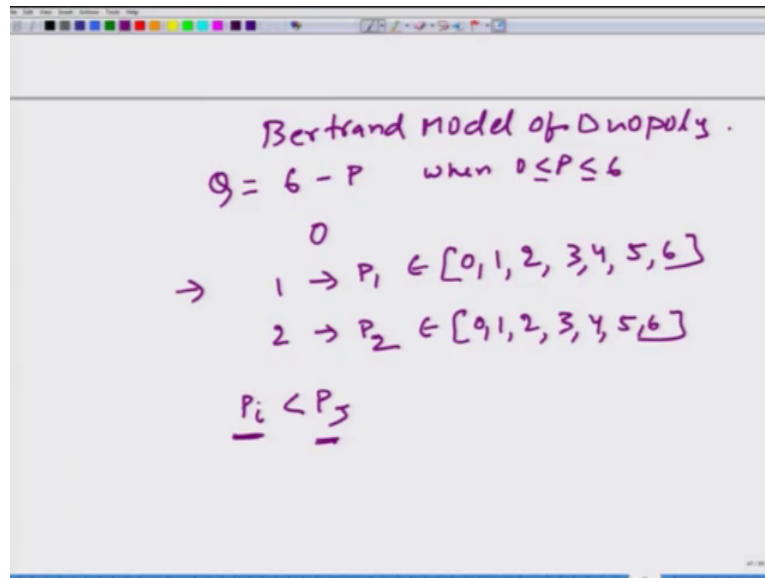


An Introduction to Microeconomics
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Lecture – 140
Bertrand Model of Duopoly

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Bertrand Model of Duopoly .
 $Q = 6 - P$ when $0 \leq P \leq 6$
 0
 $\rightarrow 1 \rightarrow P_1 \in [0, 1, 2, 3, 4, 5, 6]$
 $2 \rightarrow P_2 \in [0, 1, 2, 3, 4, 5, 6]$
 $\underline{P_i} < \underline{P_j}$

Let us talk about Bertrand model of duopoly. And let us take here a simple linear market demand function. And earlier whenever I said market demand function, I wrote inverse demand function, but for this particular problem we are going to write the demand function. So, what is the demand function? It is q is equal to 6 minus P , when P is between 0 and 6. Otherwise, the quantity demanded is equal to 0. We have 2 firms firm 1 and firm 2, both the firms they decide their price.

So, firm 1 decides P_1 and firm 2 decides P_2 , P_1 has to be we have already done for the continuous cases, in Cournot model of duopoly and Stackelman, a Stackelberg model of duopoly. In this particular case, we will limit our self to only integer prices so that we get a table rather than we have to use a differentiation like we did earlier. So, we have here 0 1 2 3 4 and 5 and 6. This is what we would do, and here also P_2 has to be one of these numbers.

Now, the thing is, if p_i is less than p_j of course, i is either one or 2 or j is 1 or 2 so, let us say here P_1 and here we have P_2 . So, if P_1 is less than P_2 everyone in the economy is

going to buy it from firm 1, ok. Also, we have to understand what happens when P 1 is equal to P 2, when both the firms have the same price. What we assumed here in this particular case that they would divide the market evenly.

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	0	1	2	3	4	5	6
0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
1	0,0	2.5, 2.5	5,0	5,0	5,0	5,0	5,0
2	0,0	0,5	4,4	8,0	8,0	8,0	8,0
3	0,0	0,5	0,8	7.5, 7.5	3,0	3,0	3,0
4	0,0	0,5	0,8	0,3	4,4	8,0	8,0
5	0,0	0,5	0,8	0,3	0,8	2.5, 2.5	5,0
6	0,0	0,5	0,8	0,3	0,8	0,5	0,0

So, I have already done the table here, the table would give us the total revenue for the firm. We will also make another simplifying assumption that the cost is equal to 0.

