Calculus of One Real Variable
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Lecture - 20
Integration-III: Newton and Leibnitz Style

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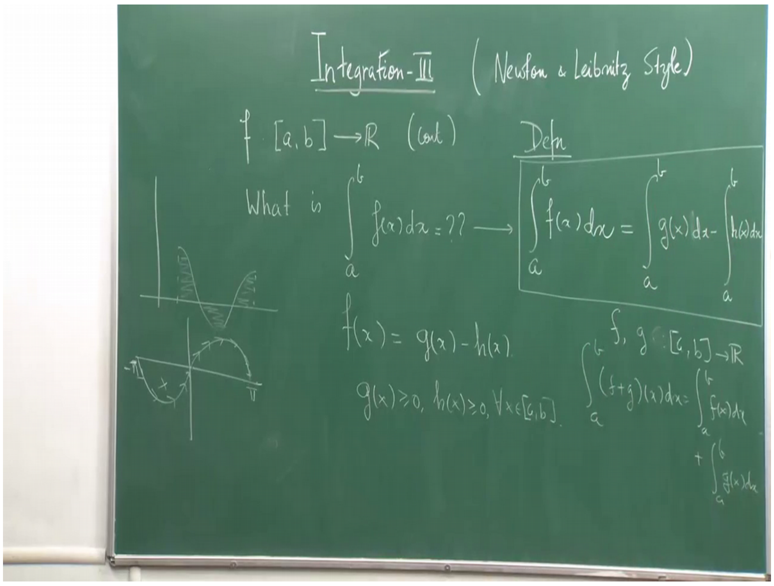

How strange we are still in the 16th, 17th century do not worry we will move soon to the nineteenth century, but let us work on what we had gained in the last 2 lectures. It will show also the ingenuity of these people the one of the early founders of modern science and so we start with this question that if f is a say a continuous function then what is the meaning of this? We are not really bothered with the sign of f at this time, f could be something like this between a and b. So, what does this, what is the meaning of this what do you mean by area under the curve?

So, here when you have know; how to define now what we have to do is to make a definition. So, you have to understand definitions are not logically derived; propositions are derived logically once you know you put in certain definitions. So, a definition is something which everybody understands and is for everybody’s convenience. So, how do you define this one? See take any real number, whatever 2, 3 minus 3, minus 5 what whatever any fraction then. So, if you take any negative number, you can always add a positive number, so that the sum becomes positive. So, every number can we express as the difference, difference of 2 non-negative numbers.

So, similarly a given any function f x, I can only express this function f x as a difference between 2 functions g x and h x, where g x is greater than equal to 0; and h x is greater than equal to 0 for all x in a, b. So, these are non-negative functions, you can always do that. And the integral now this integral this integral a to b, f x d x, so this is the definition is now given as integral is the difference of these 2 integrals of these 2 quantities. Now, we know how to evaluate the integral of this we know how to evaluate the integral or non-negative function. So, if I take the difference of the integrals we define them as a difference the integral of a, b; it is a definition by the way please understand this. So, if I define it in this way a lot of things starts making sense.

So, what it says that if f is having g x and h x as your components that is f x is equal to f x is g x minus h x. So, the integral of f x the area under f x plus the area under f x means that you first calculate the area under g x and then calculate the area under h x and subtract it. So, anything with a negative sign has a negative area that sort of idea is in here. Those who know some integration I will just look for example, if I look at the sine function oh sorry, so I am looking at a sine function, sine function from minus pi to plus pi. So, if I integrate this many of you know what we will come to this many of those who know calculus, it is only for those who know calculus you know what is the answer of this.

So, if I put a as minus pi and if I put b as plus pi, you know what would be the answer of this, answer would be 0. So, what does it mean is the area under the curve 0. Of course, this area is 1, and this area is 1 that is 1 minus 1. So, the modulus of the area the absolute value of the area is 2, if you sum the 2 areas, but if you just define the area under the curve with that sense it will become actually 0.

Now, once I have defined this then any formula that we would know for a function of negative or non-negative variable non-negative function can be done for these functions. For example, now if you take f and g, for f and g's are functions from a, b to R, does not matter what are what their stuff are when a non-negative or not non-negative this would again be true it does not matter.

