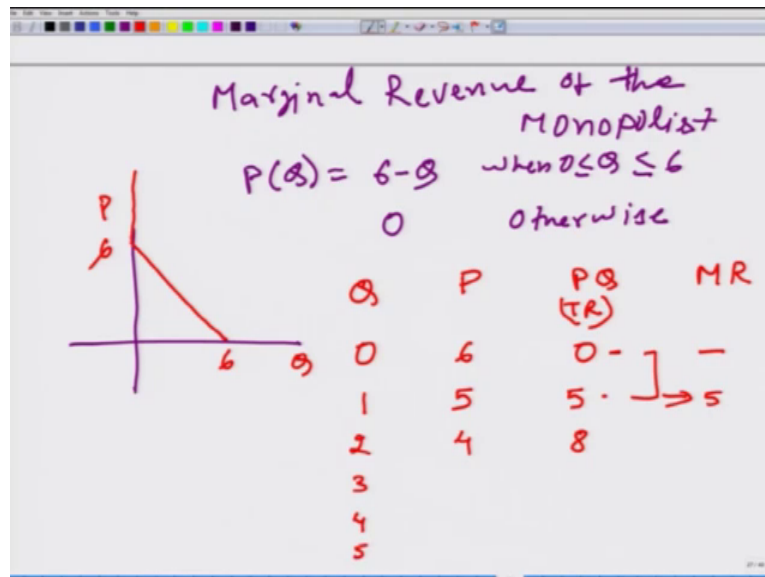


**An Introduction to Microeconomics**  
**Prof. Vimal Kumar**  
**Department of Economic Sciences**  
**Indian Institute of Technology, Kanpur**

**Lecture - 124**  
**Marginal Revenue of the Monopolist**

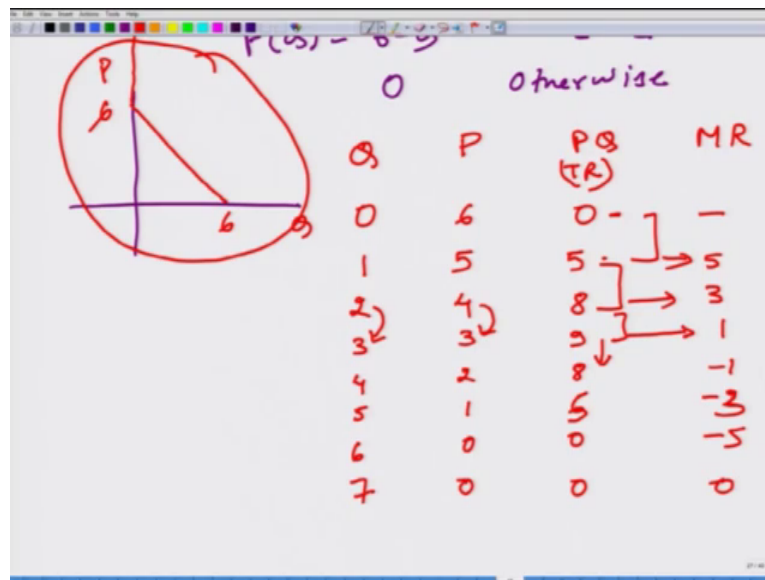
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Let us talk about the marginal revenue of the monopolist. And let us take a very simple example to begin with, what we have that inverse demand function or demand function colloquially speaking is given by 6 minus Q. When Q is less than or equal to 6 or greater than or equal to 0; otherwise 0; what it means that demand function looks like this. And here we have P, here we have Q, this is 6, this is 6, ok.

So, let us what should we do? We can write here Q we can write here P, we can write here P multiplied by Q, which is nothing but the total revenue, and then we can write the marginal revenue. Let us say when Q is 0, then this formula gives that 6 minus 0. So, P should be 6, and 6 multiplied by 0 is 0. So, total revenue is going to be equal to 0 and we cannot calculate the marginal revenue here. Let us say when Q changes from 0 to 1, P is now 6 minus 1. So, it has to be equal to 5. So, P multiplied by Q is going to be equal to 5, and the marginal revenue is going to equal to 5. So, here total revenue is 5, here it is 0. So, difference is 5 that is the marginal revenue, and 2 then 4 similarly here 8, and we will fill up the last column towards the end.

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So, 3, 4, 5, 6 and 7 let us do when here of course, it would decrease linearly. So, it is going to be 3. So, idea is very simple because if you take Q on the left-hand side so, you get P plus Q which is equal to 6. So, when Q is 3 P has to be 3 and the total revenue is 9, here 2 8 here one 6 here 0, 0 and 7 of course, that formula fails because price cannot be negative.

So, if Q is greater than 6, then P is 0 and it is 0. And now we can calculate the marginal revenue, marginal revenue is going to be equal to 3, because here the difference is 3, then marginal revenue is going to be equal to 1, then this is minus 1 now marginal revenue has come down from 9 to 8, and then minus 2, and then oh sorry this has to be 5 not 6. And so, here marginal revenue is minus 3, here minus 5 and then 0 and so on.

