Probability and Stochastics for finance-II
Prof. Joydeep Dutta
Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

Lecture - 01
Fundamentals of Interest Rate

So, we are starting our course call probability and stochastics for finance part 2. So this course would largely be the course on mathematical finance, quantitative finance. And this course would depend on the first part. So those who are not attended at the first part would find the link to the first part in the course page. The interesting fact is that, we will not start immediately from very advanced mathematics, which was our main stained in the last course, but we would rather choose a very simple step. We would take up some vary basics idea of the financial market, and try to study them of course, using mathematics. And then when we will come to the study of derivative securities, the pricing of derivative securities in a financial market, like auction and other things like futures powers bonds etcetera. Then we will pick up from where we had left in the last course.

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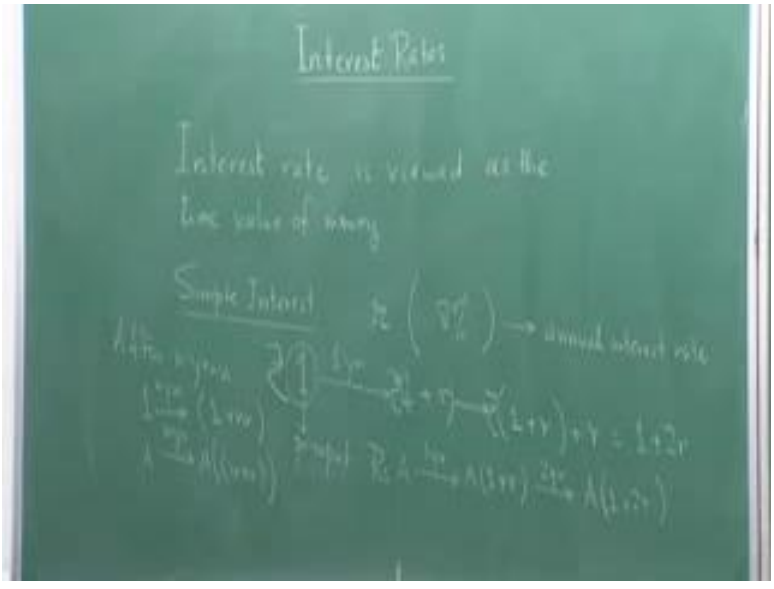

So, essentially this has 2 parts. This courses has this this second part is essentially 2 part the easy part which we study from a book call Investment Science by D G Luenberger. And the second part was we talk about derivative securities. We can go head and really

do more advance and several books would be involved at the time will you later on so here we would go largely by this book so one of the most important financial instrument that we come across in our daily lifes in is interest rates so everybody is link with banks. Most of the people who have some income is link with banks. And one of the things which is crucial to an economy is this notion of an interest rate for financial market. This notion of an interest rate is of supreme importance so what is an interest rate why do we pay something call interest rate so this is this is a sought of financial instrument so financial principle by which financial market works.

So, interest rate is viewed as the time value of money so what does it mean that, it is a time value of money. It means the following: it says that guys have a look I borrow you say 100 rupees today. And you would take you say that you would hold it for 1 year or 10 months and give me 10, 10 rupees each. Is this a good proposal that you will give me 10 rupees each? Wow to a general social emotional set up it would be an absolutely perfect proposal, but if we are talking about the financial like a bank give me you 100 rupees and you want to give in 10, 10 rupees each, they would charge you something call interest rate because you are holding their 100 rupees for 10 months which they have not been able to invest somewhere else so now, you also understand that, when you deposit a money in a bank account and you keep it holding for some 10 months, 5, 8 months, 1 year. You see your money grows a little bit it has some more money than you are actually put it. Here at the bank is paying the interest rate because when you deposit your money in your bank account you are essentially giving a loan to the bank. And hence the bank is now holding money and he is the bank is paying you back a premium which comes in a form of interest rates.

