An Introduction to Microeconomics
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Lecture – 105
Short Run Vs. Long Run Cost Minimization

Ok let us start with the cost minimization.

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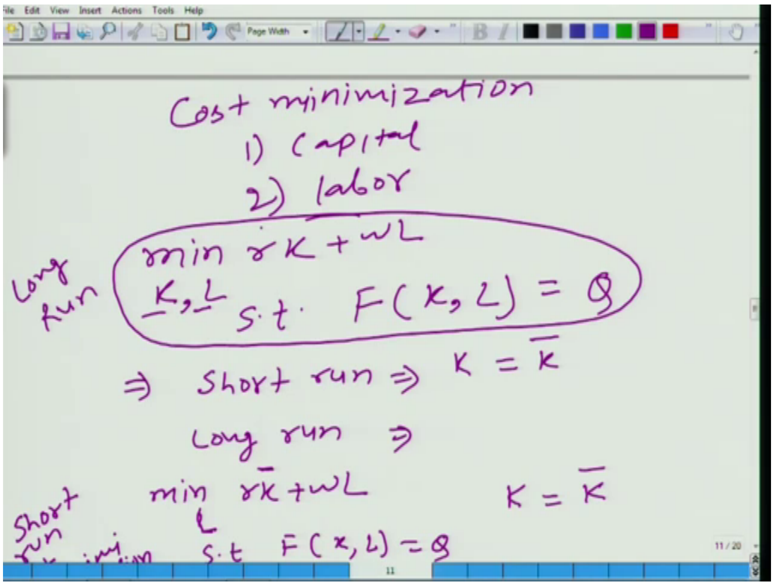

Mization and what do we mean by cost minimization? I have already said that, we will have two factors of production one capital and labor. So, our cost minimization problem would be, minimum of K, L such that r K. Such that we minimize r K plus w L with respect to K and L.

And what is r; r is payment to 1 unit of capital. That is range to capital and w is wage to capital and of course, what is the condition because if we do not impose condition then we can simply say 0 K 0 L cost will be minimized, but the idea is to produce at least q amount of output. So, that the cost of minimizing Q amount of output is minimized. So, the constraint is, such that F of K L is at least Q and so far we have been taking it as that let us say that we are producing just exactly Q amount ok.

Now, there are 2 problems here, one we can think of the short run, the problem in short run and the problem in long run. Short run is when we are not allowed to vary at least one factor of production and here we have only 2 factors of production. So, let us say that we are not allowed to vary K and let us say K is fixed at K bar. I mean what is long run? Long run we can vary all the factors of production.

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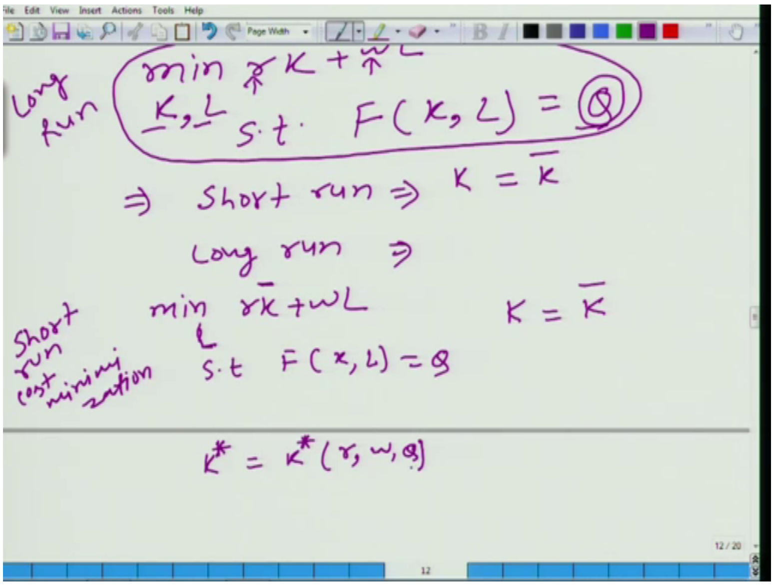

So, again let us look at it what would be the cost minimization problem in the short run, the same problem minimize r K plus w L with respect to K and L. We have to select K and L such that we are able to produce Q amount of output at the minimum possible cost, but in the short run we will have additional constraint, that is K has to be equal to K bar. This is the problem in the short run; this is short run cost minimization problem. .