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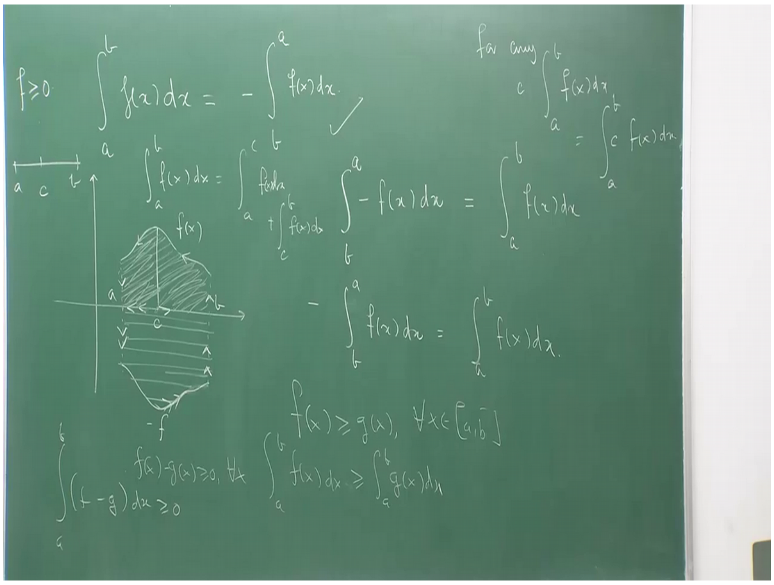

Now, yesterday we had showed that if I multiply a function by a positive number, a positive number will come out of the integral; actually it will be true for any number. Now, it is very important and you must have seen in calculus books a formula like this. Now, what does this formula mean? This is a crucial formula; you use in many, many places what does this formula mean? So, these were all developed through intuition now how do, but I do not think I have to really look up look up the historical sources that they had really thought about it. It was thought like that that if I am changing a to b, there is a or basically what I am trying to do is if I take the negative of this function, so if I instead of f x you might take minus f x all the if they if we say a non-negative then minus f x is totally negative.

And then I would you know I would. So, suppose first let us just keep f, f to be greater than equal to 0; f x to f to be greater than equal to 0, so over a, b. Now, what I will do if I have f is positive and minus f would be negative finding the negative area, area of a function which is negative values is just like. So, there I do not go from b to a, there I go from a to b, I go in the opposite way because it is a opposite thing we have it is reflected. So, it should be from b to a, and the area should be negative of the area that we have computed for the non-negative thing. So, they will tell you all this very fast, you will be definitely confused.

When mathematicians work all these confusing ideas come in their head, there is a single intuitive way they try to juggle with ideas and place something and try to find out which something that works and makes a little bit sense. So, let me tell you how in a modern way we should think about it. So, I will ask you to think about prove these things for this formula at least for any type of function, negative, non-negative does not matter negative non-negative you already know, whatever be the function this is true. Just apply this definition and do it.

Now, what happens, so this is my function f say, and now this is the area. We now developed a convention specially with the growth of vector calculus, we have developed a convention that what do you mean that an area is non-negative, the value of the area is non-negative. Of course, you will say come on; how can it be negative, this is the area of some closed object, how you understand it is a quantity of amount of a measure of a shape, I mean measure of the space content inside it or part of the plain concentrated here. So, how can it been negative, but in mathematics a lot of thing goes by convention.

Because unless conventions are bought in on specific situations many things like the one above the internal above cannot be completely justified that is my belief that you work mathematics is all our convention. If you read the book of Timothy Gower's, Timothy Gower's is a fields medalist, you must have known that many of you must know that fields medalist or the most distinguish awards in mathematics given to mathematicians under forty who does power breaking work. And Timothy Gowers is a very famous English mathematician who pure mathematician rather if you want to call him in Cambridge. And he was a field medal winner, and he has written a very small book called mathematics a very short introduction, and there he emphasized this issue of conventions in mathematics.

So, when shall I call the area positive or non-negative, so if I start walking on the boundary of the area, so I am walking on the boundary of the area, right; this is the boundary this is the boundary. And if whether walking on the boundary of the area in a counterclockwise fashion means I am walking in this way, this is my walking path. So, I am walking in a counterclockwise fashion in this way. When I am walking in a counterclockwise fashion, the area should always lie on my left; if it always lies on my left, then I will call the area to be non-negative, the areas area would be non-negative.

Now, if I now come to so this is my a and b. So, start from a, and go to b. Now, just take the just reflect this function the graph of the function on the x-axis. So, what you will get? So, this is nothing but minus f right maybe if the graph should look a bit like this whatever. So, this is nothing but minus f. So, now, I write the area in this way. Now, if I want to say that the area is non-negative, when will I consider this area non-negative, if I start walking on the boundary in a counterclockwise direction means here my walking direction should be this. So, where should I start my walk from b to a, I cannot go from a to b. Then wherever go from a, I will come like this and I have to come back to a.

