An Introduction to Microeconomics
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Lecture – 38
Convexity of Consumption Set

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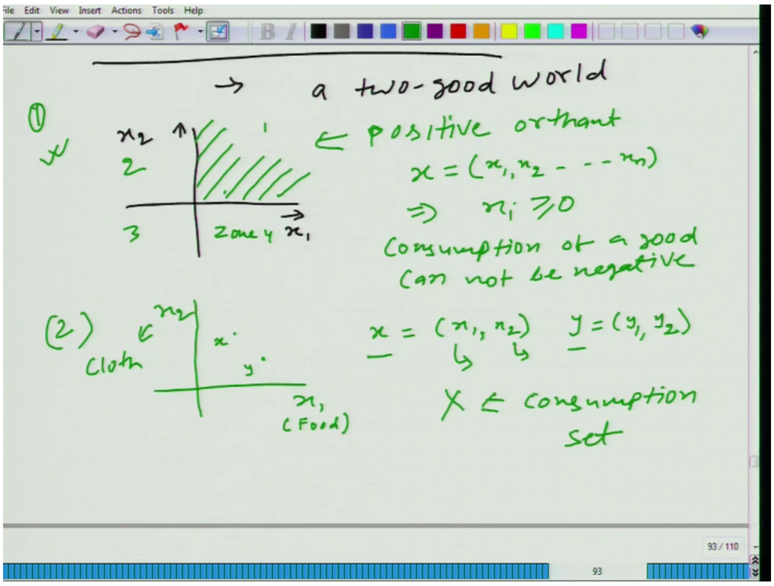

Second that I want to talk about is let us take, now I am not going to describe the whole space only the non negative part here is x 1 and here is x 2. Let us say we have two bundles bundle x and bundle y. Bundle x is equal to x 1 comma x 2, it means x 1 amount of first good and x 2 amount of second good. For example, x 1 is giving you just I am making it up that x 1 is giving you quantity of food and x 2 is giving you the quantity of cloth.

So, when I say x bundle, bundle x, that contains x 1 amount of food an x 2 amount of cloth. So, when we have, when we take these two bundles bundle x and bundle y a bundle x let us say that the consumption set is denoted by capital X this is the consumption set. And then what can you describe this x for me, what is this x in this notation, this graphical notation.

Student: The set of all bundle which are affordable.

Not affordable, remember you cannot when you are talking about consumption set you cannot talk about affordability. Remember the first building block is consumption set here we are not deterred by affordability. Whatever our mind can conceive here of course, you say if mind is allowed to conceive mind would conceive more than two goods food and cloth, but right now I am talking it is an imaginary world that I am talking about in that imaginary world you have only two goods, food and cloth. So, what would be the consumption set? Yeah or in other word a consumption set would be a set of all the bundles which contained non-negative amount of food and cloth. So, now, we are taking x and y these two bundles.

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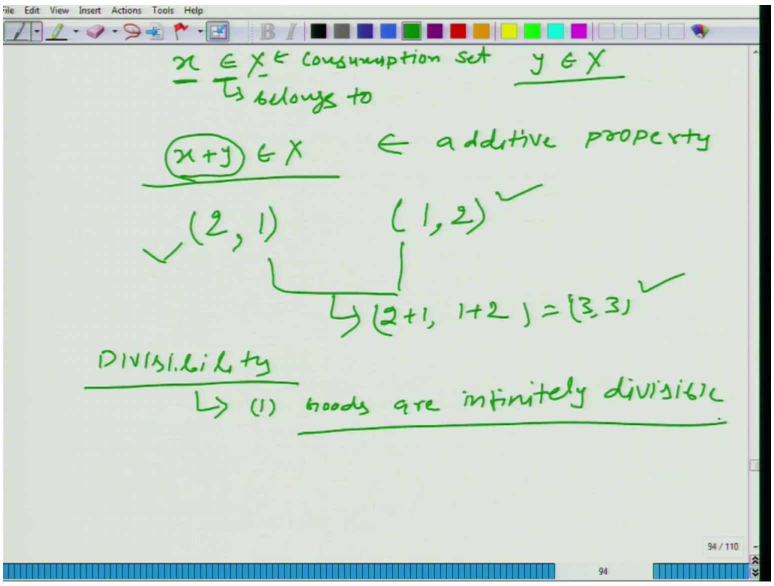