So, what the firms are interested in is maximizing revenue. So, what we have here is firm 1. And on this side, we have firm 2. Firm 1 can have either price 0, 1, 2, 3, 4, 5, 6. Similarly, firm 2 can have price either as 0 or 1 or 2 or 3 or 4 or 5 or 6. So, if both of them have price is equal to 0, what would be the market demand? The market demand is going to be equal to 6 minus 0, means 6 unit would be sold in the market, ok. So, 6 minus 0 let me write here 6 minus 0 is 6. And they would evenly split the market what it means, that firm 1 will be able to sell 3 units, and firm 2 will be able to sell 3 units. What would be the total revenue for the firm 1? And remember because we have assumed that there is no cost involved.

So, firms are interested in maximizing their total revenue. Anyway, profit is equal to total revenue minus total cost, total cost happens to be equal to 0. So, profit is equal to total revenue so, firms again let me repeat firms are interested in maximizing their total revenue. So, they both sell 3 units, but the what would be the market price? Market price is 0, it means their total revenue is going to be equal to 0. Both the firms and the first

number indicates the total revenue for the firm 1, and the second number indicates the total revenue for the firm 2, both the firms make 0.

Now, let us move to this particular box. It looks that we have 49 boxes, but do not worry it is not that difficult to fill in. So, we will do for few boxes, and then you would see that how quickly we can fill it up. So, what is happening here in this box, firm 1 has price 0 and firm 2 has price 1. It means firm 1 has the lower price so, everyone will go to the firm 1. What would be the market price? Because firm 1 has the lower price the market price is again going to be equal to 0.

So, no matter what, how many units firm sells it is total revenue is going to be equal to 0. And firm 2 is not able to sell any unit. So, again the total revenue of firm 2 is going to be equal to 0. So, all these boxes we will have 0 comma 0 as the total revenue. And same is the case here in the this particular column, because in this in these boxes what is going to happen? That firm 2 would have the lower price which is equal to 0. So, market price is again going to be equal to 0, and thus no one will make any profit.

So, this is the way we get it. Now how about in this box? Because now the price that they have both have one. So, market price is going to be equal to 1, and they will sell 6 minus 1 5 units. Out of 5 units, half would go to firm 1 and half would go to firm 2. So, each of them would sell 2.5 units. When the price is 1, and they sell 2.5 units, how much is going to be the total revenue? 2.5 multiplied by 1, they both will have 2.5 and 2.5.

Now, what is going to happen in these boxes? In these boxes firm 1 would have higher price than firm 2. So, firm 2 would capture the whole market, and firm 2 is selling it is product at 1. So, how many units again firm 2 would be able to sell? Coming back to formula that quantity sold is 6 minus P. So, P happens to be 1 so, firm 2 will sell 5 units. And 5 units at the price of 1, it means total revenue is going to be 5 for firm 2. And of course, firm 1 would not be able to sell anything. So, it is going to be 0 comma 5, 0 comma 5, 0 comma 5, 0 comma 5, and here it is symmetric.

So, firm 1 is going to earn 5, and firm 2 is going to earn 0. Now we come to this box. Now the price is 2 so, the 4 units would be sold, total revenue is going to be 4 multiplied by 2, which is 8, they would both split it evenly so, they both are going to make 4 comma 4. And in these boxes firm 2 would capture the whole market, and get the total

profit. So, 0 comma 8, 0 comma 8, 0 comma 8, 0 comma 8 and here 8 comma 0, 8 comma 0, 8 comma 0, 8 comma 0.

What is going to happen in this box? They both proposed price as 3. So, market price is going to be 3, how many units they would sell? 3 units in total so, total revenue is going to be 3 multiplied by 3 9, and they both would split it evenly. So, 4.5 comma 4.5 and here the whole market is captured by firm 2, and here all the market is captured by firm 1 so, this is the way we keep on filling. What is going to happen in this box? They would sell 2 units, why? Because they both have the price of 4. So, minimum of 4 is 4, market price is 4. So, their sell 2 unit, 1 by firm 1 and 1 by firm 2.

So, again they make 4 multiplied by one that would be the total revenue for one firm. So, 4 comma 4, what is happening in these 2 boxes? Firm 1 is proposing 5, firm 2 is proposing 4, it means the market price is going to be equal to 4, firm 2 would be able to sell 2 units according to the formula. So, the revenue the firm 2 will have 8, and firm 1 will have 0. So, 0 comma 8, 8 comma 0, 8 comma 0. Now what happens in this box? This box again they have their minimum price is equal to 5, they would be able to sell one unit, they would split it half and half, they both would sell half unit.