So, basically I start from b, so I go from b to a, but you see technically if you make just a reflection of this whole thing from very basic transformation geometry may be taught in many schools nowadays. In CBSE, there I have seen them teaching a reflections I have seen in my daughter's own daughter's schoolbook, they teach reflections, which is of course, necessary because transformations are what the key to mathematics because all of mathematics is transformations. So, everything is for changing one point to another functions.

So, what is happening is at this area though I have my drawing might be bad, but this area because if you reflect the area when you reflect the graph you reflect the area that this area quantitatively remains the same. But now what happens is that if I say integral b to a of minus f x d x, minus f x d x, this value is same as integral a to b f x d x, because I have just made a reflection on the x axis where x axis is acting as the mirror.

So, what does it mean by the formula that I can take it out. So, essentially it simply means that if I now instead of, so what is this. So, now, these 2 areas are same clear. And what does this mean, what is the meaning of now this is a non-negative area in a way I have given. So, what is the, so what would be the negative of this area negative of this area would be walking in the clockwise direction. So, if you are walking in the clockwise direction, you will actually go from a to b. So, negative of the area from b to a walking in the clockwise direction. So, if you look at now walking in the clockwise direction from b to a like this then my area would be negative. So, but that area numerically must be same as the area from a to b and that is the logic behind this.

So, this is an important result and there are some several ones like for any c does not matter what is c, x has to be real number. Another result is the following that if 2 functions f and g is such that this is bigger; and integral a to b f x d x is bigger than integral a to b g x d x. See, this comes out of a very simple fact that f x and g x f x minus g x is greater than equal to 0 for all x. So, we are going by the convention that if I move from a to b, if a is less than b, and if I move a is strictly less than b if I move from a towards b, I am actually moving in counterclockwise direction when a to b when the function is non-negative.

And in that case what is happening is that by convention the integral is one of the area is 0. So, if I even if you take this convention it simply means f minus g I am just shorthand shortening this thing is greater than equal to 0 that is all and then you simply apply the this definition. So, f minus g is some function h which has no sign, it is a unsigned right and then you use this definition and then just go through. There are several other formulas I am not going to write all the formulas that ok if you take some number c in between then I can first calculate the area from this to this. So, the whole area is nothing but area from I calculate the area from this to this, and the area from this to this basically it says that it is true for any function, but it simply says that integral a to b f x d x.

So, if you take any c which is lying between a and b then this is nothing but obviously, it could be equal also that does not matter suppose I take a c between a and b then I can write this as a to c f x d x plus c to b f x d x. Sometimes these are required because maybe the computation becomes simpler when you put in a c and all those things. So, these are some very basic ideas of the integral. And I am not here to give you tremendous amount of examples, again I reiterate that this is a course maybe I should rather calling it by this bland name of a calculus for one variables which immediately makes one thing that this will prepare him or her for an examination.

But I should rather have called this course for as the concepts in the calculus of one variable, one might thing that one variable is very simple, but let me tell you that when you talk about one variable, you are essentially talking about elementary things you have very less tools with you. But that does not mean always the concepts are so simple. For example, this is absolutely not a simple concept.

For example, I will tell you why elegant I write this equation minus pi to pi. So, if I am integrating from minus pi to pi, and if I want to move in a clockwise, if I am find one move in a counterclockwise direction, so I should move like this. So, I can actually break from minus pi to 0, and 0 to pi, but from 0 to pi again you know that negativeness would come the way that has been defined. So, then when you add them add the 2 negative things they will cancel.

So, what is happening, here I am in the clockwise direction. So, when the area is on my left, I am on in a clockwise direction, but here I mean sorry I mean the counterclockwise direction. But here when I am going from here to here, but I am when I am going from here to here the my area is on the right and I am in a clockwise sense not counter clock I am not in a counterclockwise sense I am in a clockwise sense, and then the area is negative. So, when you add them they cancel out. So, in that sense, so it is fine.

Let us now talk up forget about all these and now we come to something very very important which is now known as the fundamental theorem of calculus. For next week's lecture would be completely on Riemann integration and its generalization. So, we spend 2 weeks on integration which is pretty good and then some applications of integrals in the next week one or 2 classes and then going gone to other things. Last 2 weeks would be on power series and Taylor's theorem.