So, what I am saying if x it is important that you learn about notation this notation is means belongs to and small x denotes an element in this set in this consumption set fine. So, if x is belongs to x it means x is a bundle which can be conceived in x that is the capital X in the consumption bundle and similarly we have y in the x. So, we are talking about two bundles. Then x plus y should also be an element in capital X and this property is called additive property. Rather than using notation let me use numbers I am saying that it is conceivable to consume 2 units of food and 1 unit of cloth and also it is possible to have 1 unit of food and 2 units of cloth then you should also be able to conceive 2 plus 1 comma 1 plus 2 means 3 comma 3, 3 units of food and 3 units of cloths that is also a possibility.

You should be able to conceive these are the regularity condition that we are imposing because you can say I am not because you are not allowed to select your consumption set randomly or any way you want it should follow some restriction you it should it should be constrained by some of the criteria. And these are the criteria that if you are able to think about this bundle and this bundle then you should also be able to think about the 3 comma 3 bundle or in symbol if x is a conceivable bundle y is the conceivable bundle then their sum should also be a conceivable bundle is it clear.

Now, third property that I want to talk about is divisibility we just talked about additive property and third is divisibility. So, for that one a junction that we make is goods are infinitely divisible. Of course, these are mathematical properties, but in economics we use mathematics intensively to make our life simpler mathematics is used as a language the language of economics, and it helps us immensely. So, that is why we are making these assumptions. So, what we have here is the goods are infinitely divisible.

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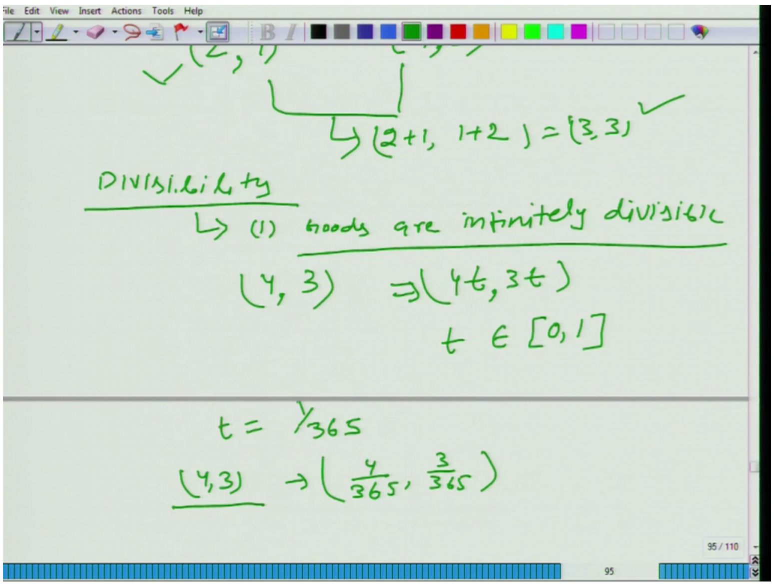

I will give it then, let us say if we have a bundle 4 comma 3 means 4 unit of food and 3 units of cloth if we are able to conceive then we should also be able to conceive 4 t comma 3 t where t is where t is a number between 0 to 1. And for example, we can take t is equal to 1 by 1000 or let us take does not matter you can take 1 by 1000.

But let us take here 1 by 365 just for the sake of example what does it mean that if it is possible if it is, if you can conceive 4 comma 3 then you should also be able to conceive

4 by 365 and 3 by 365 bundle. If this is 4 comma 3 4 units of food and 3 units of cloth is conceivable then 4 divided by 365 comma 3 divided by 365 is also a conceivable bundle. Why you know does it make sense, let us take cloth you cannot consume half of a cloth let us say if this cloth the that we have on y axis it is number of shirts that you have or number of you know earlier if we are talking about primitive economy we they have piece of cloth that they would use to put on their soldiers or they were that they would wrap around their waist. So, in that sense it does not make sense to talk about half of that cloth.

