

**Probability and Stochastics for finance-II**  
**Prof. Joydeep Dutta**  
**Department of Humanities and Social Sciences**  
**Indian Institute of Technology, Kanpur**

**Lecture - 12**  
**The Binomial Model**  
**(Lox-Ross-Rubenstein Model) [No Arbitrage Pricing]**

We are going to start our discussion option pricing our goal is a final, a black (Refer Time: 00:19) formula which is popular thing is a part of financial of finance and we are going to start on the very simplest model actually use in practice it is called the binomial model or Cox Ross Rubenstein model or CRR model the people who are started with this and gives you no arbitrage pricing that the price that you quoted in the market would be free of arbitrage.

So, the price of what price the price one have option we will come to all of them. So, here we start with the scenario where there is a market with one asset and you are also access to money market means risk, free asset means bank or something like that you can put money in fixed deposit, is one the market as one asset.

(Refer Slide Time: 01:12)



So, very idealize system this are the models are built in finance morals of building physics you take the most idealized system and try to build your model here in research we also take the simplest situation and try to analyze. First then going into much more

complex situations one asset and money market this is the scenario you have money market is a bank something like that banking.

So, this is the market that we are going to study. So, suppose asset means this asset is a risky asset this is not risk free asset risk free asset is this one. Now in this procedure we will just consider two periods we will invest money, now and get some money period time period  $t$  equal to 0 we invest some money and the get something at the end right that the period 1. So, let  $s_0$  is the price of the risky asset at time  $t$  equal to 0. So, you paid  $s_0$  equal to buy this asset and  $t$  equal to 1 we assume that there will be only 2 possibilities the price either goes up or price either goes down you toss the coin, if it is head the price will goes up if you toss up it is tail the price goes down.

So, which we write  $S_1^h$  and  $S_1^t$   $S_1$  is to denote the time period  $t$  equal to 1, we will just look at this period the trading is only just for one period we want 1 month, 2 month, does not matter time period  $t$  equal to 0 time period  $t$  equal to 1 and we make very, very simple assumption that  $u$   $S_1^h$  can be is  $u S_0$  and this  $d$  of  $S_0$  where  $u$  is strictly bigger than  $d$  right means the price would increase basically you should always have  $u$  bigger than  $d$  less than 1 that is the price either becomes more  $s_0$  or it reduces from  $s_0$  of course, if  $d$  is bigger than  $u$  you can switch your coins my head you can call the head as tail as head. So, this is the basic scenario basic model now because of the money market because you have accessed money market you can barrow from the money market and you can put money in the money market. So, they offer you by this  $r$  interest return rate of return or interest rate. So, if you put 1 rupee at time 0 at time 1 you will get  $1 + r$  rupee that is it.

Now, what is arbitrage we have already spoken our arbitrage here and as well as the previous part of this lectures the arbitrage is a way of making money without making any investment I can make a 0 investment put money in the market and I have a rule of positive. Probability of not or positive probability of actual making money, that is something very important. So, arbitrage is arbitrage means 0 investment plus 0 investment would imply positive probability let me tell you how suppose I buy 10 pens for 30 rupees for each. So, I get 300 rupees. So, 3 rupees invest get 30 rupees then, I come and sell those pens 35 rupees each. So, I have 300, I get 350 rupees.

So, I cover my investment 300 rupees and 50 rupees extra. So, I made 50 rupees without

investing actually anything. So, that is called an arbitrage market system routinely happen, but when an arbitrage starts occurring the arbitragers will start there trading heavily and that will be very importantly observed actually lot of people do arbitrage in they look at the b s c index and n s c index they look at the price in stock BSC and NSC.

So, buy one get exchange select another exchange and make money just like I took pens, but the market will always very soon adjust both the prices and both the stock exchanges will become same I have experience of this I was trying to buy things buy a water bottle on the railway station. So, they were asking 20 rupees for a cold bottle. So, then I go and check with another one he was asking for 15 rupees. So, I came back to the other one lot of had a lot of crowds. So, I came back to the older shop and I told people do not buy here the other one is giving at 15 rupees. So, everybody started to leave and he says no wait please, I am giving you the thing at 15 rupees the prices of money is equalizes. So, market does not allow arbitrage. So, these are the very important rule of the market. So, here we have UDR 3 quantities given to us what should be the relation among them. So, that I will guaranty this fact that they will be no arbitrage the relation between them of course, is. So, this is my no arbitrage condition we will say why no arbitrage condition.