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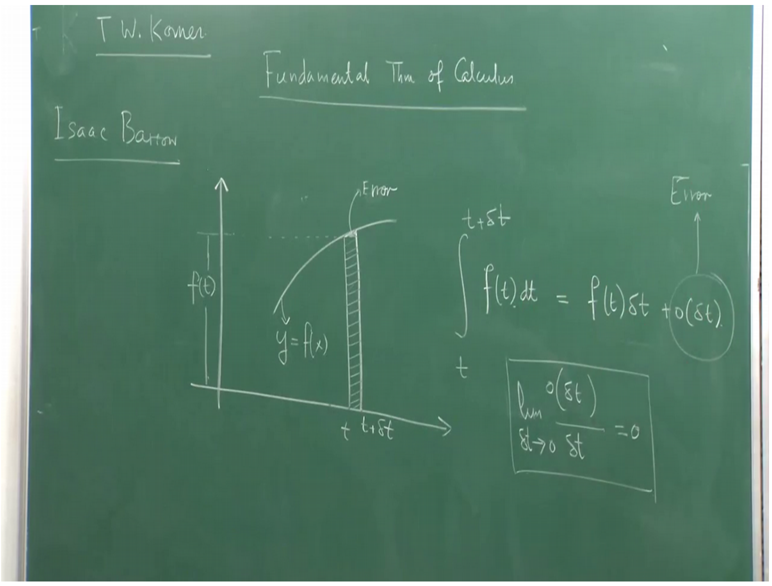

So, in this description, I will follow from Korner, Fundamental Theorem of Calculus. I follow this from the fifth book of T W Korner. T W Korner is not a fantastic writer of mathematics, but he is also very famous mathematician of a times who is currently the professor of Fourier analysis at Cambridge and has written fantastic books is I someone quite motivated by his way of thinking because a intuitions are always a guide to conceptualization and that is what he has done. So, what fundamental theorem of calculus is? So, these Greeks known knows about areas and tangents and all these things, but what is the link between area and tangent.

This link was only established in the 17th century. A large credit goes to Newton and Leibniz for analytically establishing it and trying to prove the result that is trying to show what are the deep connection between the notion of area and the notion of tangency or notion of tangency is same as a notion of derivative, so the link between area and derivatives. And in some sense area is nothing but integration, so it is trying to tell you about the link between differentiation and integration. So, the 2 fundamental concept of calculus are linked through this result. So, this is one of the most fundamental most important things that we are going to now discuss.

So, we are going to start discussing the fundamental theorem of the calculus. So, as I told you this greats like Newton and Leibniz had actually gave us what is essentially the relation between the derivative and the integral, and the a relation between area and tangents which are known geometrically. The geometrical idea which is essentially this one was understood by Isaac Barrow, Isaac barrow was a theological and a mathematician at Cambridge was the master of Trinity College, and also was a first location professor of mathematics in Cambridge. And positional location professor of mathematics is at this point one of the most prestigious or the most prestigious positions a chair professorship in mathematics in the world. And Barrow was impressed by the brilliance of Newton that he actually gave over that chair and handed it over to Isaac Newton who held it for a long time. In fact, Steven Hawkins about whom many people would know because of his works in popular science other than his work in physics that here also still few years ago location professor of mathematics at Cambridge.

So, Isaac Barrow had this geometrical idea which was later on expounded by Newton and Leibniz and these, but Newton and Leibniz essentially thought that this would be just a kind of you know, this is just a kind of curiosity possibly a result which is there and that is all. But only when they Cauchy may be a Euler before that specially nineteenth century when the mathematicians were like Delavan upon, so whenever having a relook at the nature of the calculus, then they had thought about these things more deeply and they found that this result is an fabulous one because it allows you to actually compute integrals. This is essentially not just a curiosity; this is essentially computation.

And let us see what it means. So, if you have a curve y equal to f x which is continuous and then if you look at it is from t, if I move it will be delta t here delta t have drawn positive you could take it negative also. So, what would have happened if I just take on f t, if I take this I construct the rectangle and the whole area under the curve on this base del t is nothing but the area of this rectangle plus this shaded part, the extra part which is not covered in f t, f t into del t. So, this plus this, so this is the error term, this is the error.

So, what this error term has a speciality, because you know where delta t collapses to 0, then this integral should collapse to 0, and everything on this side should also collapse to 0. So, which means somehow that this delta t where this delta t should move to 0 much faster than t; and which can be expressed by this fact that this term this o of delta t term which is also function of delta t. Because it depend on how much near your 2 t goes to 0 faster than delta t; otherwise this would have gone to infinity or something else. So, here because this goes to 0 faster the limit should be 0 that is the exactly the meaning of this error the error should vanish very first as delta t moves to 0 and then only you will get what you at delta t in the limit you will get what you require.