So, why we are talking, forget about half here I am saying you can take even one 365 or 1 by 1000 or 1 by 24 or 1 by 10 any fraction you can take and that is also a possibility how can we do that. So, see here basically one justification one can give that we are taking here 365 deliberately I took thinking that 365 is days in a year although this year we have 366 days. So, it does not matter you can change this to 366.

So, let us say you consume these food and cloth just you know once or the 4 unit or 3 unit you consumed in the whole year now you are talking about for a day. So, on average you consumed 4 by 365 comma 3 by 365. So, we are taking average for a day these 4 and 3 are unit it can be in thousand it can be in 100s does not matter. So, you can in this fashion you can achieve all the fraction. So, just it depends how we define our problem. So, sometimes its good idea to tinker with the problem you can you express the problem in some other word the problem will not change, but the solution will become much simpler.

So, that is why we are making this sort of assumption. So, it would help us to solve consumption problem later on in much more easier manner. So, one justification you just take the average over a longer period and you will get number in fraction. So, and this is allowed we say this property is called divisibility, divisibility.

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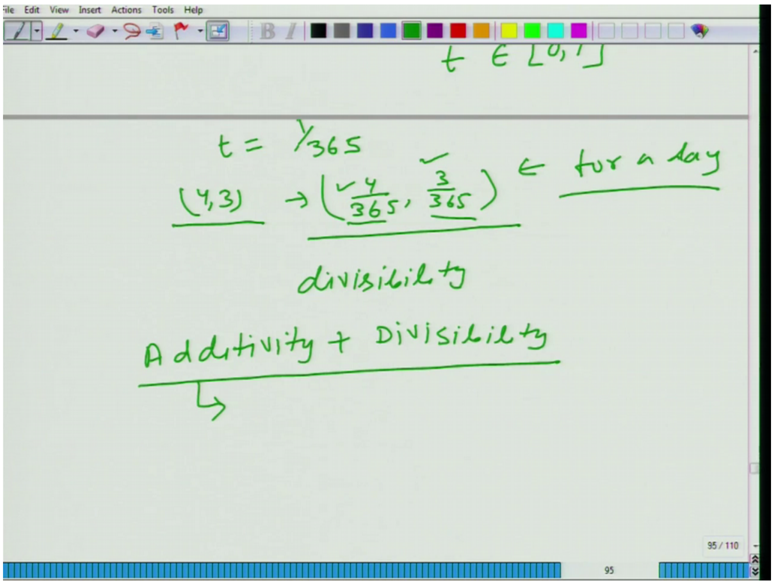

Now, what I am going to do I am going to combine this additivity with divisibility. Additively, additivity means that we can if we can combine two independent bundles then we should be able to conceive that conceived them jointly also and divisibility is just if we can we are able to think about a bundle then any fraction of that bundle can also be thought about fine that is what we are saying.

So, if we add these two what we get is convexity and that is a very nice and important property as we progress in this course you will see this word convexity again and again. So, it is very important that you understand what does convexity mean not just in the context of consumption theory, but in general you should understand what does convexity mean.

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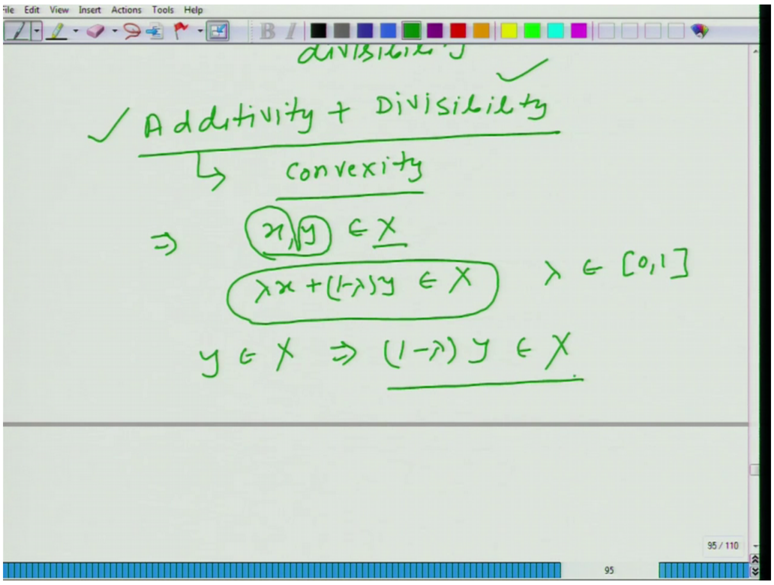

