An Introduction to Microeconomics
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Lecture – 46
Properties of Preferences: Continuity

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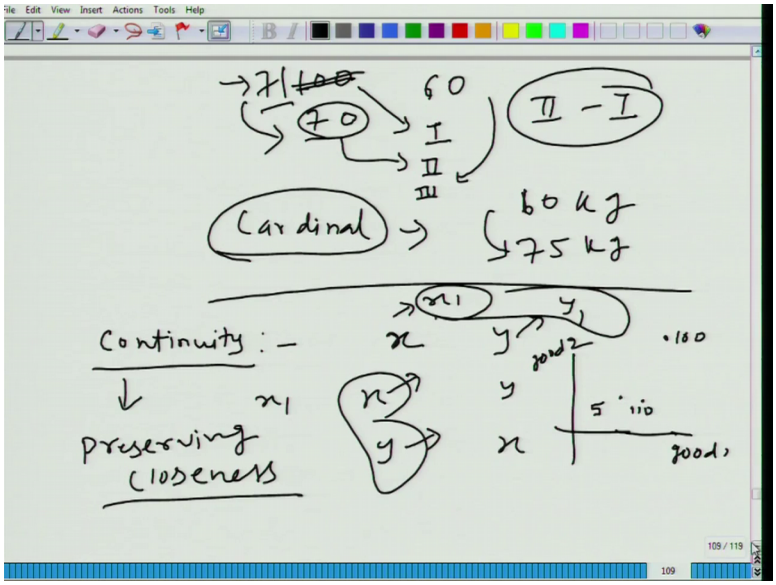

Now, as we have seen that when we have finite number of consumption bundle in the consumption set, it is very easy to rank we do not need any more we can rank them. But what we have remember go back to your, the definition of the properties of consumption set. What does it imply? That consumption set would have infinite number of.

Student: Bundles.

Bundles. How long will you go on ranking?

Student: Therefore, we have utility functions.

So, we know, but utility function again even from if you want to look at the prospective of utility function then again we should have some particular properties in the utility function. Properties such as continuity, you might you have heard about these in mathematics continuity and differentiability. Why do we need these properties? So, that we can use the optimization technique that we are going to learn shortly even in this

course. So, you can utilize them because using calculus people have developed optimization technique. So, utility function should be continuous in nature.

But so far we have not talked about the utility function should be continues and so in that sense we have not talked about the ranking of infinite number of bundles its it would be herculean task. Remember when we are talking about preferences we are not talking about the budget set, we are not talking about the feasibility set. So, whenever we are talking about preferences we want to rank all the bundles in the consumption set and if you have infinite number of bundles then just using the earlier three properties it would be impossible to rank all the bundles, we can try, but we will keep on going and going because we have infinite number of bundle.

So, we look for certain specific properties. We are not saying that everyone should have, everyone's preferences should satisfy this particular property, but if it satisfies the two set we are going to learn in the future would be applicable. If this property is not satisfied that some of the tools that we will learn in the future cannot be used. We will have to go back to the first principle. I am not saying that we cannot solve the problem, if you have done calculus and calculus we are going to use extensively remember whenever you have to differentiate you have certain thumb rules. But whenever you get a new function what do you do go back to the first principle.

Student: Principles

So, similarly something we do not call something first principle here, but the idea is very similar. So, we are developing we are trying to develop certain tool kits which would be applicable when your preference satisfies these certain properties. I am not saying that there in the world everyone whoever exist has this kind of preference, no I am not saying that. But if you have a preference which does not satisfy these property then either you will have to go back to the first principle and if they are inconsistent then you cannot even talk about it fine. So, the new properties that we are going to talk about is continuity. And what is continuity?

Student: You can define a function at each point.

You can define a function at each point.

Student: There is no break with the fixed value, at a fixed value.

That is not continuity.

Student: But the definition, sir in Maths what we learnt is that we draw its graph without lifting a pen without that is.

That is very basic definition that you learnt in class eight class ninth that when you can draw a function.

Student: Two value.

Of course then in that case function has to be a two dimensional function without lifting your pen when the function is continuous. That I am pretty certain that you must have come across more rigors definition that is not the idea right now, but what here what we are talking about these two are relative the definition of continuity in function. And here what we are saying let us take two bundles x and y and let us say x and y are ranked in a particular fashion either x is ranked above y or y is ranked above x, does not matter.

