An Introduction to Microeconomics
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Lecture - 79
More on APL and MPL

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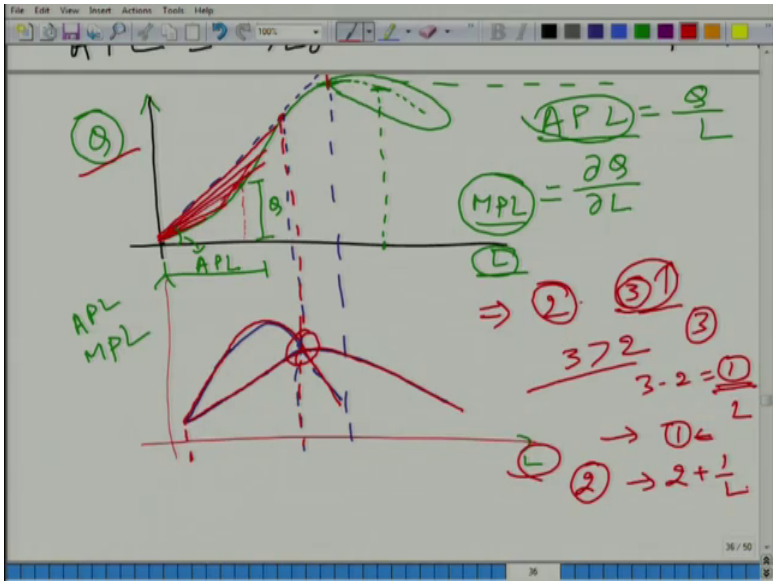

Ok let us take this graph again on y axis we have and on x axis we have labour input and of course, we are taking capital as fixed. So, we are not putting capital on any axis; so, what we have here is that output is going up as labour is increasing till this point and then its starts decreasing. That is way when we say when we talk about production function we draw here this function in dotted line; why? Because production function gets the maximum output possible given the amount of input.

Student: Input.

And we are allowed to freely dispose of as one of the assumptions and one of the properties that we have talked about earlier. That if let say we are here of course, we a if we are here and if we take this function it gives the production label as this much while we can produce higher products higher output by freely disposing of some of the labour input. So, that is way if we want to talk about the production function it should be like this ok, but that is not the point. Now here we are talking about if we take this point just

arbitrarily I am taking a point just to remind you if we draw line from origin to this point on the output curve; the slope of this line is slope of this line is equal to ?

Student: Average product of.

Average product of labour; why? Because this is Q at this point and this is L an average product of labour is.

Student: Q by L.

Q by L, but if we take the slope of this output curve at this point then what is this the slope is equal to?

Student: Marginal.

Partial derivative of Q with respect to L and this is nothing, but marginal product of labour. So, on this curve at any point if we have this curve; we can figure out APL as well as MPL. Although here the process is reversed we already knew APL and MPL an using that we had drawn this curve, but what I am saying when you have an output curve with respect to one of the inputs; we can always figure out average product with respect to that input and that input here is labour. And of course, also we can figure out marginal product of labour fine.

And if you notice at till this point that average product of labour is increasing ok; it is increasing and after this point it starts decreasing fine ok. Now, what we can do we take another graph where L is still on x axis, but on y axis we are not plotting q, but we plot APL and MPL two different curves. So, what is happening? Till this point if we just take of course, it is rough, it is not very exact what is happening till this point average product of labour is increasing and after this point it starts decreasing. And if we see the value of MPL at this point if you look at the value that is labour input labour is equal to 1; APL and MPL both are equal.

But at this point rate of increase of MPL is higher than rate of increase of APL. Or in other word MPL is increasing at faster rate than APL; so, it goes up and then it starts coming down again its not to the scale it is very rough what is remarkable is here at the point where APL is.

Student: (Refer Time: 04:41)

Highest the maximum at that point MPL curves; MPL curve cuts the.

Student: APL curve

APL curve from above. That is also very important from above ok. So, is it because the way I have given the I have put the value; the way I have given the table or its true all the time that where ever APL is maximized MPL is equal to APL.