What is important here to notice and of course, our reference point is perfectly competitive market. What happens in the perfectly competitive market? No matter how many units, this firm supplies it always supplies at P star which is the equilib market price. So, it takes market price as a given, because it is very, very simple that this firm is very, very small in comparison to whole market.

So, it cannot influence the prices in the market. As opposed to a firm in the perfectly competitive market, if we pay attention to the monopolist, it is the only firm operating in the market. So, if you wants to sell, more units let us say from 2 to 3 unit, then it has to decrease the price from 4 to 3, otherwise consumers would not buy these many units.

So, it is very, very clear from this curve, which is for the market, and as there is only one firm in the market this is for also the monopolist. So, if quantity if firm is interested in, selling more quantities it has to decrease the price, ok. Or we can see it other way also that if firm sets a very, very high price, then it is able to sell very small number of output in the market. And if firm sets a very low price it would be able to sell large number of outputs in the market ok. So, this thing this thing we can do it mathematically also.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, there are some numbers: 7, 0, 0, 0. Below that, the price function is given as  $P = \frac{a - bQ}{b}$  for  $0 \leq Q \leq \frac{a}{b}$  and  $a > 0, b > 0$ . For  $Q > \frac{a}{b}$ , the price is 0, labeled as 'otherwise'. The total revenue function is derived as  $TR = PQ = (a - bQ)Q = aQ - bQ^2$ . The marginal revenue function is then derived as  $MR(Q) = \frac{TR(Q+\Delta) - TR(Q)}{\Delta} = \frac{a(Q+\Delta) - b(Q+\Delta)^2 - aQ + bQ^2}{\Delta}$ .

And let us do it for a simple case, that P is equal to a minus b Q. The example that we just did a is equal to 6 and b is equal to 1, but here we are doing it for a general case where a is greater than 0 and b is also greater than 0. And this formula is only applicable if Q is between 0 to b by a.

Otherwise, P is going to be 0. So, if we want to calculate the total revenue, what it would be? P multiplied by Q, P happens to be a function of Q. So, we can write P as a minus b Q multiplied by Q, and we are writing it for only this range, because otherwise P becomes equal to 0, and if you multiply Q with 0 you get 0, but you should be careful about it that this formula works only in this particular range.

So, this is a Q minus b Q square. What we have here is that the total revenue for Q is equal to a Q minus b Q square. What is the marginal revenue? Marginal revenue is that how much is the change in the revenue if you produce one more unit. But why should we

talk about one more unit, we can also talk about small increase in that is delta, small increase in revenue when you produce delta amount more.

And this we can write so, what would be the marginal revenue? Marginal revenue at Q is going to be equal to total revenue at these 2-different level divided by delta, ok. And it comes out to be a b plus delta minus b, Q plus delta whole square minus a Q, oh sorry, this has to be, this has to be Q minus b Q square divided by delta.

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$$\begin{aligned}
 TR &= PQ = (a - bQ)Q = aQ - bQ^2 \\
 TR(Q) &= aQ - bQ^2 \\
 TR(Q+\Delta) &= a(Q+\Delta) - b(Q+\Delta)^2 \\
 MR(Q) &= \frac{TR(Q+\Delta) - TR(Q)}{\Delta} \\
 &= \frac{a(Q+\Delta) - b(Q+\Delta)^2 - aQ + bQ^2}{\Delta} \\
 &= \frac{a\Delta - b(Q^2 + 2Q\Delta + \Delta^2) + bQ^2}{\Delta} \\
 &= \frac{a - 2bQ - b\Delta}{1}
 \end{aligned}$$

So, this a Q will cancel here, what we get is a delta minus b Q square 2 Q delta plus delta square. And from here we will get minus Q square. So, this will get cancelled divided by delta. So, a minus 2 b Q minus b delta, this is what we kept. And if we take this delta to be very, very small if this delta goes to 0, we get a minus 2 b Q.

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$$\begin{aligned}
 &= \frac{a\Delta - b(Q^2 + 2Q\Delta + \Delta^2) - Q^2}{\Delta} \\
 &= \frac{a - 2bQ - b\Delta}{\Delta} \quad \Delta \rightarrow 0 \\
 MR(Q) &= a - 2bQ \\
 TR &= (a - bQ)Q = aQ - bQ^2 \\
 \frac{dTR}{dQ} &= a - 2bQ = \underline{a - 2bQ}
 \end{aligned}$$

So, the marginal revenue at Q is equal to a minus 2 b Q. This is just a long way of doing thing, if you are familiar with the calculus what you could have done? That you could have written that total revenue is a minus b Q multiplied by Q, and you could have differentiated this total revenue with respect to Q, and what would you have obtained this is here a Q minus b Q square.

