**Probability and Stochastics for finance-II**
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Lecture - 02
Fixed income securities

So, welcome to the second lecture. And we have proceeding on our journey to learn some mathematical finance, but I promised yesterday that I will talk to you about something related to the internal rate of return or internal interest rate. I will give your result which you can try to prove using basic facts about equations and at the end of this talk so today we have going to talk about a very important class of financial instruments called a fixed income security.

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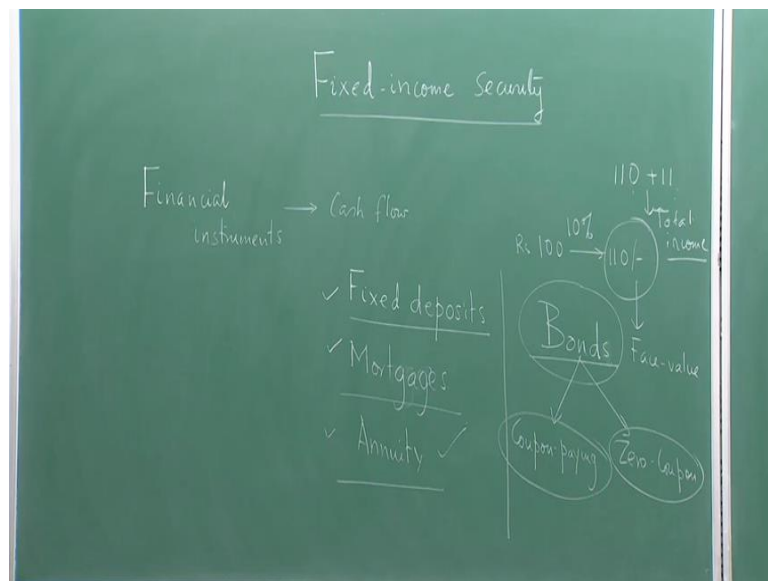
Now there are 2 parts here that we here have to look into. That what is fixed what you mean by fixed income and do you mean by security. You see there is something called a financial instrument. We use financial instruments to generate a cash flow so financial instruments will lead to something called a cash flow. And fixed income financial instruments are the ones where you are guaranteed a certain amount of income if you put in certain amount of money now. And by the word security we used very often financial markets, but the real meaning of security is any financial instrument that can be traded in the market freely is called a security so very important class of fixed income security that

you would all know is called say fixed deposit in a bank. In sometimes also called certificate of deposits in the US so in our country it is called fixed deposits so fixed deposits mean, you give the bank sum amount of money and it holds a money for a 3-year period or 4-year period or 10-year period, and then at the end of that period which is called the maturity date, they will provide you some money. In between they can keep on paying you a little bit of interest or a percentage of the final thing that you are supposed to get.

So, what you generate is essentially a cash flow. And fixed deposits guarantee you that at the end of the maturity you will get this amount of money so that is what we do in banks and most people would like to keep the money in fixed deposits this a certain growth of money. Well like me show you that this a certain growth is not at the same that you have when you actually buy a share buy shares in a stock market so fixed deposits or may fixed income securities fixed deposit is one of the fixed income securities allows your money to grow and so therefore, there is an interest rate associated with that growth. So the banks take your money and invest it somewhere else and as a result of holding your money it gives you some more money at the end wherein another important class of fixed income financial instrument is mortgages. So if you want to buy say big house when you want to buy a house usually have to mortgage sum amount of money or say you have to mortgage all your fixed deposits to the bank, so though the bank will it own all your fixed deposits. For example, an on that basis they will give you some amount of money as a loan and which you are going to pay at a monthly rate of installment. So the bank gets a cash flow so banks get some money from this sort of home loans or mortgage loans.