Student: It is true all the time

It is true all the time why? if you just look at it let say average product of capital is 2 and marginal product of capital at that point is 3; what does it mean? That if you add one more unit of labour increase in output because of this new labour would be equal to.

Student: 3.

3 while average is 2.

Student: 2.

Of course 3 is greater than 2. So, in numerator you are adding 1 and in as a result in in denominator you are adding 1, but a numerator as a result 3 is being. Added and that is of course, more than?

Student: 2.

2. So, average per person would go up ok. So, this what you can say 3 minus 2; 1 will be distributed over all the labour. So, for every one average for every average contribution would go up from 2 to 2 plus.

Student: 1 by L.

1 by L.

Student: Sir can also say this by the by the way that when average is maximum; it means we are adding something do it be makes it maximum and after that it will decrease. So, at

that point of time MPL that is the del change in labour should it should be less than average should be less than the average else it else it will increase again.

No, but it I am talking about in general at all the point.

Student: I am saying for all the points.

Student: Whenever average is maximum a.

Not whenever average is maximum.

Student: Average is maximum (Refer Time: 06:51).

So, when you are saying average is maximum average is maximum at only one point let us look at it here average is maximum only at this point.

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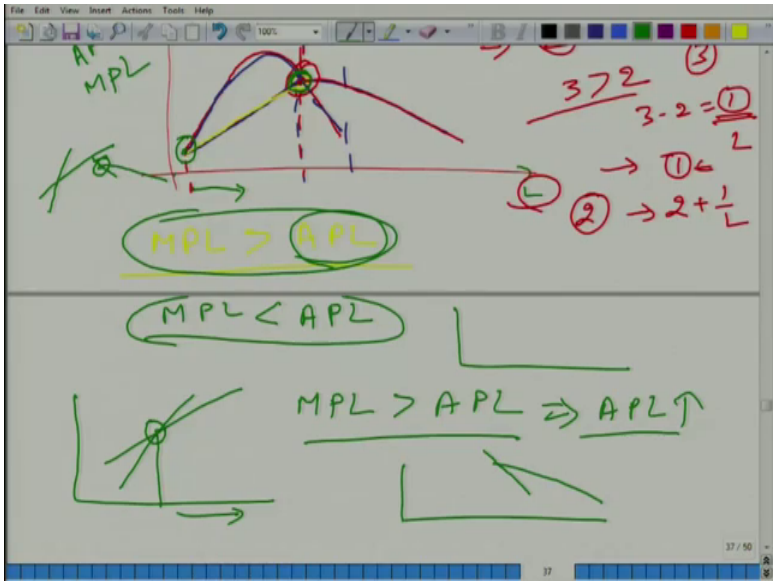

Student: Yes sir.

Not everywhere.

Student: So, sir.

But we are talking about all the positions where MPL that is marginal product of labour is greater than average product of labour.

Student: Sir what I am saying is that MPL should cut the APL graph at that point I am saying the reason for that.

Student: It is because the average is maximum; that means, we a we a we are adding something in that average which is maximum which has ablated maximum and the rest of the things are lesser than that term that is why the average.

Student: Decreasing after that.

Whole maximum what you are saying is true.

Student: Yes sir.

But in general it is true for all the point that whenever MPL is greater than APL on average it is adding average contribution of all the labours are going up.

Student: (Refer Time: 07:42).

That is why average product of labour will increase.

Student: Yes sir.

And to.

Student: Every new labour is better than the previous labour.

Not better than that incremental.

Student: (Refer Time: 07:55).

Contribution.

Student: Contribution.

Is more than the average contribution of all the previous workers.

Student: Workers.

And that is why the new workers new workers contribution would add on the average contribution of all the other.

Student: Workers.

Workers and that is way average product of labour will increase ok. And similarly here marginal product of labour is whenever you have marginal product of labour less than; whenever you have marginal product of labour less than average product of labour; what will happen? Again the incremental contribution is less than the average that is why it would decrease the average of all the workers.

