

An Introduction to Microeconomics
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Lecture – 134
Monopoly: Example

So, let us take a look at an example in which we have a firm which operates into two different markets.

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Handwritten notes on a whiteboard:

- (H) → Home → the monopolist
- (W) World → Perfectly competitive market
- ⇒ $P_H = 120 - \frac{Q_H}{10}$
- $P_W = 80$
- MC(Q) = $50 + \frac{Q}{10}$
- $C(Q) = 50Q + \frac{Q^2}{5}$

So, let us say the market 1 market is home market and in this market this form acts as the monopolist. The same form is also able to sell its product in the world market and we can assume that there are many forms supplying their product in the world market. So, there it is the market environment is perfectly competitive market.

So, let us say that the demand in home let us say home in short we will denote it by H and world we will denote it by W. So, at home the market demand function is given by P_H which is price at home 120 minus Q_H by 10 as long as this Q_H happens to be less than 1200 otherwise the price would be 0. And in the market world market because it is perfectly competitive let us say the market price is 80.

So, this form is monopolist in home market and one of the many forms in the world market. And let us say that the cost is let us say that the marginal cost to produce Q unit

of output is 50 plus Q by 10 or we can say that cost is 50 Q plus Q square by 5 this should give us this marginal cost, ok. So, it is very very clear that there is no fixed cost in this particular problem oh sorry, this should be this should have been 20. Now, it is correct ok.

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Handwritten mathematical equations on a whiteboard:

$$\Rightarrow P_H = 120 - \frac{Q_H}{10}$$

$$P_W = 80$$

$$MC(Q) = 50 + \frac{Q}{10}$$

$$C(Q) = 50Q + \frac{Q^2}{20}$$

$$\pi = TR - TC \quad Q = Q_H + Q_W$$

$$= TR_H + TR_W - TC$$

$$= P_H Q_H + P_W Q_W - C(Q_H + Q_W)$$

Now, what we need to do is to do the profit maximization for this firm. So, what would be the profit maximization profit? First it is going to equal to total revenue minus total cost, but total revenue is coming from two different markets, one is in the home market and another from the world market. A total cost let us say the production takes place in the same factory ok.

So, how much is the total revenue at home? Of course, the unit that it would be able to sell would depend on what is the price of its product in the market. So, it is going to be P_H multiplied by Q_H , here we are going to have P_W multiplied by Q_W and minus C of Q which happens to be $Q_H + Q_W$, so Q_H plus Q_W .

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The image shows a whiteboard with handwritten mathematical equations. The top equation is $= P_H Q_H + P_W Q_W - C(Q_H + Q_W)$. The second equation is $= (120 - \frac{Q_H}{10}) Q_H + 80 Q_W - C(Q_H + Q_W)$, where $80 Q_W$ is circled. Below this is the text "Profit maximizing condition" followed by $\frac{MR_H}{Q_H} - MC(Q) = 0$. A horizontal line separates this from the next part, which shows $MR_W(Q_W) - MC(Q) = 0$ and $\Rightarrow MR_H(Q_H) = MR_W(Q_W) = MC(Q_H + Q_W)$.

So, let us proceed let us write it. What is P_H ? P_H here is function of quantity. So, this is 120 minus Q_H divided by 10 multiplied by Q_H . P_W is fixed because the world market all the forms are able to sell their product at price 80. So, $80 Q_W$ minus let us keep it as $C Q_H$ plus Q_W .

Now, this firm has to select the amount of Q_H as well as of Q_W when the firm selects Q_H and Q_W gets automatically determined. So, what would be the first order condition? First to obtain the first order condition this profit has to be differentiated with respect to Q_H . So, what we get is or in other word what we get if we do that the profit maximizing condition is going to be is going to be marginal revenue in market H because when we differentiate total revenue in market H with respect to H what do we get? We get marginal revenue. It simply means that extra revenue that this firm gets when it produces one more unit.

And this does not depend on Q_H . So, when we differentiate this term with respect to Q_H we get 0 then we have to differentiate this C with respect to Q_H , but notice C is a function of Q_H plus Q_W . So, what we can do? We can differentiate this C with respect to Q and what do we get? We get marginal cost this we have obtained at Q_H , but marginal cost we are obtaining at Q . Why? Because when Q_H increases by 1 unit while keeping the Q_W fixed Q_H plus Q_W also increases by 1 unit. So, if we differentiate. So, what we are doing if you are familiar with calculus first we are differentiating C with

respect to Q and then again Q with respect to Q_H because Q is a simple linear function of Q_H what we get here is just one and this should be equal to 0.