So, why should this be a no arbitrage thing let us see let us look at it. So, let us see whether if I do not follow this we get this thing arbitrage come suppose  $d$  is strictly bigger then one plus  $r$  then what would happen. So,  $s_0$  into  $d$  sorry equal to,  $s_0$  into  $d$  they should be greater than equal to I am taking the converse this does not happen greater than equal to happens. So, what happens here I can go to the bank and barrow money at interest rate I can ask it is not of money taking that, it is not amount of money I go and buy that stock by that unit is this is price of one share. So, buy that one share at  $s_0$  and then when it comes I sell it at either  $s_0 d$  or  $S_0 U$ , but suppose I sell it at  $s_0$ , if  $S_0 U$  comes which means because  $u$  is strictly bigger than  $d$   $S_0 U$  is strictly bigger than  $s_0 (1 + r)$ . So, here head can come with probability  $p$  and tail can come with probability  $q$  while  $p + q$  is; obviously, equal to 1. So, I dint write it could half also just or not.

So, if  $s_0 d$  comes and  $s_0$  and if  $s_0 d$  comes  $S_1 t$  comes and it is equal to  $s_0$  into  $1 + r$  then I have no problem I still can. So,  $s_0$  into  $1 + r$  is the money has to give back to the bank. So,  $s_0 d$  is equal to  $S_0 (1 + r)$  I get the  $s_0$  and give it to the bank. So, I had no loss, but if  $S_0 U$  comes  $S_0 U$  is strictly greater than  $S_0 (1 + r)$  I can still give that money to the bank and get some profit. So,  $S_0 U$  comes  $S_0 U$  minus  $S_0 (1 + r)$  this is

the money I can make with probability  $p$  probability  $p$  this is the money I can make. So, I have invested nothing and I making money with I have a positive probability on making the money making some money. So, if this happens arbitrage as happens you can argue out this part what could happen if this happens right.

Now, here our job would be to find the price of a European call option. So, I will now explain what is a European call option we have described in the part one of this course, but I still do now specially for the new comers.

(Refer Slide Time: 11:34)



So, now we are going to talk about European call option what is a European call option in the European call option in this simple set up suppose I have a stock and I own that particular asset and I come to which I bought with the amount or whatever I or I may not even have the stock right the idea is that, I come in contract with you say and say at time one I am ready to sale the stock at the price  $k$  and where we would assume or it will make much more sense. So, I sell the option to somebody that is called a call option I am ready to sell the stock when, you are selling the stock it is called call option. When you are buying a put option, I am ready to I am coming to contract which is called option contract that I am ready seller of the option is ready to sell the stock to the buyer of the option at  $t$  time equal to 1 at the given fixed price which their agreed price  $k$  which also called the strike price.

Now, the person who buys the option as a right to, but the stock at time 1 with the price  $k$

is under no obligation to buy the stock. So, the buyer of the option has the right, but not the obligation. So, suppose it is  $S_1$  that is my final price then I will buy cheaper from the market why would I take it from a higher price from, but if it is at higher price then I or seller of the option buy the think of the market at the higher price and selling at lower price is incurring the loss of  $S_1 - h$  is minus  $k$ . So, that amount of loss will come to me. So, since I am exposed to this while you are not exposed to this is contract involves premium because I exposed to this.

So, that I would have to possibly have to sell stock at the much cheaper price because, what you will do you will buy the stock from nearer cheaper price and sell it higher price in market and make that money which I will be losing I cannot do anything because I am obligated now to sell you the stock I am under obligation, but the person, who is actually holding option contract is under obligation I am the seller of the option is under obligation. So, the value of the option of time 1 which is let us write as  $V_1$  is the max of  $S_1$  I am not writing  $S_1 - k$  I am just giving a dot here and 0.

So, if it is heads  $v$  on  $h$  the; obviously, the max is  $S_1 - h - k$ , but it is tail is 0 sorry  $t$  because if it is tail this is negative and this is 0. So, this value of the option this is what of option of the value of the option not for the guy, who sold option, but who is holding the contract is value what for him is the value of the option we are suppose to find  $V_0$ ,  $V_0$ ,  $V_0$  is the value of the option time 0 and that is the price the option seller will ask from it is buyer. So, what will I do with this money that is called the option price  $V_0$  is call the option price.

So, what shall I do this money what is my intention what I suppose to do with this money right I am suppose to now take this money and invest in the market. So, I can invest the part of this money in buying the stock part of this money in put it in the bank. So, at the period time  $t$  equal to 1 the total asset. So, I create a portfolio one of the stock one with the fixed deposit and my risk free asset. So, the time  $t$  one the total value of my portfolio should be exactly equal to the value of the option. So, whatever loss I would in occur will selling the option. If I have to sell is hedge or taken care by the money that I be get by actually trading market with this option price.

So, this is this process is called hedging the short position the short position when you are selling when you seller of the option that is seller of the option will sell the stock you

are in short position when you buy the stock in a wrong position hedging the short position is also called the method of replication this hedging is a fundamental fact in a finance market all this investment executives investment bankers investment gurus the real job is to hedge the exposure to this. So, if you are some kind of studying options then you are exposing to that. So, who sells them and buy the option options are not bought by ordinary people like us going to the market and they are buying options are bought by very big companies for examples air lines coming to option contracts with oil companies because oil prices fluctuate right.

