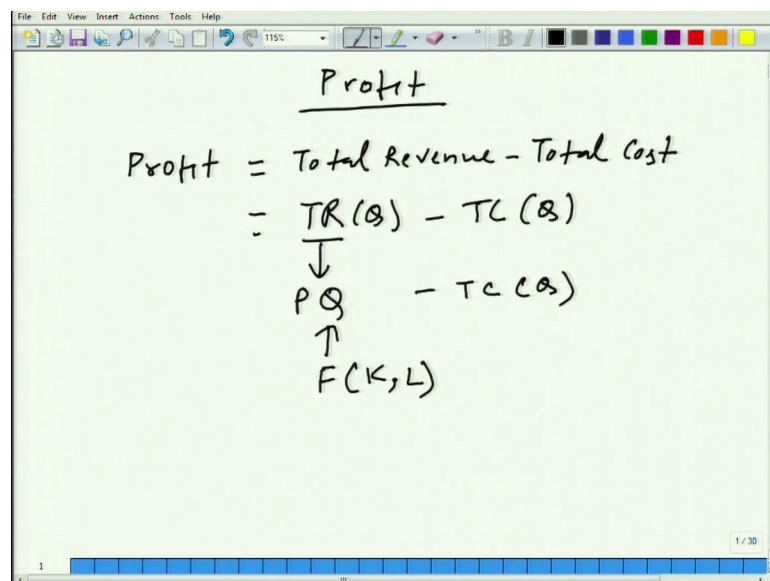


**An Introduction to Microeconomics**  
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**Indian Institute of Technology, Kanpur**

**Lecture - 109**  
**Profit Maximization**

Today, we are going to talk about profit maximization. We have already talked about cost, total cost, cost curve, and cost function and also cost minimization.

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Profit

$$\begin{aligned} \text{Profit} &= \text{Total Revenue} - \text{Total Cost} \\ &= TR(Q) - TC(Q) \\ &\quad \downarrow \qquad \qquad \downarrow \\ &PQ \qquad \qquad - TC(Q) \\ &\quad \uparrow \\ &F(K, L) \end{aligned}$$

Today we are going to talk about profit. And what do we mean by profit what is profit simply profit is a difference between total revenue what you earn that is given by total revenue and total cost what you spent fine. Let us represent it by TRQ, and this is T C Q TR stands for total revenue, and TC stands for total cost.

Just let us understand what is revenue? What do we mean by revenue? When you sell a good 1, unit of a good you get paid how much do you get paid equivalent to it is price. So, if you sell n number of goods how much will you get paid and multiplied by p. So, let us say that we are able to sell because here we are talking about total revenue for Q good. So, it is going to be P multiplied by Q, and we have already studied total cost.

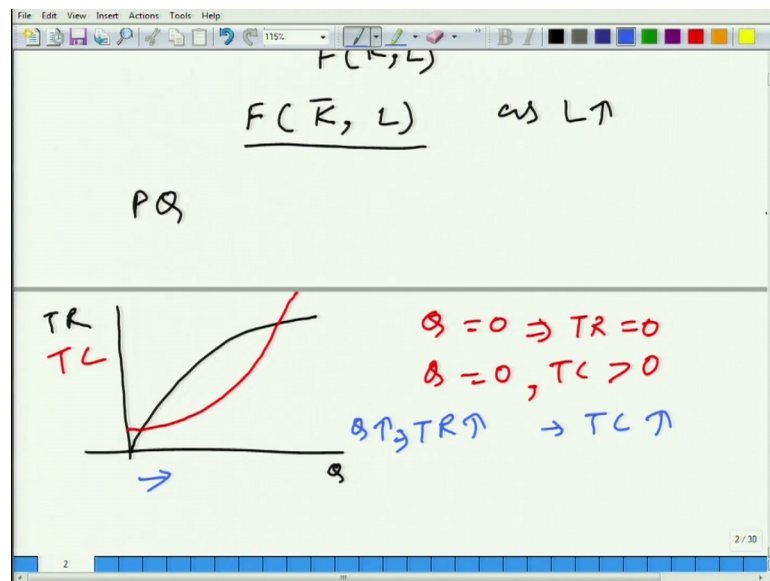
And what is Q? Q denotes for a firm it is output firm sells it is output. So, Q denotes output and how do we get this output what does the firm do remember that technology

when we started talking about firm we talked about technology and we said that a firm transform inputs into some sort of output. So,  $Q$  is equal to let us say we have used this notation this is not the only notation, but this is what we have used  $Q$  is equal to  $F$  of  $K$  and  $L$   $K$  represents capital and  $L$  represents.

Student: Labour.