So, interest rate of course, in real life borrowing the rate at which the interest rate for borrowing money from a bank and the interest rate for depositing money in the bank are different, but we will here mathematically when we model will talk about something call ideal bank were these 2 are same so let us going to the very basic thing so the something is called simple interest. In simple interest what the bank does is to announce an interest rate r so it is given in percentage would be say 8 percent, which is sought of interest rate on the fixed deposits now in India so if I have 1 rupee now so anyway this is an annual interest rate so if I have 1 rupee now, then after 1 year this money becomes 1 plus r. Now suppose you still hold the money for the next 1 year so what they will simply do, they

will simply this is called a principal and r is the interest so these principle so you have kept in the symbol rupee 1 year. And this is the rupee you are getting back which is 1 plus r. Now what they will in the simple interest in are you what the people will do the bank people they will never apply the interest rate on this money. They will apply the interest rate on given principle again add whatever you have now got that interest rate would be added to this in a simple interest in have you.

So, the money you get is 1 plus r plus r so it is 1 plus 2 r so here is very basic because if you now keep say an amount of rupees A so after 1 year this becomes A into 1 plus r because 1 rupee becomes 1 plus r so A rupee becomes A into 1 plus r simple unitary mathematics. It is nothing a basic arithmetic. And then up to 2 years it becomes and so, on so after n years so 1 rupee becomes 1 plus n r so A rupee naturally becomes A into 1 plus n r, but these you see the growth of the money is linear. Linear in terms of r. Suppose you want must faster growth, with everybody interested in the faster growth of money. The bank is interested when you give you a loan to take much more money from you then it is actually loaned you. While you when you deposit money he does not want to give you as much money as you think you should get. That is why that is why the borrowing rates and depositing rates interest rates are different so in ideas set of be the same because that is why the math becomes easier.

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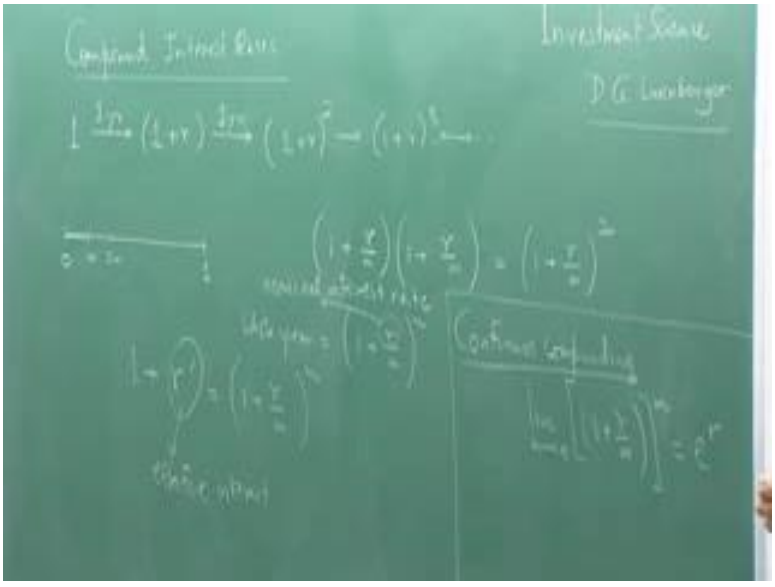

So, that is why the bankers invented, what is call compounding compound interest rates so what is compound interest rates? What this guys say after 1 year your principal as now become 1 plus r now instead of charging interest rate of 1 rupee the original principal, I will now charge interest on the new principal, so 1 rupee now becomes 1 plus r after 1 year and then after 2 years in 1 year more 1 year so basically 2 years so here also I should write 1 year so after 1 year more it becomes 1 plus r into 1 plus r so this is now the capital A, A into 1 plus r so in 1 plus r square and so and so forth. Sorry so that is the flows a cash. Now banks are clever enough if they loan, from you they do not do compounding on yearly basics. They do compounding on a monthly basis credit. Cards absolutely it has a compounding on a 20 days' basis right.