Student: Workers.

And thus average product of labour will.

Student: (Refer Time: 08:43).

Starts decreasing; so, where when maximum what does it mean? Maximum is a kind of a stationary point; at maximum you know the when at which point the average product of labour is maximized where this is one of the requirement that average product of labour should be stationary. So, when it can be stationary? When it will not change where it is equal to marginal product of labour that is why at maximum average marginal product of labour inters curve intersect the average product of.

Student: (Refer Time: 09:22).

Labor curve now the another point it why should cut the average product curve from above not from bellow. What I mean see there are two intersection point one is here and one is here to green.

So, at this point also average product of labour is equal to marginal product of labour. And at this point also average product of labour is equal to marginal product of labour that is why we need to a specify what is the difference between at this point and this point.

Here marginal product of labour curve is cutting average product of labour curve from bellow. So, when L is increasing it is moving like this average product of curve is like this and marginal product curve is like this. So, it is cutting from bellow while here it is opposite.

It is cutting from above. So, that is another requirement why should be cutting from above?

Student: Sir because initially like to we said that diminishing marginal product of labour. If the marginal product of labour diminishing and keep free (Refer Time: 10:42) adding labour then marginal product of labour could.

That is look at look at it little bite mathematically see what is happening when it is cutting.

Student: Like you said including the criteria of free disposing when we have obtain.

It has nothing to do with free disposing.

Student: Sir when you obtain when you obtain like we using the computer example when we have 7 people for 7 computers and when we

No I am not saying I am not saying for this specific example; what is happening in general. Let us look at where its cutting from below here it is cutting from below; what is happening it here? That beyond this point beyond this point MPL is above APL ok. So, it means that what will be its impact? We have just discussed that APL is APL will increase.

Student: (Refer Time: 11:37).

APL will increase. So, if APL is increasing by increasing labour; it cannot attend maximum here.

Fine while what is happening here? Here it is cutting from it sorry.

Student: (Refer Time: 11:58) is it possible for MPL to cut this from below?

Here it is cutting from here it is.

Student: But it is APL is just some of MPLs of end anythings. So, if MPL is cutting from below that is mean it is some will.

MPL is not some of all the APLs.

Student: Sir it is just.

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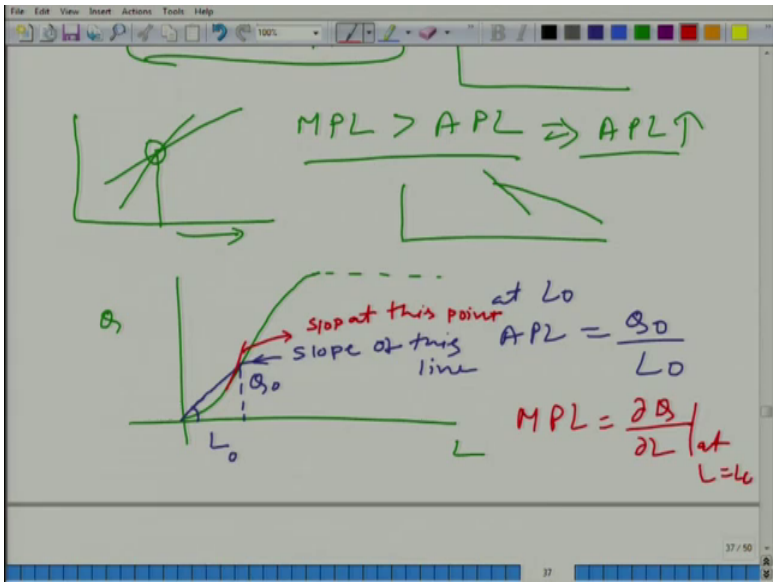

It is not; it is not let me just tell you clarify it to you; here is total quantity curve here is the total when we what we are doing is here we have Q, here we have L fine. And what is APL? APL is let us say APL at this point APL at this point is this angle because here we have Q, here we have L at this label. So, at L naught APL is Q naught by L naught; in other word the slope of slope of this line and what is MPL? MPL is?