And in long run the earlier problem that we wrote, was the cost minimization in the long run. Long run here we can vary both capital and labor. So, here again coming back to the short run. Now we are saying K is equal to K bar. So, now, we are not allowed to vary this K. So, minimization is only with respect to L and here K is given as K bar fine. We have already learned; let us look at the long run cost minimization problem.

Long run cost minimization problem is this, minimizing r K plus w L with respect to K and L such that, we are able to produce Q amount of output. So, what we have here is r is given by the market. Rent is determined by the market. We are assuming that the form is not that big that it would be able to dictate the rent in the market.

So, r is given in the market, W is also given and Q is also given that we want to produce Q amount of output. What we have the freedom; we have freedom to choose K and L. So, if we solve the optimization problem, what do we get? This K will be a function of r, w and Q. Q we have to determine, L is again going to be a function of r, w and Q.

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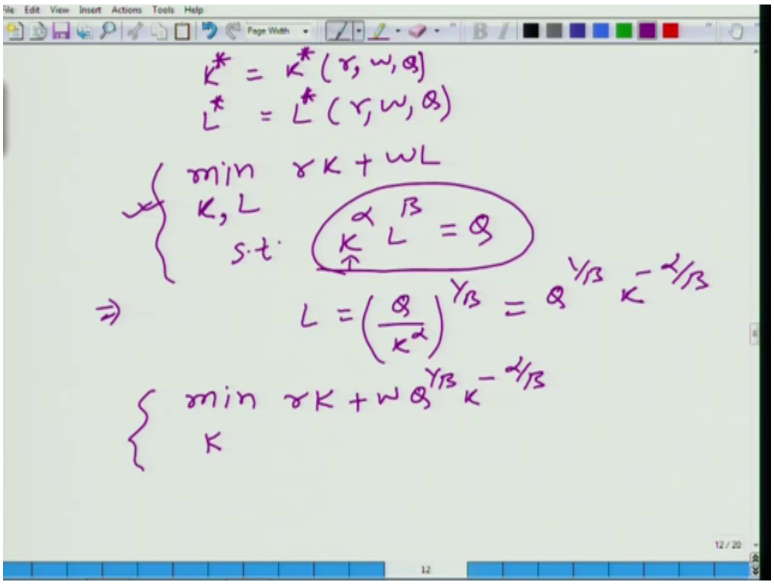

The easy way to go about it just to verify, we can check we can take a particular functional form and we do the cost minimization and it is a good idea just for an exercise that we take Cobb-Douglas production function. So, what we have here is minimum of get minimum of r K of w L by varying K and L, such that K to the power alpha L to the power beta is equal to Q and we can transform this problem, because this is given. We can transform this problem basically from here it looks like we have 2 variables K and L.

But we have an equation in K and L. So, if we fix K, L will automatically be determined if you want to produce Q unit of output. So, we can express here L in terms of Q and K and what do we get Q to the power 1 by beta and K to the power minus alpha by beta. And now the problem we have is minimize with respect to K, only r K because we can express L in terms of Q and K where Q is given Q is the parameter ok.

So, basically what we have here is Q to the power 1 by beta K to the power minus alpha by beta. So, basically now from here we have constrained minimization where we have unconstrained minimization, this is an easier problem to solve and when we solve it how can we solve it? We will write the first order condition and what do we get r if we differentiate with respect to K, we will get this we differentiate with respect to K K, we will get r.