So, suppose you have a whole set of years. This 1-year period and then they can divide the whole period in to small m blocks. Now each of these m blocks small 0 plus m, 0 plus 2 m, m, 2 m, and so on. So each of the 2 blocks they do not charge with interest rate r, but the fraction of the interest rate for that particular block. So what they will now do is that we have 1 rupee and at the end of mth 1 the first mth block you will be charge 1 plus r to the power m right. And us there are the second mth blocks will be charge 1 by r to the power m again. So this is will become a first 2, 2 such consecutive blocks. They have taken your money now become this basically that is what happens in the loan. That is why your credit card you know things goes very fast.

So, now what they can do is a following. Now you have m periods in a whole year. Than in the whole year, for whole year, this is what you really have to pay back; to the sorry, whole year you have to pay back to the bank 1 plus r by m to the power m. Because m blocks the whole years has been divided to m could be 12 we could be every month right, but it will 20 days also. You can imagine what 20 days how many blocks they would mean pretty high. That is what credit card us. They gave you 20 repayment time if you do not interest, it put on interest put on interest put if you do not repay suppose you repay some x amounts whatever is left interest is put on. So basically we it is assume that you have not repaid back anything, this is what your interest is of course, you repay that things will do down.

So, this this calculation what has been done is with the ideas and if you are taking a loan you have not really repaid back. Right now this is what you will accumulate. Now r is the simple interest rate which is already announce like 8 percent, but now the question is

truly have you given have you really been charge at the rate 8 percent simple interest. So this 1 r is called a nominal interest rate which you always announce by the bank. I am charging you this interest rate. Nominal interest rate, but I bet you will see exercises that if you really want to find what is the actual interest rate he has charge to be, if I look at the whole thing and simple interest at that after 1 year he has charged been 1 plus r, but he has compounded at every mth interval right. So select r dash with an actual some sought of interest rate, which is been implied on the 1-rupee loan or deposit whatever 1-rupee loan which will finally, give me what the bank has charge me. You always find this r dash would be bigger, than r dash would always be bigger than r. So this is call the effective interest rate

So, I am bank is telling whether I will give you 10 percent interest rate of 2 there actually over all yearly they are charging you much more. So that the whole idea this is call effective interest rate. So compounding is a very clever way to increase your money, so when that is why when they will charge you, when they will give you a loan their interest rate charge would be higher a personal interest rate charging a bank is 15 percent, but if you deposited a fixed deposit it, you fixed your money in the bank for certain period of years, they will give you at a rate of 6 percent 7 percent because there also they have to compound the whole thing. So in that if you fix your money for whole 1 year this is what you will get. So that is why when you deposit your money they as give you so less interested rate was the effective interest rate is always higher. That is the whole point. You can try out tried out with some example which will give. Also there is something call compounding at continues rate means my mth blocks I met m blocks.

So, m is say 20. 20 whole years is you have into 20 blocks, m could be 50, m could be 30, m could be 100 m could, I can be charging you at every 5 minutes, but I can be charging you. So there is this idea in bank specifically that what happens if you charge at a continuous rate. Though it does not really happen in banks; they actually have some time period, but mathematical point of view, what would happen if you do continuous compounding. What would happen if you do continuous compounding? Continuous compounding is a dangers thing because if says basically the money is that I should get or bank should get of the end of one year is this. And you know, what is the answer to this? What is the answer the answer to this is e to the power r? You see though we started with very basic inventory method they have come to pretty important limit in calculus.

So, many banks effectively use this formula. They put use especially where you are having loan they put your exponential compounding. And that is amazing! That is why when you fail to give money to repay back your credit card loans, you suddenly see that you really have to pay much more than what you thought. It would be difficult to paying an interest at 40 percent which is ab absurd basically, but that that what it happen. This is the reason because they will make them very high. They will charge you for very less time. Now for example, I could also do something else. I could do the compounding in a slightly different way. I will just rub this parent to the compounding on this spot. So I can do the compounding in some slightly different way.