Student: (Refer Time: 13:19).

Slope at this point. So, MPL is not integration of APL MPL is?

Student: (Refer Time: 13:34).

At this point at L is equal to L naught ; is it clear? Fine now it is clear why it has to cut from a buff.

Student: Yes sir.

Because at that is the stationary you cannot get the stationary point if it cuts the APL form below.

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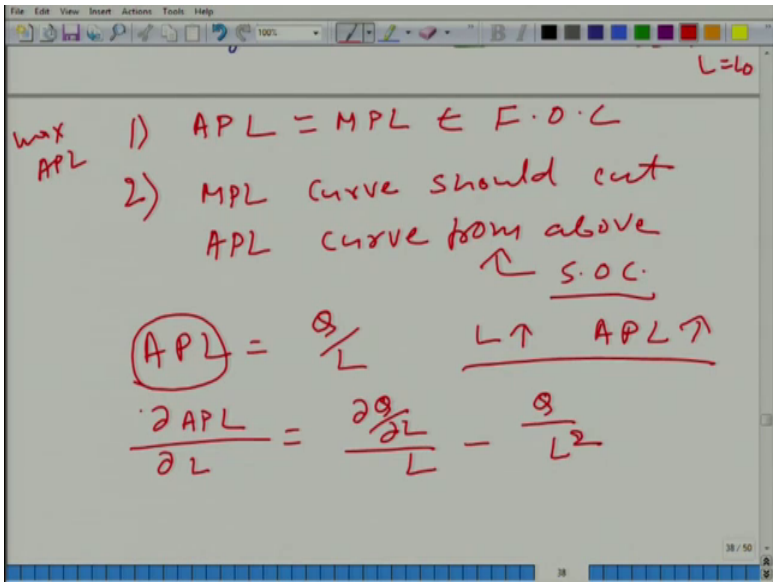

Fine and that is the reason. So, two requirements one that is where APL is maximized max APL; APL should be equal to MPL fine and MPL curve should cut should cut APL curve from above. If you know calculus you would notice that this is basically first order condition and this is second order condition, let us look at it let us do it mathematically what is APL?

Student: Q by L.

Q by L fine let us differentiate the APL; what do we mean by that? If L increases APL increases what does it mean? It means that the derivative of APL with respect to L is?

Student: Positive.

Positive that is what is means. So, let say we have only one variable what can we right it?

Student: (Refer Time: 15:19).

What can we right it this is nothing, but.

Student: Q by L square minus Q by L square plus d Q by d L into 1 by L.

This is Q by L square; you can say would partial derivative sign thinking that there may be more than 1 input fine is it clear? Take 1 by L common.

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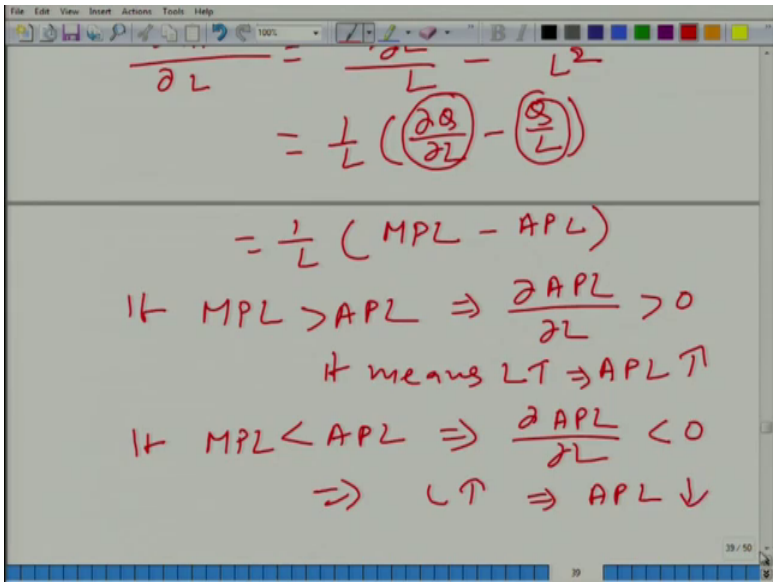

Student: (Refer Time: 15:47).