So, what convexity is let me give you the definition and then I will give you the example. Convexity is very simple if we take two bundles let us say x and y in the consumption set we are picking any two bundles in the consumption set, and what we are saying that if x and y are in the x means if these two individual bundles belong to this consumption set then this particular combination should also belong to the consumption set and where lambda is anything between 0 and 1 this is the convexity property.

We can prove it very easily because this is not a new thing we are using these two additivity and divisibility. Let us see how can we prove it very simple. Y is in the consumption set fine, so by definition one minus lambda y will also be in the consumption set if we are able to think about y we should be able to think about any fraction of y and this is one of those fractions.

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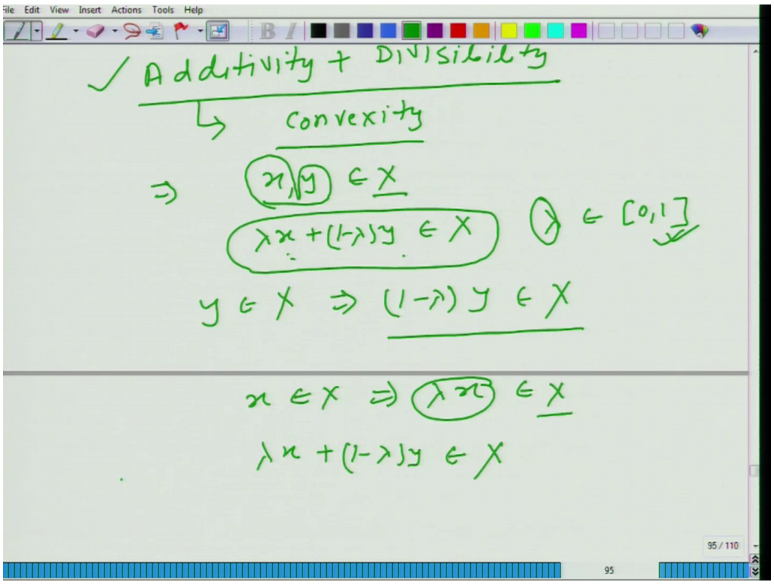

So, this is in the consumption set similarly if we are able to think about the consumption bundle x then we should also be able to think about any fraction of x and now forget about x and y what we know that lambda x conceivable bundle. So, it belongs to x and 1 minus lambda y is another of conceivable bundle. So, it belongs to the consumption set.

So, using the additivity property lambda x plus 1 minus lambda y also belong to x and since what we said 1 minus lambda and lambda should be fractions. So, they would be fraction only if lambda is between 0 and 1 that is why we are putting this restriction. So, if x and y are two consumption bundles then any linear, this is linear combination x and y are combined in linear fashion.

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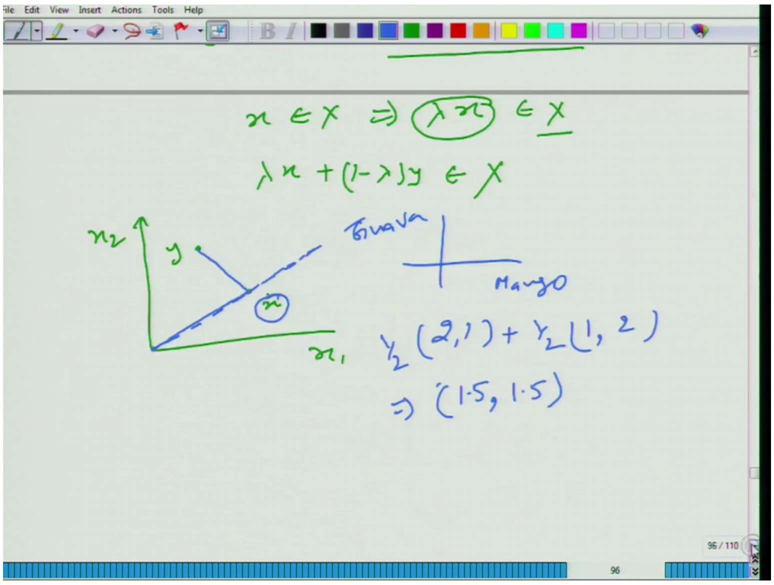

In the graph let us look at in the graph here we have x 2 here we have x 1 and we are taking two bundle x, x and here is we have y.