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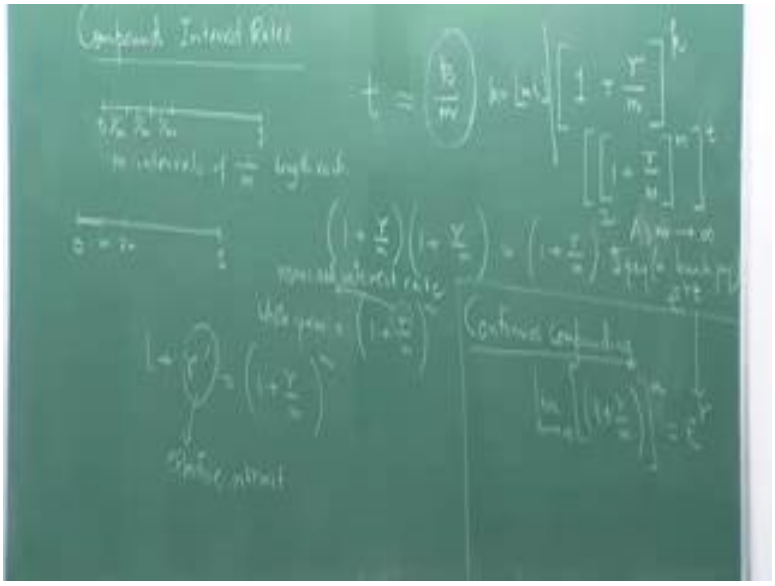

So, here the compounding is done in the following way, is that I can divide for example, the 1 year period into intervals each of a fix length 1 by m. And then I want to know how much money the bank should take from me, if I invested 1 rupee or you should give it to me 5 invested 1 rupee or taken a loan of 1 rupee. Then you should take from me at the time period t, t equal to some arbitrary time t. Right it is not just some given 1 year or 2 year or so some arbitrary time t is interval is 1 by m. So basically now I am divided this into small intervals. So 0 1, here so each of this wave divided this in to small intervals 1 by m each and let me divided in to m intervals so I will have divided the whole 1year period into m intervals of 1 by m length each. So if m is 12, basically I have divided by them into 1 month; each right 1 by 12, so I multiply by the 12 so get 1 year.

Now, what happens? If I want know at any given time t, what is the amount of money I should pay back to the bank or the bank should give it to me. And t should be expressed as some time period k by m. So the time t should be some k approximately some k into 1 by m. Do an effective calculation. So the m periods, so basically what I will now do the amount of money I have to pay back at the end of 1, 1 period is 1 plus r by m because m is the number of periods. Now what we have to do is that so I do it for k periods right, here I am done it for m periods I would have done it for 2 periods 3 periods 4 periods so suppose I have done for k periods, at this k is the period so t is the time any time I have given. So any time t are, fixed any arbitrary time in the year. So how many periods I have crossed so we take basically the lowest integral 1, but the higher integral 1 basically it should be some sought of this sought f things or rather a if rather it should be k should be m t these sought of things.

So, now I have this is the amount I should take for that k period right, but what you k, k is actually some sought of m into t, so I will just assume that I been able to get a k is upper one and m t has been chosen such a way that I am getting integer for the time in. So now, the m goes to infinity. How much should I pay, how much should the bank ask from me at the end of the time t? You should ask from me as t to course t tense to infinity. I pay or bank pays e to the power or t. Thus the limit of this is e to the power r t m tend to infinity of course, I thank you as m tends to infinity, so as I increase this number of partition I as make the partition smaller and smaller. Which mean I am making these lengths on a I am making m in the number of m is larger and larger hence this partition submits smaller and smaller and smaller, means I am asking you from money at almost every interval. Then this is very second possibly then, this is the amount essentially have to pay at the end of time t at t is equal to 1 the full whole year then I pay e to the power r. So in t is 1 you pay to the power on.

So, if t is t is less than 1 you pay e to the power r t. So it a less now will talk about something call present value. Present value future value and their relationships. We will first talk about what is called a present value, the present value of a certain amount of money. So this is a very important concept and it to be seen throughout or study of finance of course, we have doing quantitative things it is mathematical modeling were idealization would come in which may seem to be abstract they are has been abstract from the reality of course, you cannot modal reality with mathematics because when if

you look at physics. You know so difficult to model reality so complex and here that because there is issue of human interaction. Real modeling is really a tough game and that is the real issue of in economics how to more use more effective mathematical models about finance is still much more organize and so mathematical model can to work much better.