And as a result what would you get? 1 by L and what is this?

Student: MPL.

This is MPL at that point and what is this?

Student: APL.

APL just leave it here. So, now, it is very clear if MPL is greater than APL what it means? That partial derivative of APL with respect to labour is?

Student: Positive.

Greater than 0; it means it means L goes up.

Student: APL goes up.

APL goes up fine is it clear ok? Now, if MPL is less than APL what it means? The partial derivative of average product of labour with respect to labour is negative and it implies L goes up APL.

Student: Goes down.

Goes down and third would be MPL is equal to APL and that is how we get the?

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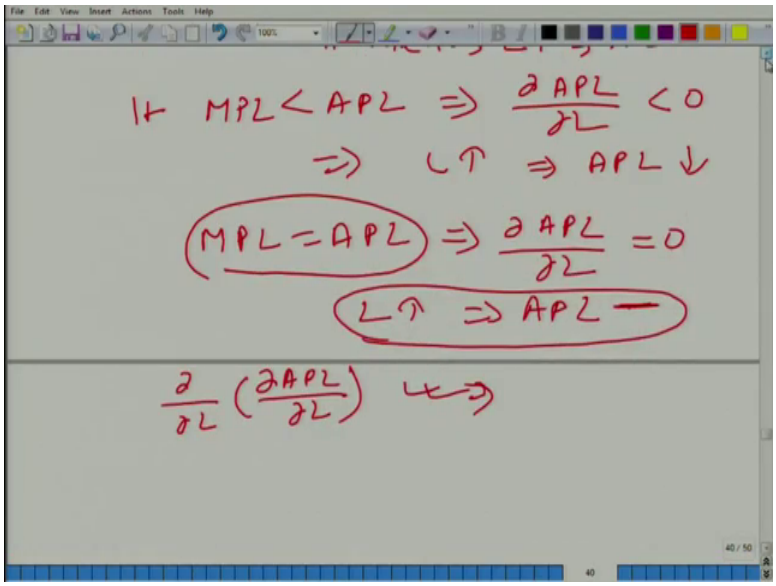

Student: (Refer Time: 17:09).

Stationary point.

Student: (Refer Time: 17:11).

L goes up momentarily not at all label; just at that point that L goes up APL remains the same no change. Remember if you no calculus although we have already discuss the graphically, but using calculus it just helps you get little more information. So, what is really happening? That this is one of the requirement that MPL is equal to APL then APL may have maximum or.

Student: Minimum.

Minimum or one of the critical points the boundary points fine. So, we cannot say that it would be the maximum; what we have learnt? We have learnt that there are two points where they are equal this point and this point, but only here we have maximum. The second requirement that I am not going to discuss, but if you no calculus we have obtained the first order condition you can obtain the second order condition. And how can you obtain the second order condition? Just let me tell you how you can obtain the

second order condition differentiate average product of labour with respect to labour twice not just once.

So, now, what you need to do is you need to obtain this and this will give you the second order condition. The interpretation; the graphical interpretation would be the requirement that it gives you would be just that MPL curves should cut the APL curve from.

Student: Above.

Above fine it is clear it is very very important. And also you should not get confuse because this is the common mistake when people say what if MPL is decreasing? What would happened to the APL? Can you say that let say that MPL is decreasing what would happened to APL? We cannot make any comment why? Because let us look at this zone here in this zone in this zone MPL is decreasing.

But?

Student: APL is.

APL is increasing.

Student: Increasing.

And in this zone MPL is decreasing and APL is also decreasing. So, we cannot say anything about it; we can comment about APL given the information about MPL only when we can compare the values of MPL and APL ok. So, be very wary about this is typical question that you get in exam fine.