So, now, syou might say do you have drawn it for delta t and you written it down. What about minus delta t? You need to convince yourself about that I will write in the notes I why this is true for any delta t, any delta t, does not matter what is the delta t, my delta t is a negative that is why t is t to t minus delta t. So, t to t minus delta t simply means minus of t minus delta t to t. So, it does not make any difference, whatever be the case t to t minus delta t some areas been calculated, it is exactly the this formula will actually hold. It does not at all mean anything; it does not at all mean anything. So, t to t plus delta t or t to t minus delta t, this relationship holds, a delta t sign at all has no consequence on this result, and that is something not so easy to prove. And that is I would leave it to you, but I will do it in the notes because here what is happening is that you really have to use the continuity of the function f.

Now, define now you want to integrate from a to t, do not integrate from a to b, but integrate from a to t, where you can vary the t. So, t is some number between a and b. So, you are integrating from a to t. But as you change t, the value of the integral will change.

So, the integral value of the integral will depend on t. So, hence we are adding it as a function of t. Now, obviously, if you write f of t plus delta t, you will write it like this and you know that if t is now you will say ok, now if t is a number between t and so if t any number between a and t plus delta t or it could be a to t minus delta t. So, we have to do things with the negative sign then in that case.

So, whatever it is this formula would actually hold. So, if t take three points a, b, c, so whatever those three point this breakups are always valid and that is what I told you. Obviously, gave you by writing a c lying between a and b, but that does not matter you see if I take a function an area like this, so a, b suppose the function is still defined and this is my c. Then what is integral a to b. So, I can take this whole area and then from I can subtract this area from b to c.

So, here I can simply write integral of a to b is integral of a to c minus integral of b to c or what is integral of minus b to c, it is a to b f d x, I am writing in a very bad way very bad about I guess. But this is nothing but integral of sorry a to c, c to b. So, it does not matter where you the c placed, now these are holds true. So, that is exactly why I have writing it in this fashion.

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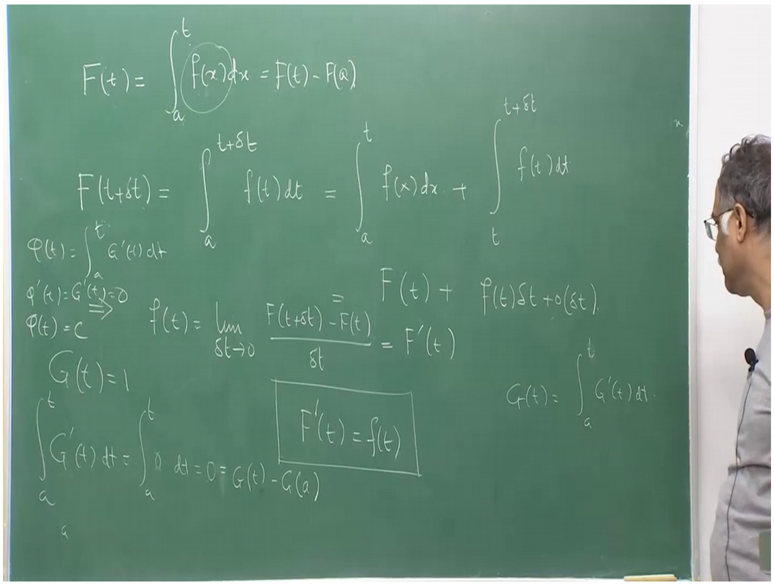

But this is nothing like f of t by definition and this is this term that we have written down from the geometry on the graph. Now, we already have told you the nature of this function the error, and then if you take the limit, then this limit by very simple definition is nothing but f dash t. Because the limit exists because this is equal to f t my f t is the finite valued function, but if this limit exists this is nothing but the derivative of f at t. So, f dash t equal to f t. So, the question is, is integration the reverse process of differentiation?

There is a tremendous tendency to think like that, that integration is a reverse process of differentiation that is ok. If I have basically I have to if I only integrate this I have to find a function whose f dash t should be f. So, basically if I keep capital F dash here capital F dash here this would be f t. So, if so this must be F t. So, this will be F dash t obviously. So, basically if I differentiate the value of the integral I get back the function itself. So, say integral mean the inverse process of differentiation.