Now, what it means that half multiplied by 5 would be revenue for one from 2.5 comma 2.5. And here it is going to be 0 comma 5 and 5 comma 0, and using the similar logic here we are going to get 0 comma 0. Now we have already learned how to obtain the Nash equilibrium. We see that if firm 1 believes that firm 2 is producing z going charging price 0 then what should it do no matter what it should do what it does it would always have 0 profit. So, all are the best responses and let me use a different color to indicate that all are the best responses.

If firm 1 thinks that firm 2 is going to price it is product at 1, what is the best response? It should also price it is product at 1, because if it prices it is product at 0, the total revenue is going to be equal to 0, if it prices it is product and higher than 1, then again profit is going to be equal to 0. So, the best response is 1. And this process we continue, here the best response is again one. Here the best response is 2, here the best response is 3. What I mean here? That when firm thinks firm 1 things that firm 2 is going to price it is product at 4, the best response is 3, because 3 gives the 9 unit of total revenue all other pricing strategy they gave lower total revenue.

So, the best is 3, and here again we see the best is 3. And here again the best is 3. And similarly, we can do for the firm 2. What we get here, that if firm 2 thinks that firm 1 is going to produce at 0, all pricing strategies are the best responses, why? Because no matter what firm 2 selects, the total revenue is going to be equal to 0. And here the best strategy is one, if firm 2 thinks that firm 1 is going to price it is product at 2, then the best response is 2. And if firm 2 thinks that firm 1 is going to price it is product at 3, the best response is 3.

And if firm 2 thinks that firm 1 is going to price it is product at 4, the best response is again 3. Let us see this is the 3, and it keeps on at 3. So, what we see here basically in this box, they are playing the best responses of each other. Only if the economy is in this box, then only there is an equilibrium. Otherwise, one of the firm would have an incentive to deviate. So, they both would produce their 2.5 units, and they both would propose to sell it at 1, this is what the Bertrand market of duopoly is.

What happens in the Bertrand market of duopoly? That we see that if we had done for the continuous case what we would have found that the price becomes lower and lower and it reaches to the marginal cost. Think about the continuous case, ok, just for a moment let us look at the continuous case here.

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2	0,0	0,5	0,8	1,5; 1,5	3,0	3,0	3,0
3	0,0	0,5	0,8	0,9	4,4	8,0	8,0
4	0,0	0,5	0,8	0,9	0,8	2,5; 2,5	5,0
Σ	0,0	0,5	0,8	0,9	0,8	1,5	0,0
6,	0,0	0,5	0,8	0,9	0,8	1,5	0,0

$q = a - P$
 $P = P - \epsilon$
 $P_1 = P_2$

What we have the quantity is given by this particular formula, ok. How we decide the P? If we take the lower of the 2 values, if firm 1 proposes P_1 and firm 2 proposes P_2 , we

have to see which one is the lower. And this lower will be taken here because the firm which has the lower price would capture the whole market. And if P_1 is equal to P_2 , then we will use the same, and we would say both the firms would capture half of the market ok.

Now, what happens if a firm proposes price P , what the other firms would do? Other firms would charge slightly lower price. And by charging slightly lower price it would capture the whole market. Rather than sharing the profit or sharing the total revenue with the other firm it would be able to capture the market. So, both the firms would have this incentive to decrease their price. So, they would keep on decreasing their price till they hit the marginal cost barrier, they could not go below the marginal cost because if they do so, they would making loss, that the concept we learned that firm should be able to recover it is marginal cost at least then only it would produce that particular unit.

So, they cannot go below the marginal cost. So, both the firms would propose the price equal to marginal cost. What would be interesting to see is what happens when the firms have different marginal cost. Also, what happens when the firms have different level of capacity what if they cannot cater to the whole market. These are the larger questions for that you will have to get more into game theory. And this is it further, but this is it for the Bertrand model of duopoly.

Thank you.