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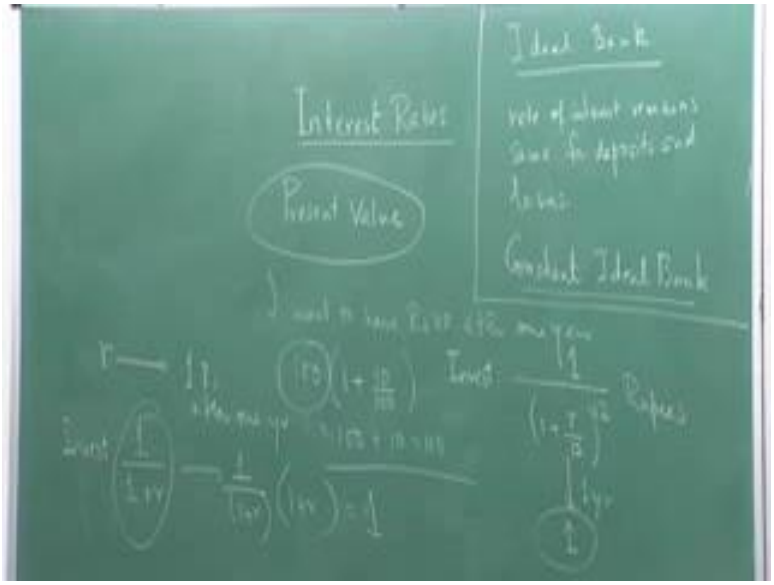

Now, I talk about present value. Present value is how much money I should invest now to get rupees 110 after 1 year. So I am going to talk about this important question. So I am, I want to have. So for very simplistic point of view if I take 100 rupees and invest with in a bank which gives me a 10 percent rate of return and in the very simple situation right.

So, 100 into 1 plus r 10 percent, so r is 10 percent so 10 by 100. So it is become equal to 100 plus 10 which is 110. So if I now we 100 rupees now pay 100 rupees to a bank and after 1 year if the bank actually gives me at a rate 10 percent I will get back 110. If you give me lesser one, I have to investor likely more money less interested is less. So these 100 rupees is what I called the present value of 110 rupees. It is so the 110 rupees after 1 year now what is 100 rupees to me if a bank gives me a 10 percent interest rate for holding the money keeping the money. Right so if my interest rate is r right and I want to have 1 rupee after 1 year what should I do? I should invest 1 by 1 plus r. Because now this money after 1 year the bank should pay me is A into 1 plus r. And that is everywhere

1 rupee so this 1 by 1 plus or quantity is called the present value of 1 rupee. So where want to get 1 rupee after 1 year. Now I have to invest 1 by 1, 1 plus 1 amount of money this is the idea so these are very important idea which is there in all over finance.

So, what do the happen if I am talking about compounding. Right so if I want to talk about compounding, if instead of this interested I talk about compound interest rate, say for 12 years, so I have m as 12 so 1 by so I have 12 periods. It so then if I invest this, invest this amount of money 1 by 1 plus r by 12 rupees if I invests now and if r be the nominal rate and this is my I have this 1 monthly compounding then I will get A after 1 year 1 rupee. So now, if the bank gives me compound interest rate this is my present value of the rupee 1 after 1 year is now what thing what worth some amount. So it is you less now something which what is you some x rupees now if you hold it in a bank because you get some positive money extra money that so you that that increase the current value of the current money. So you always hold some lesser one you know bank now which will use some higher money later on right. So this is called present value, we will now come to the discussion of what is called an ideal bank because our discussion of finance throughout whenever we talk about bank or money market to come to the concept of an ideal bank.