So, instead of what we have, we have a function then we get its derivative. So, we are telling that is so instead of other than what derivative differentiation does give me a function I will give you a derivative. So, you have might have a feeling that here what it is telling that give me a function by integrating, I will tell you whose derivative this function is, but you have to be extremely careful of such maneuvers make such thought processes.

For example, I consider the function of G t is equal to 1 right. So, let me take integral a to t is a differential function continuous G dash t d t integral a to t 0 d t which is 0 it is not equal to G t. You should think a while that what is essentially going on right. So, the issue is not really this you know what where does my problem like then. So, the function that a to t G dash t is representing what sort of function it represent, it actually represents the 0 function, it is not representing the G t function. So, you cannot say that I have got a function. So, this should be a G t because then I can just recover it defined it is a derivative of this.

Here what I am telling that this integral is giving me this function, but this integral of this is not giving me the G t function. So, you cannot say ok, this is just a reverse for differentiation. So, this sort of terminology which our teachers use very frequently in high school and even in a undergraduate colleges is absolutely wrong. Thus this is something we thought about remember here I am calculating the integral and getting a value for every t, that is a function of t. Here if I actually calculate the integral, I do not recover G t which really not an inverse process in the way you think.

Now, let us do something more. Let us tell you; what is the secret which is hidden in this formula what secret is hidden there and that that I am doing it from this book which I am pretty sure that it needs a huge thought process mathematicians big mathematicians write books. And they say it is for you know kids, it is for undergraduate kids, it is for the very, very simple books I have it is nothing its nothing is there. But I can tell you even very well trained mathematicians would have to pause to read this book the calculus for them, which was have been mentioning it from quite a lot quite something.

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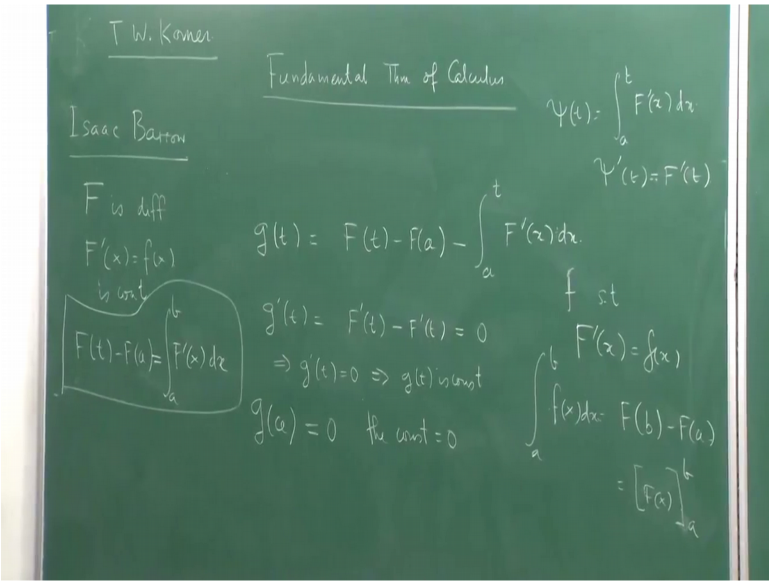

So, let me construct a function g t. Small gt which is F t minus F a minus a to t f dash x dx. So, it could be that if I can integrate this function then you can give me for every t something it to be some for capital phi t I do not write. Do not immediately think that this is F t right. Do not think just you have your lesson is already learned, do not thing that this is nothing but capital f t. So, just cancel out these g t is minus f a this will be immediately a wrong conclusion. What is very important is that here once we have integrated it, once we have integrated it, here we integrating continuous function, your f dash really have to be also continuous in order to obtain back f t.

So, let us look at it. So, suppose a derivative, so now, I am assuming that capital F. So, when I have written this I have assuming that F is differentiable and the derivative F dash x equal to f x is continuous and then this integral becomes a meaningful thing. So, now let me write that psi t. Now, what would now happen, if the integral is continuous? So, psi dash of t would be what would be F dash of t. So, what is the function capital phi whose derivative would be F dash of t that would be F t. So, this is this is something important, I would still rather tell you to keep a thought on that.