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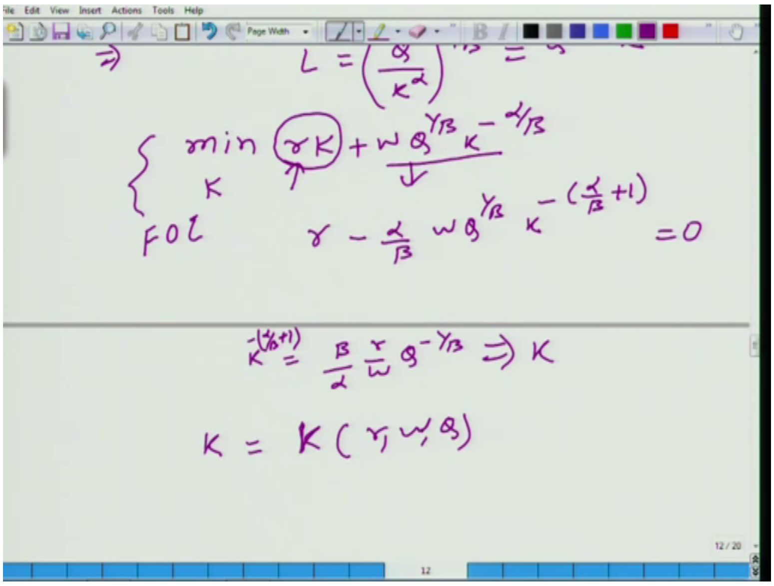

Now, here we differentiate with respect to K what do we get? K would be has to be equal to 0 and now we can express K; K as K would be equal to if we solve it, K would be equal to beta r divided by alpha Q to the power minus 1 by beta and what else, let me write it here that would be simpler fine.

So, basically what I am trying to say here is that K there should be a w also r by r by w. Basically K is expressed here as a function of r w and Q. That is what I am that is where I am coming to. That is where I want to be and of course, by solving this you can get the K the functional form K as a function of r w and Q. That you can get from here. FOC first order condition, first order condition why we are doing it? We are trying to minimize this expression.

So, if we change K let us say if we increase K what will happen? This will increase, but this will decrease and we want to figure out where this increase is equal to this decrease fine. So, basically we will get K as a function of r w and Q and similarly, we will again get L again let me put star here just to indicate this is the optimal amount of K, as a function of rent, wage and output Q fine.

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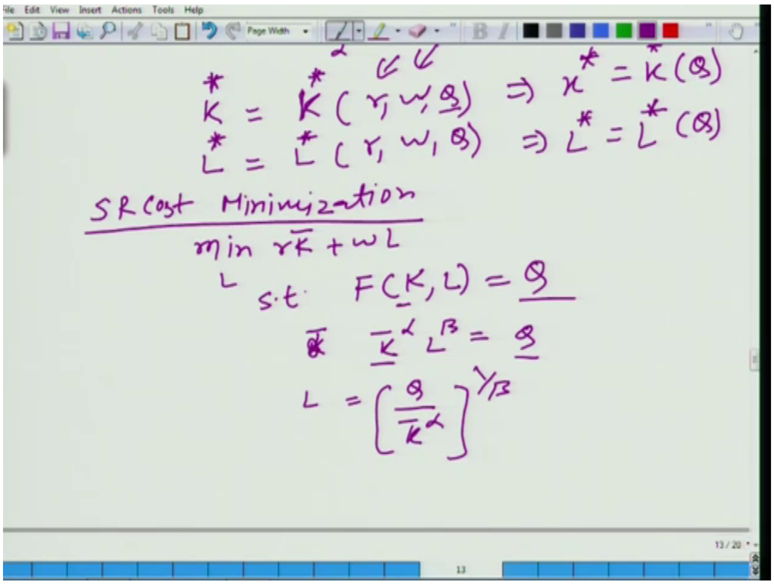

Now, look at it many times we write, many times we write that K star as a function of Q. What we mean here is simply that it is not that K star is not a function of r and w, but r and w is market given. While Q the form is deciding how many output how many units of output to produce. So, basically we are interested in K as a function of number of units produced and similarly we get L star is equal to L star as a function of Q.