Another type of fixed income deposit is a pension. Which is also called annuity, means a pension holder starting from a time 0 at time 1 he receives a fixed amount of payment and time 2 he receives another fixed amount of payment. So the fixed payments every month he receives that fixed payment over a certain period of time and that is called annuity. You will your own an annual period of time, so this called annuity. I am just giving you some examples of fixed income securities. Very important fixed income securities in the financial market are bonds. So bonds are essentially like fixed deposit where, but fixed deposits are you know certificate of deposits which the banks give you and you give your money to the bank. Bonds can be issued by a company, so bonds can

be issued by any government organization, bonds can be issued by say even what to say even a financial firm can issue it is own bonds.

So, bonds are slightly different from fixed deposit. Though the nature is almost a same, but their initial intrinsic character is quite different. And both in case in fixed deposit if you are depositing with the government bank you always sure that you will get back your money after at the time of maturity, but while for bonds your very not sure. Because bonds if issued by a private company and the company goes broke a goes bankrupt they may not be able to pay you your required amount of money. So in that case bonds are not really completely secured you know items, so one has to worry that bonds are fixed income, but not such a guarantee of income like the fixed deposit in the government bank. So the 2 types of bonds one is called coupon paying, and another is called zero coupons. So suppose I had a bond which will say pay me I will buy the bond at 100 rupees now, after one year get 1 10 rupees, get 1 10 rupees. So I had 10 percent interest rate basically. Say is given a 10 percent interest rate now.

Another might be a bond might be that I will pay you 10 percent interest rate. 10 percent of the face value as coupon means in whole year I will not only pay you 1 10 rupees I will also pay you additionally 10 percent of the total money. Right so 10 percent of the face value. So this is called a face value of bond though money that you get at maturity is called the face value. So it will say that I will pay, you 10 percent of the face value as a coupon so and I will pay it every 6 months. So the first 6 months will pay you 10 percent of 1 10. Which is how much which is 1 1 rupees. So I will pay 10 percent of 1 10 means 1 1 rupees for the whole year. So at 6 months you will pay you 5 and half rupees and then the next 6 months at the end you will pay you 5 and half rupees. So basically what will now have 1 10 plus 1 1 rupees as your face value total income, face value plus the coupon value.

Total cash flow is this. So this is called coupon paying bond is called coupon because earlier days in the bond when you are the bond certificates this coupons where attached to it, so you have to send the coupon to the issuer of bond and the guy would send you the that particular amount of money. And many government bonds are 0 coupon bonds. They do not care it is like a fixed deposit it does not care anything in between and then they will be paid this amount. So bonds can default either result to which there had been credit rating agencies like moodys and standards and poors, who rates the quality of

bonds. They do not get government bond because government bonds are highly secured bonds. Government has obligation to pay, but for private bonds you afford some of ICICI bank in India, you will see the issue a lot of bonds. So in those case you honest to really check the moody's or standard in poor setting before buying such bonds. So what we will go to move into the mathematical structure of the whole thing.

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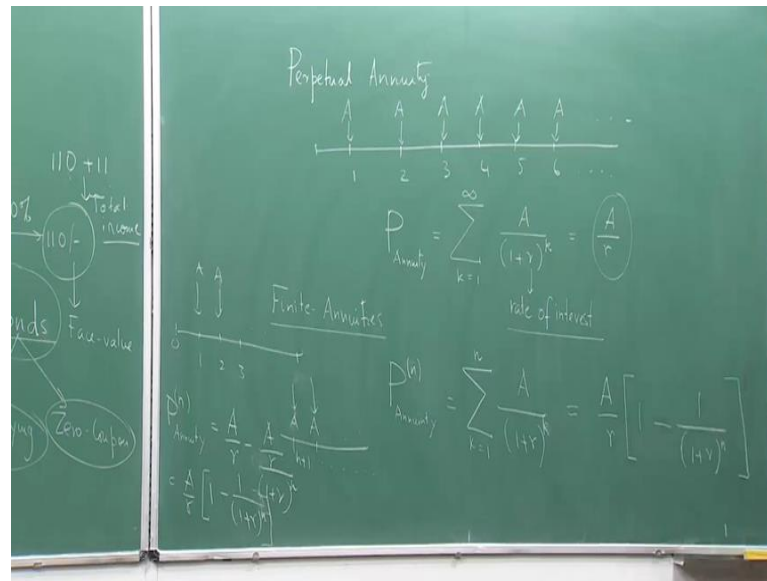
Now, let us talk about something called perpetual annuity. So we have also spoken about annuity. Annuity means every month say or a given time. So let us talk about months 0 1 2. So it is the first month, second month, third month, four months so the government a gives a pension to somebody. I receive A amount of money same. So this is called perpetual annuity. Suppose I do it for forever. The perpetual annuity is need not a feasible thing, but usually it given till the death of the person or something arrived, but perpetual annuity in some cases is the made still be retained, but perpetual annuity essentially means till the death of the person. So from a mathematical point of view what is the present value?