So, if I take G dash t is the F dash t 0 minus F dash t this will be if I integrate out it this will be left. So, actually if I had integrated this out, so you really compute G dash t, you have put the value and put 0 to g t you will get this. But if you actually integrate it out because these derivative is derivative of this is continuously 0, then this if you actually differentiate this function it give you G dash t and G dash t is 0. So, that matches right. So, this is something we have to be very, very careful. So, this is 0. So, this implies g dash t is 0, and it implies that g t is constant. Now, calculate g of a that will be what will be of 0. So, because g t is constant then this constant must be 0, right. So, the constant is equal to 0, so that simply means g t is equal to 0, so which means that F t minus F a is equal to integral a to b F dash x d x.

So, if you are know our function whose derivative is whatever you are integrating then this is the way you compute the derivative. So, all this story that has been told this little understanding that you know this is the area under the curve is rectangle plus a little error actually tells you finally, how to compute the integral. So, this was not understood on unless Cauchy started looking or at all these things that you can actually write it in that form. So, this is a very, very crucial form and this is the fundamental theorem of the calculus.

So, what is happening if you have a function f such that F dash of x is equal to f x, then what you write then integral a to b f x d x is equal to F b minus F a. So, this is sometimes also written as F x a to b in short. So, it is a computational power that once you can figure out for those cases if you figure out that if is a nice continuous function and you know another continuous function whose derivative is f x then this is exactly the way you do it you compute the integral, so that is the area that. So, your area can be computed so easily. So, this is called this capital F is called the anti derivative of smaller f.

So, let me go back to this formula, what was the flow because gt is one continuous g dash is continuous. So, flaw is that you have actually computed out g dash t here and directly then try to compute the integral. You have to observe that when you if you write this as phi t, so you write phi t is equal to integral a to b G dash t dt then it simply means that phi dash t is G dash t and phi dash t is 0. So, then phi t is nothing but constant. So, G dash and that constant is 0 basically. So, G dash t, so this integral is representing a constant and that constant is 0; it is not representing the constant one it is representing the constant 0.

So, this integral actually represent the 0 function, it does not represent I mean if you do this you will not get g t. So, it is ok g dash t is this? So, no it represents. So, integral a to bf it just you repeat represents this, this 0 function because you call it as phi t then phi dash t should be g dash t and phi t should be equal to c.

Student: 0 is actually G t minus G a.

Exactly; so, 0 is actually G t minus G a. So, if you now look at the thing if you go back and write that these are the fundamental theorem because now G t is 1; and G a is 1. Now, you might ask me [FL] what happened to this? Here I am only telling that f has this property that if you take the derivative of this function, you have to you recover this, but here what I am telling that if you take the derivative of this function and a to t then you recover G dash t. So, this function is such whose derivative is G dash t. So, G dash t is 0. So, this is actually the 0 function, it does not represent. So, phi dash t is 0 it. So, it is a function whose derivative is 0. So, you can say it can be any constant function it does not represent in specifically one because if you actually compute the integral it will not be one. So, b then you compute the integral and say that this function phi t must be the at a 0 function. So, that anomaly is removed by either by that that formula.

So, here what we have shown that why did not you write this thing as this integral. See this integral this integral is actually written as if you really compute the integral it should be F t minus F a. Well if so, but it is only possible if you have f dash t equal to f x. If you can write that as F t minus F a only the that this integral value this is actually F t minus f of a; and what is F of a here F of a here is 0. So, this is actually F t minus F a. So, this is the hidden fact that we have written and that is why you do you think that what is happened. So, why G t is not this? So, G t is 1, but here my integral is 0 understand, so that is the whole point.

So, when you look at just this you handle everything as if it is a function, but if you really want to compute it, then you have to write it in this form. Actually here also if you write G t with this particular case, now what does it mean it simply means it does not tell you that G t is actually the integral of this. This integral is nothing about G t minus G a; and in this case G a, is one a to t a because this integral which was that. So, a to t it does not tell you that integral of this a to a, if I put a to a and this integral should become 0, but that integral is not 0, it is one. So, you only a constant function you have to be very careful that when you when you evaluate this integral as integrals value is actually 0.

So, it is actually G t minus G a, so if you write G t like this g dash t is of course, G dash t is nothing this is the function, so that that function is valid that result is valid. But if you say that I do not care about the value of g at a I simply care about the value of G at t and G dash t, t is giving me this is incorrect. Because if you say if we put a here and G a here what would be G a it cannot be 0, because G a is 1, it is constant and that is exactly why you cannot say that I am evaluating G t by, when I evaluate this I get back G t. When you actually just evaluate this integral, you actually get back the 0 function. So, will you get G t minus g so that formula remains valid. And this formula that f dash x f dash t equal to t ft is also remains valid. So, these are such simple issues of life which I do not want to get into, but I end this lecture by talking about integration by parts.