So, what is an ideal bank? Ideal bank is banks were the rate of interest or remains same whether you deposit cash with it or you take a loan from it. So ideal bank is rate of interest remains same for deposits and loans. Meanwhile you have to understand that the interest rate might change, if you for example, change your period of holding. Suppose you want to our 2 year hold. You want to kept money for in a bank for 2 years. Or you have bought fee deposit for 3 years then interest rate could be something else. And if you put money for 5 years then interest rate would be something else, but now suppose you of may be put money in a fixed deposit and then interest rate is r, but you take the same money with a 3 same money as loan with the 3 years' repayment period then the interest rate should also be r. So that is a meaning of an ideal bank. So essentially the term which is of a new is called constant ideal bank, so because the rate of interest remains constant for both this cases whether it is loans or deposit.

So, these sometimes also called constant ideal bank, but we will just call ideal bank to mean that for a certain type of transaction interest rate of deposit or interest rate of taking loan reversing the proceed year is same. The way I just explain now. So there is

something called future value and present value of a cash flow stream. What is a cash flow stream? Suppose I must salaried person right, and I am getting money every year, every month actually. I am getting salary so I said this year this month I will keep x not amount of money and put it in the bank. Next year I will get next month, I will x 1 amount of money and put it in the bank. And I assume that they will give a simple interest rate they compound it fine I am just taking a very simple statement simple interest. I put some more money in the x 2 money in the bank and so and so forth.

So, something is called the, we will sobered discussion by taking about the present value over cash flow stream. And the future valve of a cash flow stream. So cash flow stream is the amount of money I am receiving. It could be that I am receiving at 1 month.

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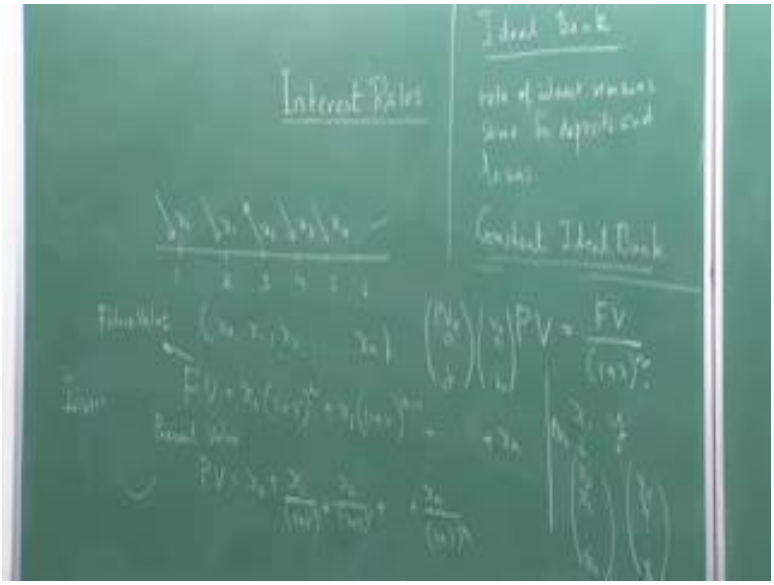

And giving in some month that could be so, it is a cash flow streams. So there is a flow of cash suppose this is my year. I am say first month, second month, third month, fourth month, fifth month, sixth month. I can go up to tenth month or any periods actually. So here have a cash flow x, not here have a cash flow x 1 it could be outside also. It could be giving money here I could have cash flow x 2 x3 x4 and so on.

So, basically I have a cash flow. So I do it for n months. So how much money I will get at the end of any months? So that is call a future value of the cash flow which is very natural if have x naught now I keeping the bank up to n months I get this money. The x 1 money I can hold up to only n minus 1 month because after n month I am taking out the

money so this will be plus x n, x n is the money were I deposited n month and then they will be no interest rate because I will take it up. Similarly, something is this call the future value. There similarly something which is called present value, right? So now suppose I have the same cash flow or would the present value of the stream, r is the interest rate so the present value is if have x naught this is my present value at zeroth period. I am zeroth period I am looking at the value so; I am expected that I will receive this amount of money right. So the value which I will receive at time 1 at time 0 it is value.