So, if I am going to make if I am a government organization, I am going to pay a pension to an employee what is the present worth of these payments that I am going to make. So if that is the money I now invest somewhere, I would be able to generate that particular money. So the present value of the annuity present value of the annuity just means this theoretically infinite who means give them for large times. Theoretically it is infinite. So

you know this end infinite geometric series and that would sum up to A by r where r is the simple rate of interest which is quoted by the current financial market is quoting and on an we have expected for example, for just for the record, let it be fixed over given time just for like a modeling. So one might ask why if this formula how it is useful. What is this meaning? How can you pay somebody for infinite time? So the present value you see is nothing, but A by r if I want to pay A. So this is the money I have to really invest now. That is coming use of this geometric infinite geometric series.

Now, let us look at finite life steam finite annuities. When you look at finite annuities then off course I will write this as p annuity up to n period. So I am writing it as n the so what is the principle value. So how much is the worth, how much I really into invest to give somebody of fixed payment say every month or n months? So that would be exactly we are not taking compounding then also bring compounding and all those things. So what we get up is a finite GP series. If you want to look into the thing as a finite GP series and you know how do sum of finite GP series then formula is, so these after summing the finite GP series. In fact, you can if you do not want to bother to look at the finite GP series is like this, then you can look at 2 perpetual annuities 2 perpetual annuities one that starts at time 1 and one that starts at time n plus 1.

So if you are going to pay so if you look at 2 perpetual annuities. One that starts at time 1 a keeps on paying and another that starts at time n plus 1, so if I will take the difference then I am looking at this. So you really do not have to bother with summing the GP. So then I can look at so p n annuity as some difference of the 2 perpetual annuities. One starting at n plus 1 and another starting at the time 1 so take this whole minus this, so this would give me a by r is the amount of present value that is what I am supposed to invest, minus the present value of the one starting at n plus 1, but I have to look at the present value at this time right time 0.

So, I have to not only A by r is the A by r is the money have to invested n plus 1. Right in order to make a payment from n plus 1 till infinity, but then what is if I want to make an investment of A by r at time n plus 1 what should be the investment now at 0. It should be A by r the A by r divided by 1 plus r to the power you have to come back n steps. So that should be the formula right? So if you look at it if you look at it that is exactly now you have to and you to come back n steps, right so you really have to come

back n steps, so you have to start your discounting. Because here A by r is the investment you make at time n plus 1 up to infinity.

So what is that how much to make that in start that is investment how much investment have to make at time now, that is that is a by 1 plus r to the power n, A by r divided by 1. Discounting this A by r itself, so then you get simply the formula which is exactly that what you get by summing finite GP series. As you see so this perpetual annuity can be used to understand this finite steam annuity. Now how much do I need to pay you know this is very important. Which among this 2 terms what I have express, interest rate is known to me so I am basically linking p with A. Right so I can also link A with p suppose I know that this is the amount of funds I have available which really you know about a comp in a company or in a government organization that is the fund which is available, then how do I set my annuity. How do I give pension to a person?