Labour and  $P$  is of course, price. So, just to understand just let us see that in the short-run although what we are talking about may be valid or may not be valid for just the short run. But let us see what happens? Let us fix one of the input and we do this in short run we say that in the short run we cannot change the capital we can vary labour, but we cannot change the capital that is the convention we have adopted in this course ok.

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Now, as  $L$  increases what happens to this? This increases, but remember we also talked about diminishing marginal return, diminishing marginal productivity, as we add one more unit of labour of course, our output increases, but the rate of increase in output decreases as we add more or more of labour.

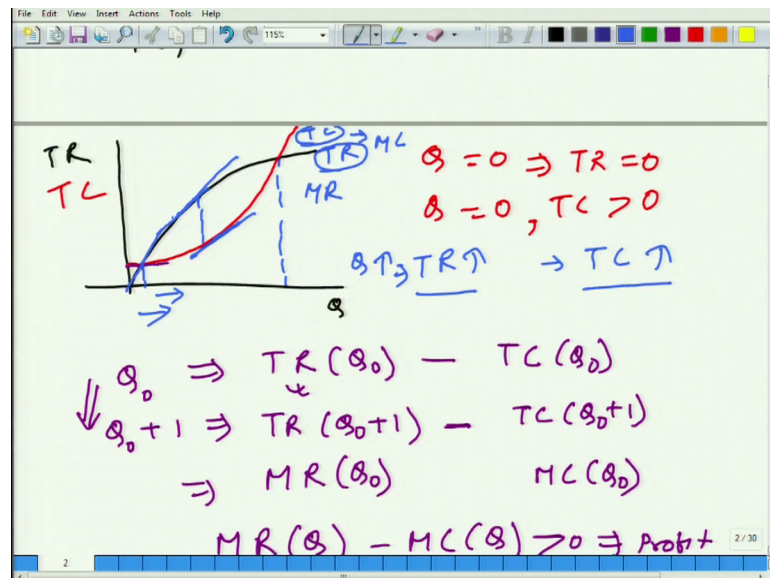
So, what I am trying to say in one variable I can draw I can draw the total revenue that is  $P$  multiplied by  $Q$ , I can draw it like this 2 way, we can understand that either we are talking about short-run. So, we are able to vary only labour and not the capital that is why we are getting like this and this is  $TR$ .

And let us continue with the short-run notion that we have developed and there of course, even though if you are not producing anything you incur some fixed cost ok. So, let us say it starts from here and it increases like this and this is just representative this is not the case for all the production processes this is just a typical of typical representation.

But the idea is we talked about that a firm would like to maximize its profit ok, firm would like to maximize its profit and we have already discussed this that firm may have several other motives also, but we are concentrating on profit motive that firm is interested in maximizing its profit.

So, what happens let us say when  $Q$  is equal to 0 your total revenue is 0, as total revenue is  $P$  multiplied by  $Q$ , but if when  $Q$  is equal to 0, total cost is typically greater than 0 why because of fixed cost ok. And as this firm produces more goods  $TR$  starts increasing and so is  $TC$  as  $Q$  increases total revenue increases as well as total cost.

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But typically what happens that the rate of increase in total revenue is more than rate of increase in total cost, typically fine ok. I am saying look at it here; what is the rate of increase in total revenue this is the slope, and how much is the rate of increase in total cost this is the slope. Typically I am saying this is more, but that is what not we are interested in at present. What we are basically talking about it that firm is interested in maximizing profit.

So, let us say firm is at present  $Q$  naught level ok, and with  $Q$  naught level you have certain total revenue and you have certain cost. So, your profit is definitely this now you increase you take the output level to  $Q$  naught plus 1 what will happen TR will increase TR will increase and that will be denoted by TR of total revenue of  $Q$  naught plus 1 unit and similarly TC will increase. Now and this will be the total profit, but now you compare these two first the difference of these two what is the difference of these two?

Student: Marginal revenue.

Marginal revenue marginal revenue is nothing, but the increase in revenue because of increase in production by 1 unit or increase in sell by 1 unit. So, this is marginal revenue at  $Q$  naught level, and similarly, here we get marginal cost at  $Q$  naught level. If marginal revenue is more than marginal cost, then what happens if we move from here to here total profit will increase, total profit will increase.

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The image shows a digital whiteboard with handwritten mathematical expressions. At the top, it lists  $\Rightarrow MR(Q_0)$  and  $MC(Q_0)$ . Below this, it states  $MR(Q) - MC(Q) > 0 \Rightarrow \text{Profit increases}$  and  $MR(Q) - MC(Q) < 0 \Rightarrow \text{Profit decreases}$ . A horizontal line separates this from the next section, which shows  $MR(Q) - MC(Q) = 0$ . Below that, it defines  $\text{Profit} = TR(Q) - TC(Q)$  and then the derivative  $\frac{d \text{Profit}}{dQ} = MR(Q) - MC(Q) = 0$ .