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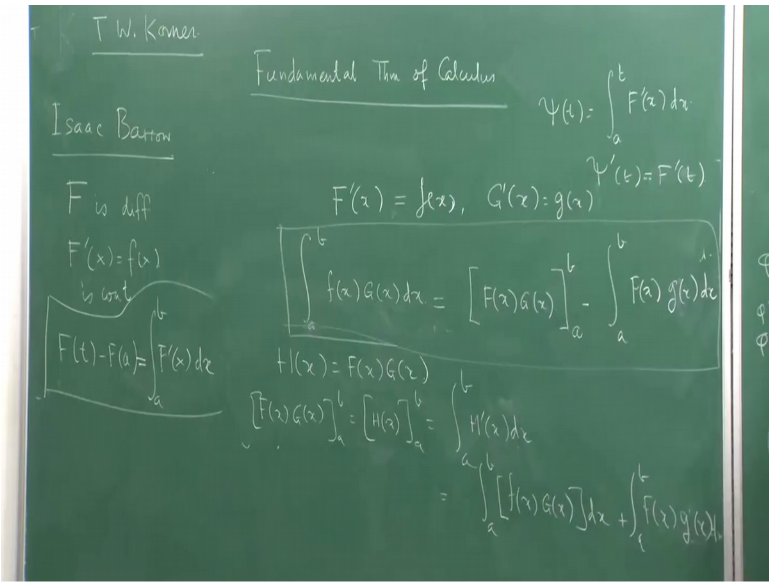

So, it comes out of a very simple calculus of product derivative of the products. F dash x equal to some f x and another G dash x equal to some g x. Actually I want to find the integral a to b f x into g x into d x, how do I do it. So, define H x is equal to G x into G x. So, F x G x from a to b which is H x from a to b is integral again by the theorem assuming every condition holds H dash x d x. And what is H dash x d x, H dash x d x by a product rule is a to b capital F dash x into G x a capital F dash x is F x F x into G x d x plus I am just writing everything directly capital F x into G dash x G dash x is g x. So, F x into g dash x. So, f x sorry F x into g x, right, where G dash x is small g x d x. So, what is this formula of mine, if I look at it from this angle, this is nothing but F x into G x minus integral a to b F x into G x d x.

So, does it rhyme with what you have learnt in your class. So, I have when I was a student I was sort of F x into g x means you see which function is integer we will keep one fixed first function integrate the first function and you do this minus integral a to b take the derivative of the second function and into the integral of the first function. So, here what is g x is the derivative of the second function. And what is F x it is a integral or anti derivative of the first function. And of course, here f x is the anti derivative. So, it is the integral of the first function and the anti derivative you can it can will also told as the indefinite integral of the first function, so anti derivative as another name indefinite integral.

So, this is by the simple application of the product rule you simply get that the differentiation formula integration while parts formula which you simply mug up when you are giving our exams. So, you never mug up things in mathematics because concepts are more important than anything else, concepts are more important than tricks, this is something very important your concepts are more important than tricks. Go home in your notebook write it down.

So, I am happy that good amount of thing has been finished, but I would again like you to think about this. This is something too crucial to understand that this function these are very, very the constant when you have a constant function you have to very careful that when you integrate g dash t of a constant function, you get the g 0 function. If you just take thing a fixed and as a function of t, you are looking at it as a function of t you get the 0 function, but that does not invalidate. So, when you are having constant function, these are very, very special cases. So, if we have a constant function which is 0 function what is the derivative of this derivative of this is g dash t which is a 0 function at every t you understand, so that is very important to understand.

So, when I for this very specific case where you think that everything has gone topsy-turvy no it has not gone topsy-turvy because everything it actually is valid. So, because if I put a to a, it is actually giving me back 0, but g a if I put this as g, g a is not 0. So, this function is not the G t function it is some other function which is the 0 function and that 0 function can be now represented as a difference of these two. Here I can write this as the function f a, f a is 0 because I have been told that this F t. I have defined this that is if you fix a and keep on changing the t you are actually getting this function values F t. And this a to b if you keep a fixed and change t and if you evaluate the integrals this is the function value you get here that is what is happening we are not getting back G t. So, you cannot tell that this is G t. So, this is something you have to keep in mind; this is a very, very important lesson.

And thus we have done our job thank you very much and now we will now move to the 19th century; 1800s into a place called (Refer Time: 57:58), I will tell you the story later on when we start the story start that story of the one integration.