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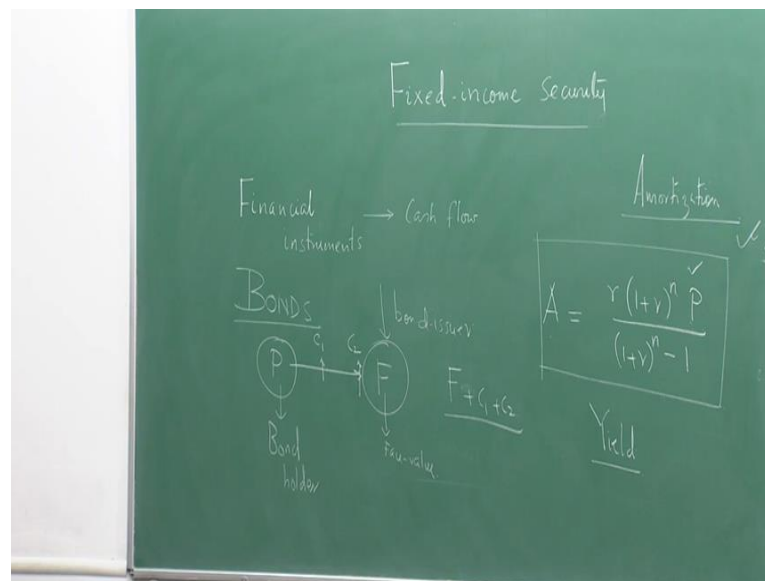
So basically then you can also write down a relationship between annuity and the principle value that is, the money that is available. Now so annuities, so from here you can calculate out the annuity as r into p, divided by r, to the 1 minus not 1 plus 1 plus r to the power n minus 1.

So, this is what is really used. You so in real organizations you will find that list of principle values list of interest rates list of annuity. Tables are made you cannot be it is not so easy to compute. This it is and periods are given so tables are already made. So

when you really sell those annuities you cannot just be sitting down and computing every time. You really have to have table based on this formula, it looks out to a very simple formula, but based on this formula huge numbers of tables are created and used in the financial market daily. So what we are now learning is, what we are learning at this movement, at this movement is essentially something related to the practical techniques used in the financial market. And for as the financial instrument that we use go more and more complex and you will see when we really take into answered into account all the things that we did in part one of the course we all come back on the board, which includes bavani and bushani to integral and everything else.

Now, what this is A or what is this A? It is telling you if I know what is how much money I have I know how much payment that I have to made every month. So this process is called amortization. So you have to pay a loan, so your current obligation is to pay a loan of this amount say x amount for a car, and you say that this is the current interest rate and if I say choose a period of 5 years, I will be able to pay this amount so that is this is exactly the way, your payment that every month that you make your payment to the banks. So those low only loan repayments that you make it the installment that you give every month to the bank say for buying a car that is actually calculated by this formula that is called an amortization loan.

So suppose I that this is my worth of the car the loan I take from the bank, this is the interest rate and this is the amount I am ready to pay every month, then what is my period. So this formula is a lot of things it is very powerful formula little thing very simple, but is a lot of information. So if I know p if I know the interest rate the bank is charging if I know what amount I want to pay, then I know what amount what are what is the total number of months or periods I have to pay and suppose n terms of to be finally, that I have to pay overall 5-month period 60s any 60. Then I will go to the bank and ask for a 5-year-old, 5-year loan on automobile. So this process is called amortization.

Now, we will study of very important class of financial instrument called bonds which we have already mentioned. Bonds as I again want to clarify a different from fixed deposits. A fixed deposit is usually issued by banks and largely by government banks also a private bank. And fixed deposits and private banks are quite like bonds because of a chance of defaulting by government bank there is no chance of defaulting you have to

pay back that money. But bonds are something on which people can default so let us now try to compute the value of the bond, the current value of the bond means why this current value is so important, mean see I am all these instruments are paying some money in the future, for which I now have to pay some money. So this present value is what I have to determine in his whole story of finance is essentially story of determining present value. There is hardly anything else all the technology or techniques so mathematical tools that you would see would actually depend on that particular thing that computing the present value. There is nothing else much then computing present values.