So, if you differentiate b Q with respect to Q; you get a and when you differentiate b Q square with respect to Q you get 2 b Q, exactly the same amount you get, ok. So, why remember in the perfectly competitive market how much was the marginal revenue, you sell one more unit you earn the market price for that particular unit. So, it was equal to P whatever is the market price and it was not in control of the firm because P was determined outside the firm in the market.

While in the monopolist setting what is happening? The marginal revenue depends on this particular firm it is level of production. So, of course, here we the example that we have taken, is example that we have taken has a very simple linear demand function, one can take more complex example where P we will take P simply as a function of Q.

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$$TR = (a - bQ)Q = aQ - bQ^2$$

$$\frac{dTR}{dQ} = a - 2bQ = a - 2bQ$$

$$TR(Q) = P(Q) \cdot Q$$

$$\frac{dTR}{dQ} = Q \frac{dP}{dQ} + P(Q) \cdot 1$$

$$= P \left[ 1 + \frac{Q}{P} \frac{dP}{dQ} \right]$$

$$= P \left[ 1 + \epsilon \right]$$

Q ↑  
P ↓

So, here it is TR at Q is equal to P as a function of Q multiplied by Q, and we differentiate this total revenue with respect to Q, what we get we get? Here they are in the multiplicative form, first we will differentiate this keep as it is first we will differentiate this, and keep this part as it is when we differentiate this with respect to P Q, what we get Q d P by d Q. And then we keep this part, and differentiate this part with respect to Q we get 1, ok.

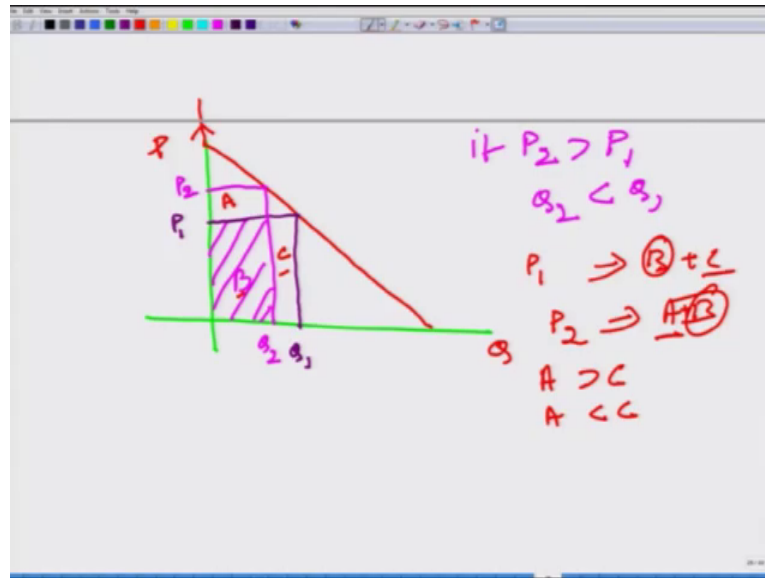
So, if we rewrite it, what we get if we take P out what we get is Q P dp by d Q. So, here, can you think 4 can you pause for a moment and think whether this is greater than P or less than P, ok. Let us look at it, what we have here, this is the P, let us P this is positive, but this particular entity is negative, if Q goes up P comes down, because remember demand is a downward sloping curve, ok. So, derivative of P with respect to Q has to be negative.

That makes this whole term negative, ok. So, depending on the value here, this term can turn out to be positive or negative, ok. So, there is a possibility that this whole term becomes negative, and it means the marginal revenue would be negative. As we have seen in this numerical example see here, the marginal revenue is minus 1 minus 3 minus 5. For a perfectly competitive market, marginal revenue would never be negative because firm would always earn the market price, ok.

So, as long as this product is selling for a positive price in the market, the firm perfectly competitive firm or in a word in what it is a short term that I am using to talk about a

firm operating in the perfectly competitive environment would always earn a positive profit, and never a negative marginal revenue. But here it turns out to be negative. Let us look at it graphically what is happening and then it would become clearer.

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So, what we have here is, let us take again simple linear demand, not because all the products have simple linear demand in the market, but it is easier to understand, ok. Here we have Q, here we have P, and let us say that let us say that earlier this is the market price P 1. Correspondingly, Q 1 units are sold; now let us say the market price goes up to P 2. As market price goes up to P 2, only Q 2 amounts are sold, if P 2 happens to be greater than P 1, then Q 2 would be less than Q 1, ok.

So, what happens what is the revenue, let us call this area B, this area can be called A, and this area can be called C. So, when price is P 1 what is the total revenue? B plus C. And when price is P 2, how much is the total revenue A plus B. So, this B is same in both the part.

What is happening? What is difference is C here and A here? If a happens to be greater than c, then of course, an increase in price would lead to an increase in revenue. But if a is less than c, then an increase in price would be accompanied by a decrease in revenue. And this clearly in the next part, you will see that this is related to the elastic price elasticity of demand.

Thank you.