon as we have our initial point K naught and L naught; we can vary K naught and L naught in a particular way and that way is given by this ok. So, what we can say that now Q is a function of t.

In this particular case; remember the current operation is L and K ok. Let us say this is the current operation. We cannot move in this direction or in this direction, what we have to do? We have to stick to this straight line we can either move in this direction or in this direction. Scaling up means that moving in this direction is scaling down means moving in this direction. So, that is represented by Q of t; Q is now a function of t and what is t? I can say t is scale of operation is it clear.

Student: Yes sir.

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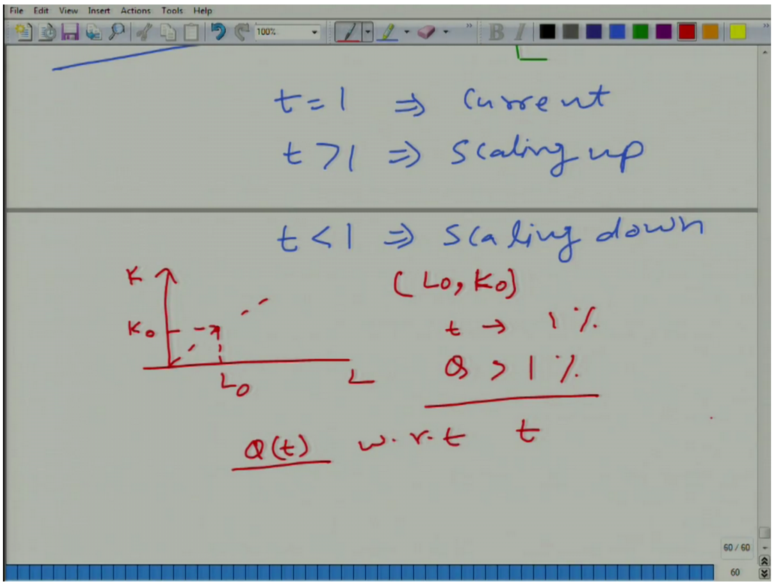

So, I can say t is equal to 1 means current; current level of operation.

Student: (Refer Time: 03:32).

T greater than 1 means we are scaling up the operation fine and t less than 1, we are scaling down is it clear fine. So, now, what we can say? What we are basically interested that; what happens what happens if we move on this line. Let us say earlier here we are L, K naught L naught, whenever we talked about return to scale we talked about globally.

But, now we are talking about what is happening at this particular level of operation and that is L naught K naught, so if we move rather than worrying about the overall range, let us worry about what is happening at this point. And we increase t by small very very small amount ok. Let us say that t is being increased here at that this 0.1 percent and then output if output goes up by more than 1 percent, what can we say; that locally at this point it is exhibiting increasing return to scale, but in a way it is wrong to say, because we have already defined increasing return to scale to be true, when such kind of thing is exhibited at complete range not just at this point, but at all the points for all the values.

So, to do that; we use another term elasticity of scale and how can we figure out; rather than talking about, what we can say? What we are interested in it; change in Q with respect to.

Student: (Refer Time: 05:33).

With respect to t; so again remember the logic that we gave earlier when we talked about elasticity; we rather than talking in absolute from what we can do? We can talk in the proportional term. So, what would be the proportional change?

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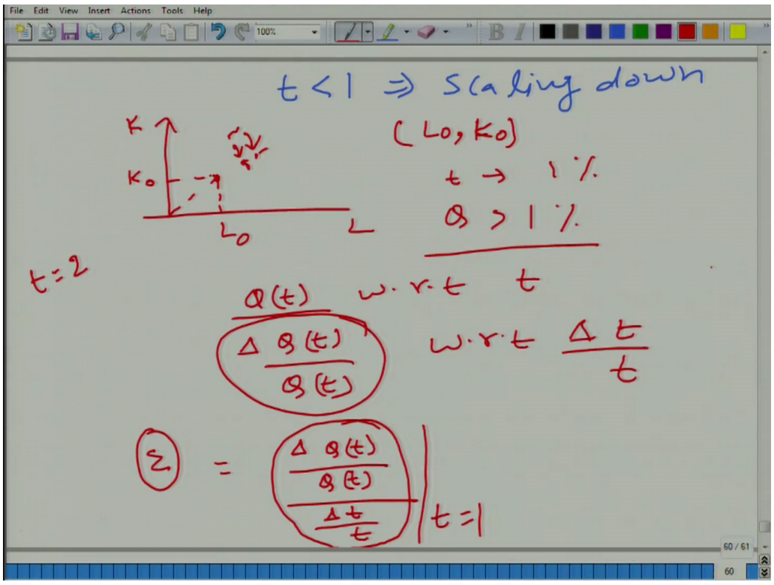

This is the proportional change in quantity with respect to proportional change in t and this is what is defined as elasticity of scale.

So, let me write it here; I can; if you want to use delta instead of d proportional rate of change in Q with respect to proportional change in t; and of course, Q here is given as function of t. This entity is defined as elasticity of scale. And this can of course; but where will be evaluated of course, it would be, if we start here start at the original point and then we start comparing here and here these two points, then we will not we are not doing it locally to do it locally, it means if you want to evaluate it at this point, what do we mean; that this entity should be evaluated at t is equal to 1. So, this is the complete definition of elasticity of scale.