So, if you now look at bonds. So bonds are that if I if some money p now, and I buy I become the bond holder. So the bond holder pays the money p now, to get an amount f which is called a face value of the bond. And this will be paid by the bond issuer. Maybe intermediately suppose I am holding it for a year, I will get a payment say c 1 or if is the coupon paying bond and I will get a payment say c2, then what I total income from the bond is f plus c 1 plus c2.

Now just like this notion of internal data of return all this stories at we are trying to say you is depending on whether all these stories at you are trying to tell you, now is actually depending on the interest rate which is prevailing in the market which is guaranteed by the federal banks or guaranteed by the reserve bank of India all these things. But there is something called internal thing internal rate of return of bond, means suppose the prevailing rate of interest is 8 percent and it is I will pay you 100 rupees if you pay me 100 rupees after 1 year I will give you 110 rupees. And I am actually paying you 10 percent, so that is not same as the current given rate of interest so which means that this is something internal to the bond structure itself and that is called a yield, yield of a bond. So we are going to now compute what is the meaning of the yield of the bond. And with the computation of the yield of a bond we will stop our discussion today.

So, what is the yield of a bond? So let is let me now do that calculation. It is not difficult to calculate and here we will you compounding the way that is really used in the financial market.
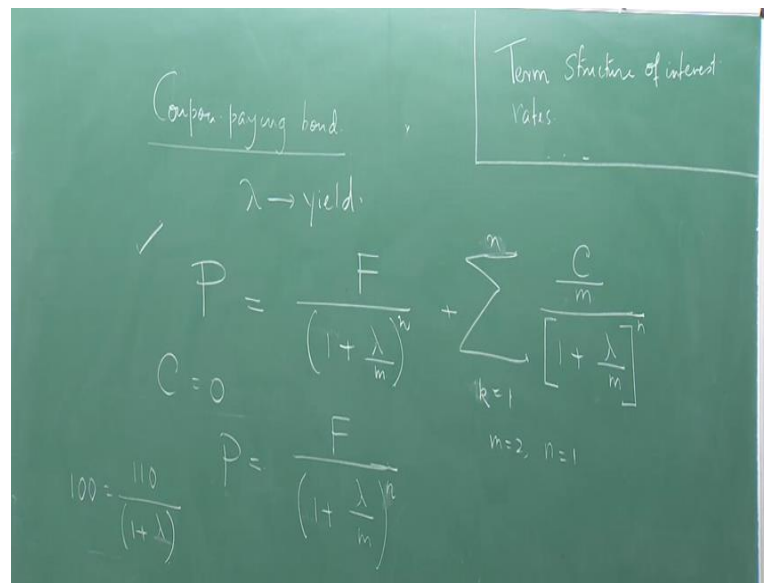
Suppose a bond has n periods to go; so one maturity will come after n periods of payment suppose I am taking a coupon paying bond; suppose, I am talking of a coupon paying bond. Now because I am talking about a coupon paying bond, and let me divide the whole thing into m periods and their n periods too, now which means that if the internal rate of interest of the bond which I called yield, which is the intern internal rate of interest suppose that is lambda, suppose that is lambda, lambda is an internal rate of interest so if this is my internal rate of interest, how much I actually have to pay if I want to by the bond.

And the bond pays a coupon value c total coupon worth c and that they pay me in m periods each worth in c by m, at every period they pay me c by m. So the face value the, to buy the bond the money that I have to pay is a face value of the bond. So I do not charge because a m periods where coupon is given, I do not charge a lambda every time and at the what the what lambda m at each period. So and I discount face value over each period, plus over the n periods I am making a coupon payment. So what is the coupon payment? The coupon payment is c by m and that is also discounted by the same interest internal interest rate of the bond. That is if have to you have to discount everything to get the current price. So this is the price that you have to pay for a coupon paying bond. If c is equal to 0 that is a 0 coupon bond. Then you are actually paying, so if you know, how many times the coupon is paid. So if the coupon is paid twice means an n is 1, sorry n is the number of periods or number of n is the number of period remaining, and m is the

number of times the coupons are paid. Suppose number of times is coupons are paid is 2 and n is 1 year 0 1 year. So you will put 2 here you will put one here.