So, if you look at this for example, here so at this money, is my present value of this x n cash flow. So now, x n is I am getting after n period. So if I multiply by 1 plus r to power n I will get back x n so that is that is why that is called a present value, x 1 is I am only holding it, for I am getting it my next month. So I just want to know how much money I should invest now, I should have now so the next month I get x 1. If I this money so next month I will have x 0 plus x 1 with me. So basically it is the total, so is the current worth of the stream of cash flow that is going to come to me. These the cash flow that is going to come to and here it is the opposite I know that these are the cash flows I are going to come to me and after n period what I am going to get, or I know that this is the cash flow there is going to come to me then what is that?

If you look at the formula very carefully, you will find there is a very interesting relation with in f v and p v. And the relation is that the present value is nothing put the future value y 1 plus rn. This present value and future value these are very important concepts. So we are not going to deal with too much of issues to day. What we are going to do is put as an exercise certain concept call internal rate of return. So internal rate of return is a very interesting concept it says that suppose I am by cash flow consists of some deposits and takes out some of bank and depositing a bank and with drawing some a bank some deposits some withdrawals from a bank.

So, basically the rate of interest is r, what rate of interest I should have if my current worth of the future flow of money is 0. So now, suppose I want to have this cash flow. This could be 0 actually. Suppose my present value is 0 p v is 0. What interest rate should I have so that the p v the present value worth present worth of this cash flow is 0? Because here you are not doing anything with the bank you are not fixing up any loan you not taking any loan not you given a fixed deposit. You are just transacting on the

account so you preset value or your transaction is actually 0, you have not done anything to a bank you just taken from your account given back to your account. So what sought of rate that is called the internal rate, so if what is the value of r for which p v is equal to 0 that is called internal rate, so that is something interesting which you give you learn teach you some exercises and there is something called the main theorem present value.

So, if you have 2 cash flow streams so x and y. So this is cash flow consists of x 0 x 1 to x n and this cash flow consist of y 0 y 1 to y n. So if you have interested r and you use these moneys either do for example, you have constantly the moneys coming to you for end month and you want to deposit the money constantly in a bank and then see how much you get or you want to calculate it is net worth. So 2 cash flow streams are set are equivalent if the present value is same. So on the given interest rate r are the constant ideal bank, if 2 cash flows are the same present values then they are equivalent. So means you can choose this cash flow or that cash flow. So means when you can transform a cash flow this, cash flow to this cash flow, means when is this same as taking this 2 cash flows. One this same as taking this cash flow or that cash flow your present worth of both the cash flows are should be same. So if present worth of both the cash flows are same we call such cash flows equivalent cash flows.

So, what I can do is the net value, so suppose I know the present then present value of the cash flow is some pvx. So I can just put pvx in my bank, and forget and do not just put anything here I will get back. And in the next cash flows 0, 0, 0, 0, 0, right? And I can put pvy here and then I can put 0, 0. I have no cash flow at all. So and then if I put that in the bank that is exactly that is exactly my present value. If you right down the present value of this and you right down suppose I was stream like this pvx 0, 0, 0, 0. The present value of this and the present value x0 x 1 this present value of this, 2 streams are equal. So this pvx is the present value of this stream, so this cash flow is same as this cash flow. So if I keep this amount of money and do not do anything the present value of that cash flow is same as present value of this particular cash flow were I am getting x naught, x 1 x 2, x n and supply.

So, with this we have some idea of interest rate. And we will start working on other aspects in the next classes. So some of the issues will come to you through little problems, may be you will talk upbeat about of fixing some securities in the coming classes whatever is left for the interest rate. So I hope you have some general idea of

what we have try to do here. Tomorrow I will start with some very basic result and this is a very important result. This is called, this is one of the major result is it is called the main theorem on internal rates of returns.

So, tomorrow I will try to give you that theorem may be give a proof. Internal rate of return happens the rate at r rate of rate of return r 1 of the interest rate or interest rate some as call the rate of return. So what is the interest rate for which this is 0? So tomorrow will start with this and then will try to do some related things. So I hope you have some idea what the whole I thing and may be will continue tomorrow.

Thank you.