So, if what we get from here that if MR at one point is greater than MC, then it means profit is increasing profit increases. As we increase the production and if MR at that point is less than marginal cost at that point profit decreases.

And what we are interested in we are interested in maximizing the profit at which point it is maximized definitely the necessary criteria is that marginal revenue at that point should be equal to marginal cost ok. At that point either profit is maximized or profit is

minimized anything can happen, but we can ensure profit is maximized how by looking at second-order condition that we will pay look at later, but this is the requirement.

So, here look at this graph here we have and what is the rate of increase in total revenue it can be given by let us say, it is total revenue curve and it is total cost curve. The slope will give us a rate of increase in revenue, what is the requirement in this graph to maximize the profit? The profit is maximized at this point.

Let us look at is it maximized at this point or this point no. In fact, at this point profit is 0, even at this point profit is 0. Why because total revenue is equal to total cost profit is total revenue minus total cost so, which is equal to 0 in this case.

So, as we move in this direction what happens now beyond this point total revenue is more than total cost and both are increasing ok, so profit starts increasing. So, as long as the rate of increase in total revenue is more than rate of increase in total cost we will get an increase in profit as we increase the production.

When we do when we get the maximum when rate of increase in total revenue that is marginal revenue is equal to rate of increase in total cost that is marginal cost both become equal and here of course, in this graph where tangent is parallel to each other or the slope of tangent is the same fine is it clear that is the basic thing.

This is very easy to see in the using calculus we can simply say that let us maximize profit which is total revenue, minus total cost. Let us maximize profit with respect to output what do we get? Marginal revenue minus marginal cost and using the first-order condition at maximum point these both should be equal and marginal revenue should be equal to marginal cost, fine.

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The image shows a whiteboard with the following handwritten text:

$$MR(Q) = MC(Q)$$

$$\frac{d^2 \text{Profit}}{dQ^2} = \frac{\frac{dMR}{dQ}}{\uparrow} - \frac{\frac{dMC}{dQ}}{\uparrow} < 0$$

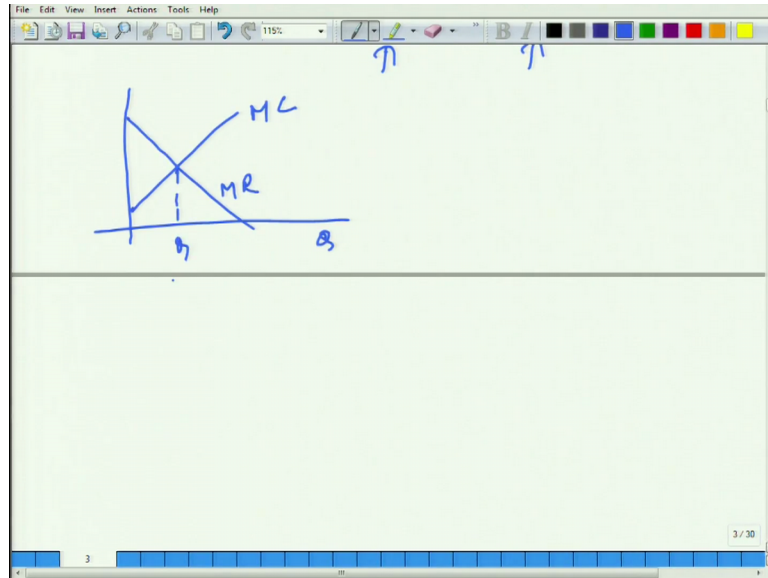
The fraction  $\frac{dMR}{dQ}$  is circled, and both  $\frac{dMC}{dQ}$  terms have an upward-pointing arrow under them.

And what is the second-order condition? If we are using calculus we can talk about second-order condition the second-order condition is it should be less than 0, if we are getting the maximum at that point. What does it mean? Rate of change in marginal revenue at that point is rate of change in marginal cost it means what is happening that beyond this point if we go back to beyond this point.

If we at this particular point the marginal cost is changing after faster rate than the rate of change in marginal revenue is it clear it means marginal revenue is becoming flatter while marginal cost is becoming steeper that is what it means let us pay attention to this equation once again.

Here look at it what is this this is rate of change in marginal revenue this is rate of change in marginal cost, the second-order condition says for the maximum this should be less than this at this point it means marginal revenue curve if we draw and we can draw the marginal revenue curve how does it look like?

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Let us think about the graph in this case.

Student: Sir, we are linearly.

It is decreasing. Now we can say linearly we can we do not know exactly I have drawn it randomly. So, we can say basically it is decreasing.

Student: Yes sir.

It is a decreasing curve and how about marginal cost it is increasing curve, this is again representative not the actual, fine. And so we get maximum at this point.