So, one plus lambda by 2 by f, that would be if this is a 100 and this is 110, this 110 and this 100, then the yield is so yield even calculate out if this is I pay 100 to a 110 rupees, so I pay a coupon twice at the interest rate, and I sorry I do not pay any coupon so because I do not pay any coupon so I pay everything at the end. So you see so m also becomes 1 and n also becomes 1. So this becomes so this lambda is same as the given interest rate, but now if I am paying come coupon say for first 6 months and then next 6 months, so it is c and then this becomes 2 and then this becomes 1 this becomes 2 and this becomes 2 this becomes n becomes 1 so if say if your m is equal to 2 and n is equal to 1 and say this is 5.5. Then your yield, your f is 1 10 this is a p, then this lambda is different from the market interest rate. It is not 10 percent interest rate right.

So, with this formula we end our discussion today, and in the next class we will talk about the term structure of interest rates which will start discussing yield cause and their behavior. So just let me give you the problem I will just give you the problem which I had said I will in the last class. So here is the problem which I want to you to solve which is independent of the assignments. So do not consider it as an assignment problem, but I would like you to solve or even check it out in the book or try to learn. This is the interesting little thing and then that we you that that we end it. So it is a theorem on the interest rate internal interest rate.

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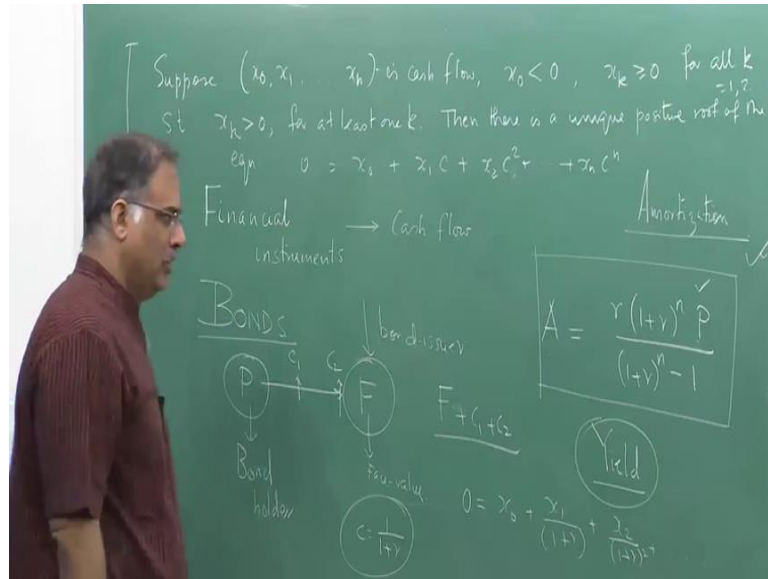
So, suppose x n is the cash flow or n periods, is the cash flow. And 1 has x naught strictly lessons you means you are paying the money and after that you are getting the money just like a 0 coupon bond just like a 0 coupon bond. Or it could be that you have may a huge deposit in the bank and then you are gradually taking of every month right that, so for all k equal to 1 2 such that, x k is strictly bigger than 0 for at least 1 k. So you could have that for every month you are supposed to take some money every month, you are not taking money may be 1 month you took some amount of money is positive. Then there is a unique positive root of the equation.

Now what is this c? C is related to the interest rate you know if you look at the internal cash flow internal rate of return that we discussed yesterday. It was like this so c is actually equal to 1 by r. So which means if this is what happening then as the unique internal rate of return that is what you really have to prove everything do it in the class was it can be thought about the problem.

So thank you and we will start discussing term structure of interest rate we will talk about bonds the what is the issue of duration and what is the how duration effects the bond prices and all those things. So we will discuss it over 2 classes.

Thank you very much.