So, what this says that whenever we take any two bundle and we draw a line let me use a different color. So, all the bundles lying on this line should also belong to this consumption set. So, in other word if we can think about let us say for let us I am just do not want I am not very particle about food and cloth, let us say now I am talking about just two good world and one good is mango and another good is guava fine. Let us say 2 comma 1 means 2 mango and 1 guava and another bundle is 1 mango and 2 guava just two I am picking.

So, it also means that lambda can be anything between 0. So, let us take half and half it also means that 1.5 mango and 1.5 guava, a bundle containing 1.5 mango and 1.5 guava also belongs to this consumption set fine. What does divisibility means? That you draw you take from origin you connect this bundle and you can extend it also further. So, if x belongs to if this x small x belongs to the consumption set capital X then all the consumption bundle that lie on a line connecting the original consumption bundle to origin would also be would also belong to the same consumption set and why did I extend because I can add x with x, I can add x with a fraction of x. So, all the bundles on this line starting from origin they belong to this consumption set fine its clear is it very abstract for you guys, its fine it is not very abstract its clear.

So, now let us focus just for time being not just on consumption set because we are interested in convexity what is the property of convexity. So, let us take any shape. Let us say this is a set, I am talking about, this can be a consumption set or this may not be a consumption set.

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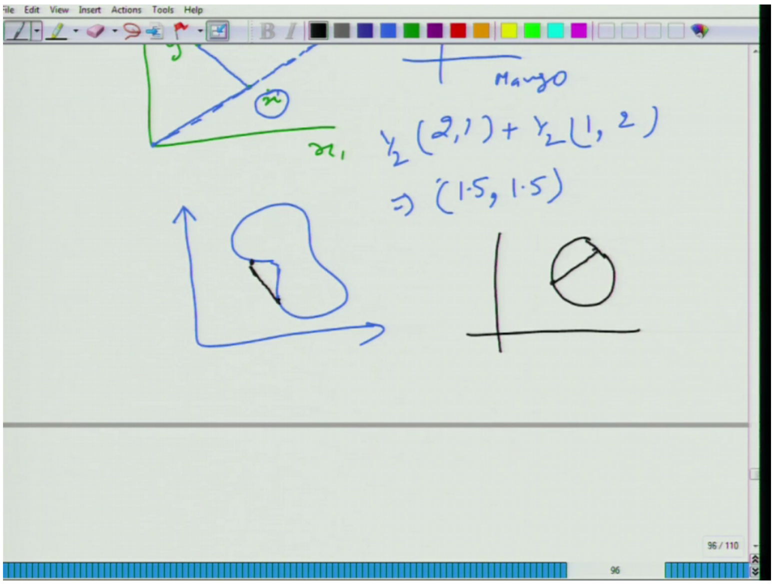

Do you think this set is a convex set? It is not convex because let us take an element here and an element here the two elements that I have selected here and here they belong to this particular set. But their linear combination and what is the linear combination if we draw a line through them. Here most of the bundles on this line do not belong to the same set that is why this set is not a convex set.

Convex set means that any two points we pick and we connect them using a line, straight line then all the elements on that line should belong to the same set then the set is convex or set exhibits the property of convexity. For example, when we take a perfect circle although it is not very perfect drawing, but you understand what I am trying to say, any point you take here, here and you connect them all the element on this line or this line completely is in the circle. So, circle is convex fine.

So, now, we have learned about some mathematical property of consumption set and those properties are that first that its non negative orthant, second additivity, third is divisibility and additivity and divisibility they imply convexity.

Now, I think we have enough information. Why I talked about consumption set because preferences are defined over consumption set, it is defined on consumption set. So, that is why I talked about consumption set.