

An Introduction to Microeconomics
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Lecture - 82
MRTS: Few Examples

Right now, let us try to calculate MRTS for three different cases. One when the production function is Cobb Douglas production function.

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Cobb - Douglas production function

$$\Rightarrow Q = K^a L^b$$

$$Q_0 = F(K_0, L_0)$$

$$Q_0 = F(K_0 + \Delta K_0, L_0 + \Delta L_0)$$

$$F(K_0, L_0) = F(K_0, L_0) + \frac{\partial F}{\partial K} \Delta K_0 + \frac{\partial F}{\partial L} \Delta L_0$$

$$\Rightarrow \frac{\partial F}{\partial K} \Delta K_0 + \frac{\partial F}{\partial L} \Delta L_0 = \Delta Q$$

So, now you should understand that Cobb Douglas is the name of the form particular form that we use; it can be use for utility, it can be used for production function let us take a particular one Q is the power a.

Student: L to the power b.

L to the power b what will be the MRTS? How can we calculate the MRTS?

Student: dQ by dL by dK b Q by dK .

So, before we do that let us take a general case what is basically happening? We are using let say to produce Q naught label we are using K naught and L naught. And now what we are talking about? We are changing K naught to K naught plus delta K naught and let us leave it leave it as positive because we know if one is incre one input is

increased, second input has to be reduced to come to the same label, but expression will bring that minus sign. So, do not worry about it $L_0 + \Delta L_0$.

So, basically what we have done? Remember in this graph we start with here and we are trying to move we are we we will change L such that that we come to the isoquant. So, we will come to here; so, earlier point is K_0 earlier point here is and the new point is of course, what we are assuming that they are on the same isoquant here we have $L_0 + \Delta L_0$ comma $K_0 + \Delta K_0$.

Student: Delta.

ΔK_0 fine now in both case this should also give us the same label of output fine. So, if you know the Taylor series expansion then it is very simple what we can do? What we can write? This is equal to $F(K_0)$.

Student: K_0 .

Comma $L_0 + \Delta L_0$.

Student: Plus (Refer Time: 02:32).

Of course, we will take the approximate because Taylor series expansion we will have infinite term that we cannot you know we will not use. So, just first terms and here what we have? Ok this will get cancelled and what we will get? d is equal to 0 or in other word what we will get ?

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The image shows a digital whiteboard with handwritten mathematical derivations. The top part shows the total differential of a production function $F(K_0, L_0)$ set to zero, leading to the equation $\frac{\partial F}{\partial K} \Delta K_0 + \frac{\partial F}{\partial L} \Delta L_0 = 0$. The bottom part shows the derivation of the Marginal Rate of Technical Substitution (MRTS) as the negative ratio of the Marginal Product of Labor (MPL) to the Marginal Product of Capital (MPK).

$$\begin{aligned} F(K_0, L_0) &= F(K_0, L_0) + \frac{\partial F}{\partial K} \Delta K_0 + \frac{\partial F}{\partial L} \Delta L_0 \\ \Rightarrow \frac{\partial F}{\partial K} \Delta K_0 + \frac{\partial F}{\partial L} \Delta L_0 &= 0 \\ \Rightarrow \frac{\Delta K_0}{\Delta L_0} &= - \frac{\frac{\partial F}{\partial L}}{\frac{\partial F}{\partial K}} = - \frac{MPL}{MPK} \\ \Rightarrow \frac{\partial K}{\partial L} &= - \frac{MPL}{MPK} = MRTS \end{aligned}$$

Student: Minus MPL by (Refer Time: 03:20).

Fine and what is this?

Student: MPL.

MPL there will be a minus sign here MP

Student: K.

K and of course, we need to take limit. So, basically what we are getting? The slope is equal to minus MPL divided by MPK divided by sorry.

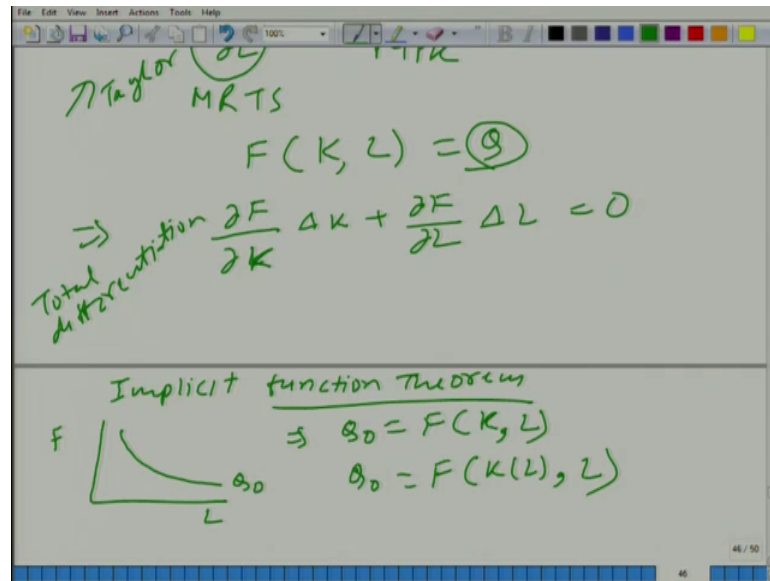
Student: MPL divided by MP guess.