Student: (Refer Time: 07:18) one point.

Go back here; what is t is equal to 1.

Student: Current.

Current if we evaluate it at t something greater than 1; 1 let us say if we talk about t greater than 1. So, then we are here and then what would it explain; this entity would explain if t; let us say you start with t is equal to 2 and we are talking about change in t ok. So, where will you move; from this point to this point probably? So, you are not talking you are not talking about this rate of change locally you are talking about again some point away from the current operation.

How did we start; that what is happening at the current point? What is the rate of change in quantity with respect to rate of change in scale? So, that is why we are taking t is equal to 1, because t is equal to 1 represents current operation is it clear.

Student: Yes sir.

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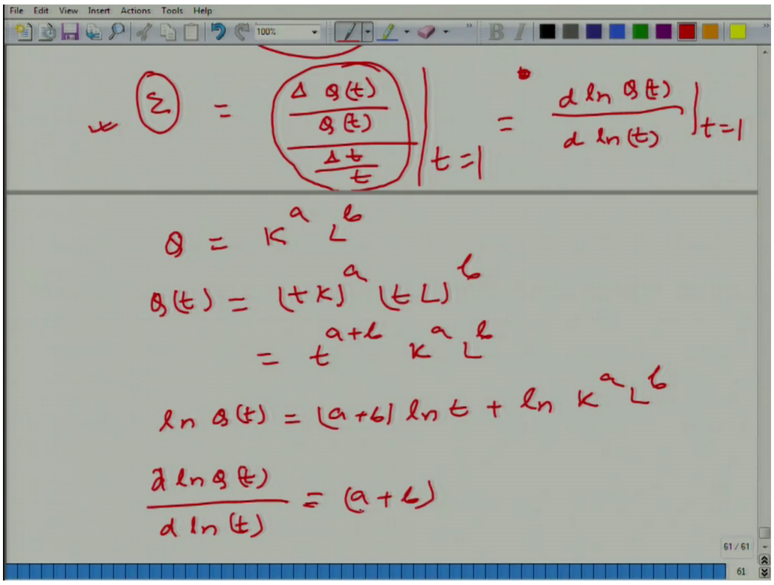

And here then we can say of course t is, if it is great equal to; let us take an example; let us take an example to evaluate; let us take Cobb Douglas function this is elasticity to scale fine. Let us take a Cobb Douglas function Q is equal to, let us say K to the power a, L to the power b. In this case local and global does not matter, because Cobb Douglas has this property ok, but we are evaluating elasticity of scale fine ok.

So, let us see what happens. First, we have to make it a function of t. So, it means t K to the power a, t L to the power b or in other words t to the power a plus b fine. This can also be written here as d ln Q of t with respect to d ln t evaluated at t is equal to 1 fine.

So, let us take log right; I am not going to decompose it, I am going to leave it as it is there is no need and differentiate it with respect to.

Student: Lnt.

Ln t and what do we get.

Student: A plus b.

A plus b. So, now, we can say locally Cobb Douglas function exhibits increasing returns to scale, when a plus b is greater.

Student: (Refer Time: 10:19).

Than 1 when a plus b is equal to 1.

Student: Constant.

It represents, it gives exhibits constant returns to scale.

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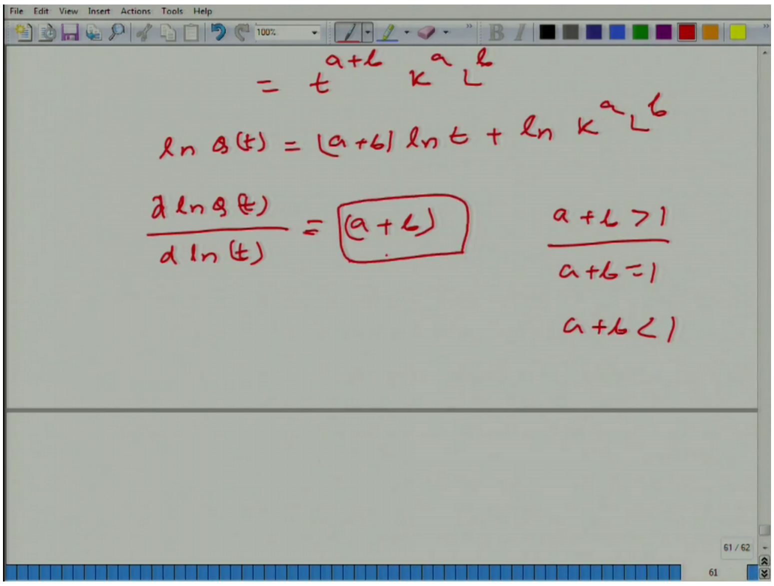

And when a plus b is less than 1 then it exhibits.

Student: Decreasing.

Decreasing returns to scale. So, t is equal to 1 is clear here of course, we did not get it as a function of t, but sometimes we may get it as a function of t fine ok. So, we can get rid of t by evaluating at t is equal to 1 fine that brings an end to this topic.