Now, let us look at it in the short run cost minimization are you following me? Fine; what is the short run cost minimization? This is trivial because we have what are we doing? K is fixed we are allowed to vary only L we are allowed to vary only L and what we have here is r K is fixed and w L, but even L we cannot vary because we need to produce Q amount of output. Q is fixed K is fixed L is automatically determined from this equation.

So, we do not need to carry out the cost minimization, because to satisfies all the constraints we have only one point and in this case, let us say what is going to happen when we take the Cobb-Douglas function, K hat is K hat alpha L to the power beta is equal to Q this is given this is given. So, we get how much L we need.

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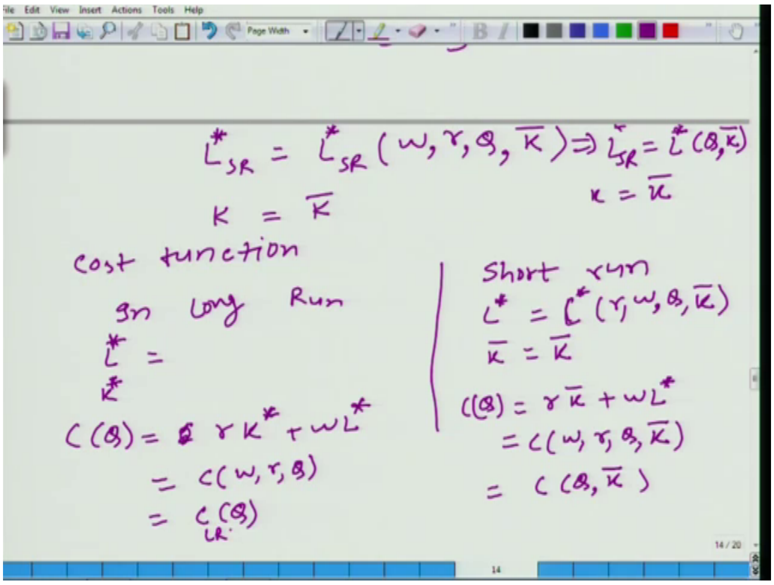

So, what are we going to get? Q K hat alpha 1 by beta there is only one point that satisfies the constraint. So here what we can say little differently that L with that we get in the short run. Let me write it here in the short run, this L star is a function of not only of w here in this case it is not a function of w.

But I am talking about a general case where it may be a function of w, it may be a function of r, it will be a function of Q and now there is another parameter that is K bar. It will be function of K bar you can still say this L is a function of w, r, Q and K bar. It is just a function where w is to the power 0 and r is to the power 0. That is it and how about K, K is given K bar.

So, here I can say here I can say in this case that this is a function of q and K bar, because K bar is the company's decision. How much capital to have to run this firm, run this business. So, this is the company's decision and what we have here is K bar fine. Now let us look at it the cost, cost function in short run and in long run not rum, but run ok. What we have here is we have already figured out L star we have already figured out K star.

So, what will be the cost function, to produce Q amount of output C of Q is going to be a function of it is going to be r K star w L star. So, we will get C as of because K star is a function of r, w and Q; w is at L star is a function of r w and Q. So, cost is going to be function of w r and Q; or here in the short we can say its function of Q. Oh this is by the way this is long run.

Now, let us look at it the short run. What do we have in the short run? L star is a function of w, r w; Q K bar K bar is a function of K bar ok. So, how about the cost, by the way cost function gives the minimum cost required to produce Q amount of output given the price of inputs in the market fine.

So, here we get r K here of course, bar w L star. Here what do we get we will get C as a function of w, r, Q comma K bar or here we can say C is a function of Q, K bar. Let us say here write long run, here it write short run. Short run is it clear mathematically?