Now, if we pick a bundle that is sufficiently close to x and let us call x 1 and a bundle that is sufficiently close to y, y 1. So, then your ranking of x 1 y 1 should not be different from ranking of.

Student: x and y.

x and y fine.

Student: Yes sir.

That is in terms of preferences.

Student: (Refer Time: 05:19), ok.

If we are now we have learnt the concept of utility it means x is assigned a number and y is assigned a number. And if we pick x one that is sufficiently close to x then the number assigned to x 1 should be very very close to.

Student: x.

Number assigned to.

Student: x

X. So, what it does that it preserve the continuity is about preserving closeness bundle which are close to each other should be ranked near each other. Let me give you an example. So, let me say here, let us take again two dimensional world good 1 and good 2 and let us take a bundle here and a bundle here and a bundle here. Let us say this bundle is assigned a number 5, this bundle is assigned a number 100. So, then this bundle should have a number nearer to.

Student: 5

5, because it is closer to.

Student: 5

5, let us say 6 or it may be four that is what that is not what we are talking about here whether it could be 6 or 4 should be close to each other. But if it is 110 then definitely.

Student: It would be (Refer Time: 06:47).

This preference is not.

Student: Continuous.

Continuous fine, is it clear.

Student: Yes sir.

So, now just one thing I want to add here. We talked about we talked about utility function and what we said, then when your preference relation satisfies completeness reflexivity and.

Student: Transitivity.

Transitivity, then we get utility functions such that it preserves the rank. Now we talked about one more property, that is continuity.

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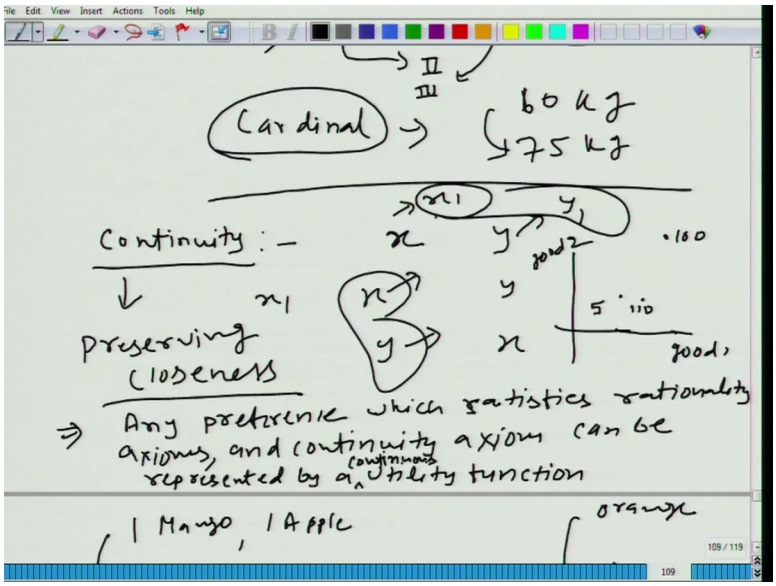

So, here the thing is that any preference which satisfies rationality axioms. What does it mean? Satisfy rationality axioms means all 3.

Student: All 3 those properties.

Completeness, reflexivity and transitivity are satisfied rationality axioms and continuity axiom can be represented by a utility function that is continuous in nature or you can say transic represented by a continuous utility function, fine. What we just talked about is that any preferences that satisfies continuity axiom or if you want to call it assumption then assumption is also fine continuity axiom and rationality axioms, there are three axioms if all these three axioms are satisfied then we say that preferences preference of a person satisfies rationality axioms.

If rationality axioms and continuity axioms are satisfied then preference of a person can be represented by a continuous utility function. And when we have already learnt that when we figure out one utility function to represent someone else preference, then what we can do? We can do we can take any monotonic transformation of that particular utility function and we will get a new function that will also be a utility function in the sense that it would represent the same persons preference, same persons test. So, utility function is not unique whether it is continuous or not fine.

So, earlier we talked about you know a continuity is nice mathematical property we can if a function is continuous and differentiable, then we can use optimization technique as I told you that we will be using some optimization technique. But now let us say you got a utility function that is continuous, but that is not differentiable.

What can we do? We can always transform a utility function using some monotonic transformation in to a differentiable function a continuous function can always be transformed in to differentiable function using monotonic transformation. So, we are not worried about differentiability of the utility function, is it clear, fine, clear.