So, they come into contracts with various oil companies right particular state can come into option contracts with electricity companies particularly industries for example, there are cast speculators. Who can buy options, but an ordinary person should not buy an option because in the real market 95 percent of option become worthless they are not actually finally, executed only 5 percent of options are executed. So, in general option is tricky subject when, you are just talking about ordinary investor, but for very big companies very big organization option is a very valuable instrument and it is called a derivative security option this term option is called a derivative security. Derivative is not derivative of calculus derivative security because, the price of this option depends on the price of the security there is risk risky asset right. So, that is why it is called derivative security.

So, why then we are pricing options what is the interesting pricing options. If I am ordinary investor is going to really use it, but a huge amount of investment in the market not just by the ordinary investor or organization. So, essentially the option pricing not only brings out of very beautiful aspects of the mathematics required to actually the price option, but this techniques are very useful to very big organization. So, investor forms are very interested in doing this option pricing because, then they get a lot of we know projects lot of money from the very big organizations very big organizations may ask investment in bank to have buy and sell options with for them and do a every things that is required. So, that is the option pricing is very important, but I would really tell that if you think, if your thinking going to the short marketing investing never if you are ordinary guy like me do not invest in do not just invest in options just to standard trading selling or buying stocks that is all.

So, that I S1 aspect of it now let us sees what we are supposed to do. So, the market in

which we can completely hedge our exposure to risky the seller of the option can completely hedge he is this are exposure to risk then such market is called complete market and the portfolio that we create with this money  $V_0$  is called a replicating portfolio because you are trying to replicate the option. So, right complete market is absolutely the fundamental notion when you are talking about European call option pricing then the black idea, but in real market may not be complete that may not be replicate the pert portfolio do of perfect hedging this is called perfect hedging that replicating portfolio the worth of the replicating portfolio is exactly same as the worth of the option.

So, it is very important to note we are going to study only complete market we are not going to make a study in complete market in complete market is the completely different ball game as completely different analysis I cannot promise you whether I can give you some examples in the incomplete market also at the very and unless complete this if there is time I will do if there is no time I beg forgiveness, but it is very important those to start with finance to understand what we can do in a complete market if we are in idealistic situation what we can do. So, today really we do not have time to do the complete analysis of how to get this option price which we will take up tomorrow tomorrows class we will take up how to do this option pricing now how to get  $V_0$ .

So, the worth of the option at time 0 is the price that you pay this is put in your head you have to put in your head that you are in a arbitrage that you have to put in your head at the market is complete we have to put in your head that you will always able to get a replicating portfolio. So, that is what makes a complete market. So, what about the put option there is something call the put option in a put option what you do is very interesting in a put option you instead of you been seller of the stock you are buyer of the stock. So, in that case scenario reverses.

So, if you coming to contract with me then I am obligated to a buy a stock from you had given a price  $k$  right at the given appointment time. So, there is expiry time for the option. So, here in this case our simple guess you enter the option contractor time  $t$  equal to 0 and time  $t$  equal to 1 the expiry time of the contract and the European call option as a important feature you can only exercise that European call at the expiry time of contract cannot do in the middle American call options can do it in the middle will not study, American call options it is pretty involved for  $t$  equal to 1. If you say this is the

expiry only  $t$  equal to 1 you can actually when you discussing you are assuming  $t$  equal to 1 is the expiry time and because there is no other possible time in our simple model.

So, this exercise as to be at only expiry time you cannot exercise your right option at any other time. So, our put option what would happen, if the price goes down you will come to me and I have to sell you at higher price that is I have to buy it price  $S_{1,t}$  instead of going to the market right. You will now buy it at lower price at  $S_{1,t}$  as an option holder and you will come and I will be obligated to buy from you higher price when, I know I could bought it easily from the market at lower price. So, I would be having a loss right. So, this whole thing will change if you have a put option right this then  $V_{1,t}$  become  $S_{1,t} - K$  right  $K - S_{1,t}$ . So,  $V_{1,t}$  becomes  $K - S_{1,t}$   $V_{1,h}$  would be 0. So, that is called a put option. So, there are some issues like put called etcetera which we are going to bother now tomorrow's job would be to see how to compute  $V_0$ .

So, our focus to tomorrow we to step by step compute  $V_0$  and then, see what are the consequences are associated with it and once we will do one period our next class we will discuss the multi period model and once we do the multi period model we will go to the continuous time of course, one can show that why this guy, now will single period model is or multi period model is important because, when the number of periods is sufficiently large it can actually replicate what is happening in the market can be modeled very well binomial pricing techniques are heavily used in the market to very easily find the option price because for  $n$  very large it tend to black price that is one of the reason that they are used in the market of course, there are many softwares in which exactly used the black price. So, after doing that make are you might feel it is slightly arbitrary jump neither arbitrary jump counting price system that the trading is going on at very small interval trading of course, does not go and we say again it as little gap, but the theoretical point of view you can consider to be continuous.

So, thank you very much for your attention. So, we will be going and trying to find the price  $V_0$  in the next class.

Thank you.