

An Introduction to Microeconomics
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Lecture – 139
Stackelberg Model of Duopoly

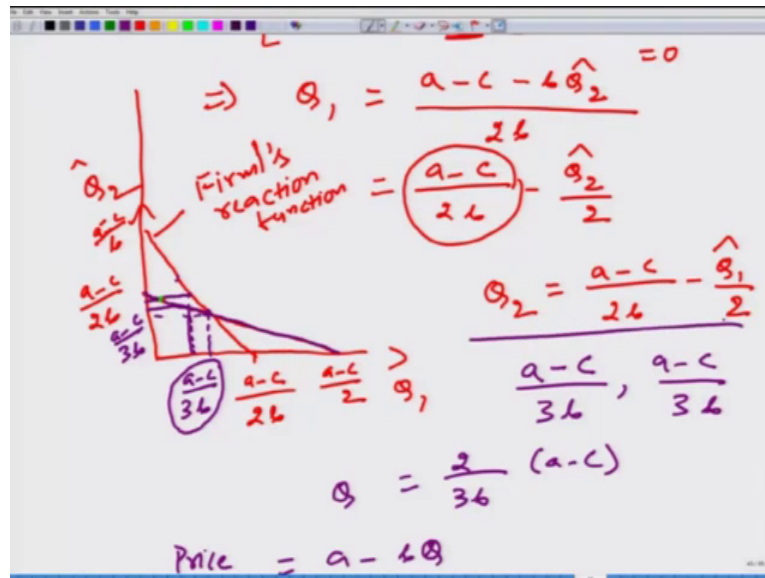
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The image shows a whiteboard with handwritten mathematical equations for the Stackelberg Model of Duopoly. The title "Stackelberg Model of Duopoly" is written at the top. Below it, the reaction function for firm 2 is given as $Q_2 = \frac{a-c}{2b} - \frac{Q_1}{2}$. The profit function for firm 1 is written as $\max_{Q_1} \left[a - b \left(Q_1 + \left(\frac{a-c}{2b} - \frac{Q_1}{2} \right) \right) \right] Q_1 - cQ_1$.

So, let us talk about Stackelberg, Stackelberg Model of Duopoly the setting is very similar to the Cournot game. The only difference is that the 2 firms do not make their quantity decision simultaneously. Here we can say that the firm 1 is a leader, what it means that; firm 1 decides first firm 2 observes what firm 1 has decided and then firm 2 makes its production decision. So, firm 1 is the leader and firm 2 is the follower.

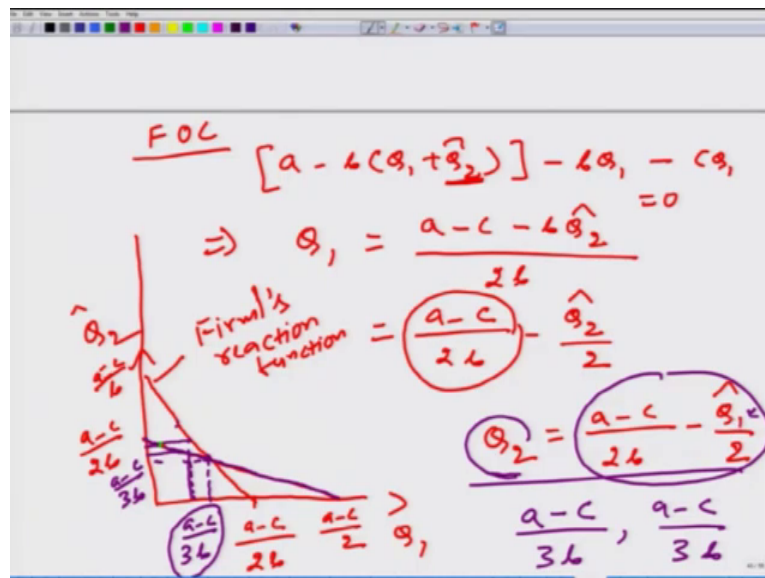
So, the optimization problem that we did earlier in the Cournot game this is exactly the same for the firm 2.

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For firm 1 we will see the change. So, what we have obtained because firm 2, here what we have done? Here firm 2 in the setting Cournot setting.

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What we have done? That firm 2 thinks that firm 1 is going to produce Q_1 and this formula gives the optimal level of production for firm 2. The only difference here is that it is not firm 2 believe anymore firm 2 knows for certain that firm 1 is going to produce Q_1 amount because firm 2 has observed how much firm 1 has produced.

So, Q_2 is going to be a function of a minus c divided by $2b$ minus let us go back Q_1 by 2. And there is no hat here because firm 1 firm 2, sorry firm 2 knows how much firm 1 has produced. Now let us think about the optimization problem or the profit maximization problem of firm 1. Firm 1 when firm 1 makes its production decision it does not exactly know what firm 2 is going to produce, but firm 1 knows that firm 2 is profit maximize. And firm 2 is going to observe how much it has produced Q_1 , and firm 2 is going to take it into account and would produce according to this formula.

This firm 1 is aware of. So, firm 1 rather than thinking that how much Q_2 would be in the market firm 1 would know that if it is equal to Q_1 Q_2 would be given by this particular equation. So, the firm 1's profit maximization problem would be maximized with respect to Q_1 a minus bQ_1 plus Q_2 and Q_2 happens to be here a minus c divided by $2b$ minus Q_1 by 2 this is what and here this is the price in the market multiplied by Q_1 minus cQ_1 . And this is the optimization problem in for the market leader in the Stackelberg problem.

Remember in Cournot problem here we could not do this because firm 1 didn't know what firm 2 is going to produce, but here even though firm 2 is going to move after firm 1 has decided, firm 1 knows that this is the formula firm 2 would be using how much to produce, just let me check that it is correct ok.

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Handwritten mathematical derivation on a whiteboard:

$$Q_1 \left[a - b \left(Q_1 + \frac{a-c}{2b} - \frac{Q_2}{2} \right) \right] - cQ_1$$

$$\Rightarrow \left[a - b \left(Q_1 + \frac{a-c}{2b} - \frac{Q_2}{2} \right) \right] - b \left(1 - \frac{1}{2} \right) Q_1 - c = 0$$

$$\Rightarrow Q_1 = \frac{a-c}{2b} \quad Q_1 + Q_2 = 3 \left(\frac{a-c}{4b} \right)$$

$$Q_2 = \frac{a-c}{4b}$$

$$Q_{\text{Cournot}} = \frac{2}{3b} (a-c)$$

$P_{\text{Stackelberg}} < P_{\text{Cournot}}$

So, if we solve this problem what would be the profit maximizing condition, $a - bQ_1 + a - c - 2bQ_1 - bQ_1 - \frac{1}{2}c$ multiplied by $Q_1 - c$ has to be equal to. So, when you solve it from here you will get the amount of Q_1 which will come out to be $\frac{a - c}{2b}$.

And when you plug it in the formula what you are going to get is Q_2 is equal to $\frac{a - c}{4b}$. So, how much is going to be $Q_1 + Q_2$? It is going to be equal to $\frac{3}{4b}$ multiplied by $a - c$. So, what happens to the total level of production? The total level of production is more than the Cournot because we see in the Cournot let me write Q_{Cournot} is $\frac{2}{3b} \frac{a - c}{2}$. So, the level of production goes up from the Cournot in a Stackelberg setting. So, when the total output is more in the market the price in the Stackelberg is going to be less than price in the Cournot market. So, that is it about Stackelberg competition.

Thank you.