We will try to explain this again using without using any calculus. And similarly what we get is MR_W at Q_W has to be equal to MC of Q it has to be equal to 0 and so therefore, we get MR_H at Q_H has to be equal to MR_W at Q_W which has to be equal to MC at Q_H plus Q_W . So, as we had explained earlier when we had the theoretical construct MR is equal to MC we have explained many times why we are getting MR_H is equal to MR_W the reason is simple that this firm can without changing the Q this firm can increase the Q_H by decreasing the Q_W , ok.

So, if let us say MR_H at Q_H happens to be more than the marginal revenue in Q world, what this firm should do? It should increase the Q_H because this would lead to an increase in the profit ok. So, in the equilibrium at the optimal level these two have to be equal, ok.

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The image shows a whiteboard with the following handwritten equations and steps:

$$\Rightarrow \textcircled{MR_H(Q_H)} = MR_W(Q_W) = MC(Q_H + Q_W)$$

$$\propto MR_H(Q_H) = 120 - \frac{2Q_H}{10} \quad \begin{cases} P = 9 - 6Q \\ MR = 9 - 24Q \end{cases}$$

$$MR_W(Q_W) = 80$$

$$\propto MC(Q) = 50 + \frac{Q}{10} = 50 + \frac{Q_H + Q_W}{10}$$

$$\Rightarrow 120 - \frac{2Q_H}{10} = 50 + \frac{Q_H + Q_W}{10} \quad \text{--- (1)}$$

$$80 = 50 + \frac{Q_H + Q_W}{10} \quad \text{--- (2)}$$

An arrow points from the 80 in equation (2) to the 50 in equation (1). Below equation (2), it is written:

$$30 \times 10 = 300 = Q_H + Q_W = Q$$

So, let us calculate how much is MR_H , this is going to be 120 minus two Q_H by W by now we are familiar that when we start with demand function a minus b Q then the MR comes out to be a minus $2b$ Q . So, this is the formula I have used.

What is going to be the marginal revenue in the world market? At all level it is fixed and it is fixed at 80, because it does not matter how many units this firm sells the market

price remains at equal to 80. How about the marginal cost? Marginal cost at Q is already given which is 50 plus Q by 10 and if we write Q is equal to Q H plus Q W this is the formulation we get. So, what we have? We have 3 unknowns, 2 unknowns Q H and Q W, and we have two formulation we can use two these two formula to obtain the Q H and Q W. What are those two formula? In the first setting that is the home setting MR marginal revenue at Q H is equal to marginal revenue at Q. So, this is the first formula that we are going to use. Here this has to be equal to this. So, 120 minus 2 Q H divided by 10 has to be equal to 50 plus Q H plus Q W is equal to 10 this is one.

And in the second in the world setting the MR marginal revenue at Q W which turns out to be 80 has to be equal to the same marginal cost 50 plus Q H plus Q W by 10. So, from second equation we can obtain the Q H plus Q W, ok. And how much do we get? If we rearrange this we take this 50 on the left hand side we get 30, and if we multiply it by 10 on both the side we get 300 on the left hand side which is equal to Q H plus Q W which is equal to Q. So, this form will produce a total of 300 units of output.

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The image shows a whiteboard with handwritten mathematical work. At the top, there is an equation: $80 = 50 + \frac{Q_H + Q_W}{10}$ with a circled 2 next to it. Below this, the total quantity is determined: $30 \times 10 = 300 = Q_H + Q_W = Q$. The marginal cost function is given as $MC(Q) = 80$. Then, the marginal revenue for home is set equal to the marginal cost: $120 - \frac{2Q_H}{10} = 80$, which leads to $Q_H = 200$. Since $Q_W = 100$, the price in home is calculated as $P_H = 120 - \frac{200}{10} = 100$ and the price in the world is $P_W = 80$. The final price equations are circled in green.

So, at this level of production how much is the marginal cost? We can plug it back the value here or we do not even need to plug it this we have already calculated when we put Q is equal to 300 we are going to get 80. So, this RHS is equal to 80 in the equation number 1. So, we can write rewrite equation number 1 as 120 minus 2 Q H divided by 10

has to be equal to 80, ok. So, it means that Q_H has to be equal to 200 and this also gives us the Q_W has to be equal to 100 why because Q_H plus Q_W is 300.

Now, we can calculate the P_H P_W we already know it is 80. How much is the P_H ? We can plug the value of Q_H in the demand function, and how much is the demand function 120 minus Q_H by 10 here Q_H is 200 , so by 10 . This comes out to be 100 . So, remember that we had derived that the market which has more elastic demand curve for the firm will have the lower price this is what we get P_W is 80 while P_H is 100 , ok. So, this is just an example numerical example to know how to calculate the forms output and forms prices when the firm is operating in more than one market.

Thank you.