Sorry just a minute let me check it ha MPL divided by MPK is it clear?

Student: Yes.

And this is how we can calculate this is what we had just discussed that this is MRTS. So, this is MRTS there is another way to do it also just mathematics you should be familiar with what we are saying is that.

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What we are saying is that with the new label again the same technique we can use ; we are producing the same amount , we are producing the same amount. Or in other word by changing del K in capital and del L in labour, we are not able to we are not changing the label of production.

So, by totally differentiating it what we can get? This is or let us say just leave it like this; this is Q and by changing Q and L we are not M is not to change the Q, but m is to keep the Q fixed. So, by if we are changing K and L what we can write that

Student: d L by d L delta K.

Delta K.

Student: Plus d F by d L (Refer Time: 05:24).

Delta L this should be.

Student: 0.

Equal to 0 and we will again reach to the same label and there are several other techniques also. The other technique that also this is called total differentiation other here we use Taylor series just you should be familiar with the technique this is Taylor sort and this is called total differentiation.

Ok and third is again we will reach to the same point Implicit function theorem. Again I am not getting into detail, but you should know look at it the graph this is L this is F and value is Q naught. So, basically what we are saying? Q naught is equal to function of capital and labour, but Q naught is fixed, Q naught is a number.

So, what we are saying that to obtain Q naught K if we are changing the L then K should be a function of L fine. So, basically this is what we are saying K as a function of L and this is L; this is your independent variable. Now, differentiate it with respect to L again you will reach to the same point, but that is immaterial.

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The image shows a digital whiteboard with the following handwritten equations:

$$MRTS = - \frac{MPL}{MPK}$$

$$Q = K^a L^b$$

$$MPL = b K^a L^{b-1}$$

$$MPK = a K^{a-1} L^b$$

$$MRTS = - \frac{b K^a L^{b-1}}{a K^{a-1} L^b} = - \frac{b K}{a L}$$

What is important is that the marginal rate of technical substitution is in this particular case minus MPL divided by.

Student: MPK.

MPK and now we have a Cobb Douglas function; Cobb Douglas function is K to the power a, L to the power b let us calculate the marginal rate of technical substitution how can we calculate? First we have to calculate.

Student: MPL.

MPL and how much is MPL? b.

Student: b K to the power.

K to the power L L.

Student: b minus.

B minus 1 and what is MPK ?

Student: a.

a.

Student: K to the (Refer Time: 07:33).

K to the power?

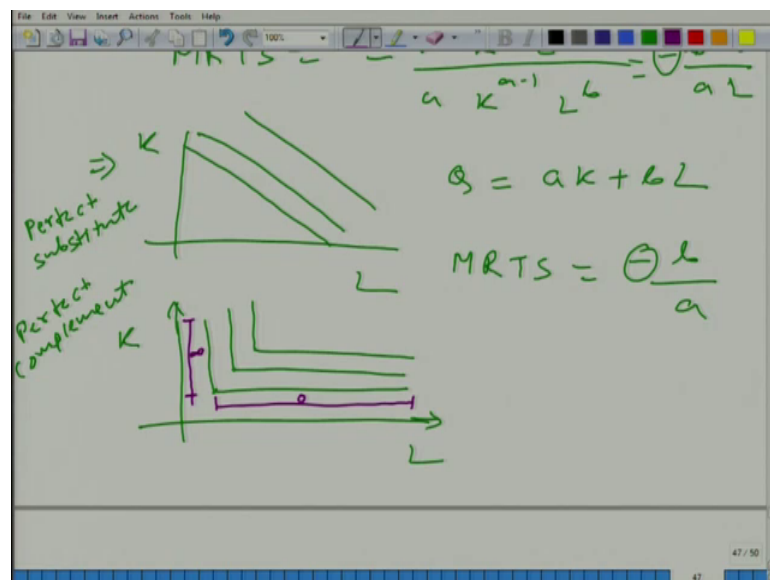
Student: a minus 1.

a minus 1.

Student: L power b.

L to the power b and now it is very simple marginal rate of technical substitution is equal to minus or in other word minus b K divided by a L is it clear? Let us take another example; earlier we talked about a production function where the both inputs are perfect substitute of one another; it means what we have is the production function is linear in.

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Student: K and L.

K and L here we have K here we have L Q is equal to a K.

Student: Plus.

b L and what would be the marginal rate of technical substitution ?

Student: Minus b by.

Minus b by a in many books you will see in many places you will see that there is no minus, but you know for yourself that you know that it has to be negative. So, you can omit this negative sign as long as it is clear in your mind that marginal rate of technical substitution cannot be positive fine ; in some book they define that it is negative of this MPL by.

Student: MPK.

MPK in some book they just divide sorry it is a described as it is simply a MPL divided by.

Student: MPK.

MPK, but does not matter fine it is clear? How about, when these two factors of production are compliment up each other.

Student: (Refer Time: 09:35).

Perfect substitute think about it.

Student: (Refer Time: 09:50).

So, what we can say at corner point we cannot.

Student: Define it.

We cannot define how about in this zone where it is horizontal.

Student: (Refer Time: 10:04).

It is 0 sometime you know just looking at graph you can figure out; here in this zone it is 0 and where it is vertical it is?

Student: Infinite.

Infinite and at corner point it is not defined.

Student: Defined.

It is clear? Ok.