

An Introduction to Microeconomics
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Lecture – 120
Profit Maximization and Returns to Scale

We are going to talk about profit maximization and its relationship with returns to scale.

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Profit Maximization & Returns to Scale

$$\max_{K,L} P f(K,L) - rK - wL$$

$$K^* = K^*(P, r, w)$$

$$L^* = L^*(P, r, w)$$

$$\pi^* = P f(K^*, L^*) - rK^* - wL^*$$

$$P f(2K^*, 2L^*) - r(2K^*) - w(2L^*)$$

$$= 2P f(K^*, L^*) - 2rK^* - 2wL^*$$

$$= 2 [P f(K^*, L^*) - rK^* - wL^*]$$

$$= 2\pi^*$$

So, profit maximization and returns to scale. So, in profit maximization firm tries to figure out an input combination which gives the maximum possible profit. And of course, when you determine the input combination the output is also determined because the firm is representative of that technology, which does nothing for the economist, but to take input and convert it into output.

So, maximize let us say with respect to the quantity the firm does that or with respect to inputs and we will continue with our canonical inputs which is capital and labor this is what firm does P it takes as given. And the technology which converts these 2 inputs into an output and then we have the return to the capital and wage to the labor. And let us say the input combination that maximizes this profit is k^* and k^* of course, is going to be a function of P , r and w . And the other part of other combination is L^* which is again a function of P , r and w , but we are going to use only k^* and L^* and we will of course, on the background we know that k^* and L^* are function of P , r and w ok.

So, basically let us say this maximize profit is π^* which is $P f(k^*, L^*) - r k^* - w L^*$, clearly this profit depends on only 3 parameters on these parameters are P , r and w . And it also depends on k^* and L^* , but k^* and L^* are in turn they depend on P , r and w ok. So, what happens? When we keep the prices it means the output price as well as input prices fixed and we double the amount of inputs, what would happen to the maximize profit?

So, this is what we are talking about what we are going to do P and f and instead of using k^* and L^* let us say we use $2k^*$ and $2L^*$. Of course, we use more of inputs we have to pay higher price to get those inputs. So, what would happen r will now be multiplied by $2k^*$? And w would be multiplied by $2L^*$ if we have constant return to scale we can take 2 out from here, and this is going to be $2 f(k^*, L^*) - 2r k^* - 2w L^*$. And we can take 2 out and this is what we are going to get $f(k^*, L^*) - r k^* - w L^*$. And notice this entity this entity is same. So, I missed P . So, we will have a P here too. So, this entity is same as this expression. So, this is clearly equal to $2\pi^*$.

So, what did we get that in case of constant return to scale if we double the combination of we if we double all the inputs and of course, not any input the inputs which or in combination maximizing the profit. So, given the prices we figure out the input combination which maximizes maximize the profit and then we double all those inputs, what is going to happen in constant return to scale? Our output will also get doubled, and then as the expression suggests profit will also get doubled what it means; that we were not maximizing the profit here because remember π^* we defined as the maximized profit means this firm given the prices P , r and w cannot obtain profit more than π^* that is what the maximize profit means.

Now let us look at the expression here at the same prices which is P , r and w now we are able to obtain $2\pi^*$, what it means; that we are able to double the profit. So, this is contradictory it only means one of these 2 things that; either we were not maximizing the profit here or this π^* is not a positive number or π^* is equal to 0, because if it is positive number only then only the profit is getting doubled, but if it is 0 then the profit is not getting double. So, from here can we say that in constant returns to scale maximize profit has to be equal to 0 because if it is not 0 you would replicate the process and you would keep on increasing the output and because it is increasing in the same proportion.

So, profit will also increase in the same proportion then the your earlier obtained profit would not be the maximize profit ok.

So, we have to be careful. Exactly same kind of expression we can obtain when we say instead of it is constant return to scale technology we have increasing returns to scale.

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$$\begin{aligned}
 & P_f(2\hat{K}, 2\hat{L}) - r2\hat{K} - w2\hat{L} \\
 &= 2P_f(\hat{K}, \hat{L}) - 2r\hat{K} - 2w\hat{L} \\
 &= 2 [P_f(\hat{K}, \hat{L}) - r\hat{K} - w\hat{L}] \\
 &= 2\pi^* \\
 \\
 & t^n P_f(\hat{K}, \hat{L}) - tr\hat{K} - tw\hat{L} \\
 &= t^n P_f(\hat{K}, \hat{L}) - tr\hat{K} - tw\hat{L} \\
 &\Rightarrow t P_f(\hat{K}, \hat{L}) - tr\hat{K} - tw\hat{L} \\
 &= t [P_f(\hat{K}, \hat{L}) - r\hat{K} - w\hat{L}] \\
 &= t\pi^* \quad \pi^* > 0
 \end{aligned}$$

And let us see here what we have is P and let us say y we should we talk about just doubling all the inputs, but taking it t times where t is greater than 1. So, what we have t k star t L star minus t r tk star minus w P L star and this is going to be equal to, let us say t to the power n where n is greater than 1, P f k star L star minus t r k star minus t w L star, which is definitely greater than t P f k star L star, why greater than? Because t and t to the power n would be greater than t, which is again we get back to the expression of the maximized profit. So, we started with a an increasing return to scale technology and we were able to increase profit by more than t times notice greater than sign here.

So, profit is increasing more than t times, which is not possible if pi star is the maximize profit. So, at only one level it is going to happen either this pi star is greater than 0 or there is another possibility that the profit maximization would not happen in such a market where firm firms technology exhibits increasing return to scale.

There are some other implications also that we should think about and one of the implication is that if you keep on increasing the size of the form whether it is in the

constant return to scale or increasing return to scale what is going to happen the firm will become. So, large that assuming that it is price taker would be wrong, it means; it will start and in the next chapter we are going to talk about scenarios, when the firms are no longer the firm or firms are no longer price taker.

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$$\begin{aligned}
 &= 2Pf(\hat{K}, \hat{L}) - 2r\hat{K} - 2w\hat{L} \\
 &= 2 [Pf(\hat{K}, \hat{L}) - r\hat{K} - w\hat{L}] \\
 &= 2\pi^* \\
 &Pf(t\hat{K}, t\hat{L}) - r t\hat{K} - w t\hat{L} \\
 &= t^n Pf(\hat{K}, \hat{L}) - t r\hat{K} - t w\hat{L} \\
 &\Rightarrow t Pf(\hat{K}, \hat{L}) - t r\hat{K} - t w\hat{L} \\
 &= t [Pf(\hat{K}, \hat{L}) - r\hat{K} - w\hat{L}] \\
 &= t\pi^* \quad \pi^* = 0
 \end{aligned}$$

1) too large to act as a price taker firm
 2) CRS not at all the levels
 3)

So, one possibility is too large to be act to act as a price taker firm. The other possibility is that the firm may not be exhibiting this constant return to scale technology at all level ok. So, as the size of the firm increases firm may no longer be exhibiting CRS ok.

So, CRS not at all the levels and the third possibility is that possibility of making profit would lead to many other firms entering the market and if many other firms enter the market, what would happen to the supply? We know from individual supply to get the market supply we have to add up all the supply curve horizontally and then we get the market supply.

So, if we keep on adding the supply the supply curve would shift rightward which would lead to a decrease in price. So, it will lead to decrease in price and as price would decrease, the profit would also decrease and this is the property we are going to exploit even in the next topic.

Thank you.