Basic Concepts in Modal Logic

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Lecture - 09
Origin of Modal Logic: Strict Implication.

Welcome back. In the last few lectures we started our journey with crash course on propositional logic and then followed by that we used semantic tablets method to solve some of the important puzzles like knits and knives and lady and tiger. Then we saw some of the limitations of classical logic particularly these are classical logic fails to differentiate bet differentiate things such as something which is actually in the case, something which is necessarily the case and something which is possibly the case there is no difference between box p diamond p and p and the second thing is that classical logic is fails to dealing with future contingent sentences.

So, these two things led us to the development of modal logic and third important thing which we have seen in the last classes there is thorough dissatisfaction with the material implication, the material implication is simply defined as p plus q is defined as not p or q or it is not the case that p is true and q is false.

So, there is thorough dissatisfaction with the material implication, especially when you use it and the day today this course in particularly we at least to counter into kind of results. So, in this lecture we will be considering some of the historical origins of modal logic and need to note that this is no way exhaustive kind of survey of modal logic. But I would like to highlight on some of the important things, which is considered to be important for us. So, I will be talking about historical overview.

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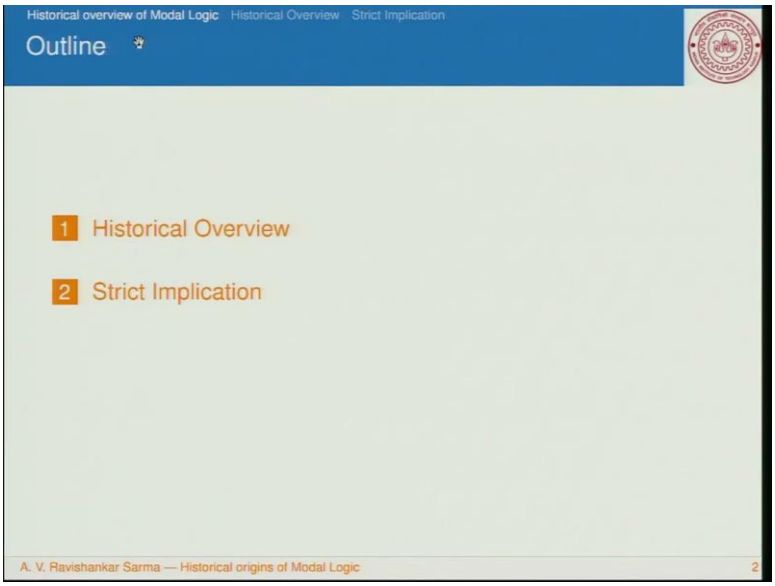

And in the next lecture I will be talking about something about strict implication because, the strict implication is on which led to the development of various modal logical system and that is not we are going to take up in the next week.

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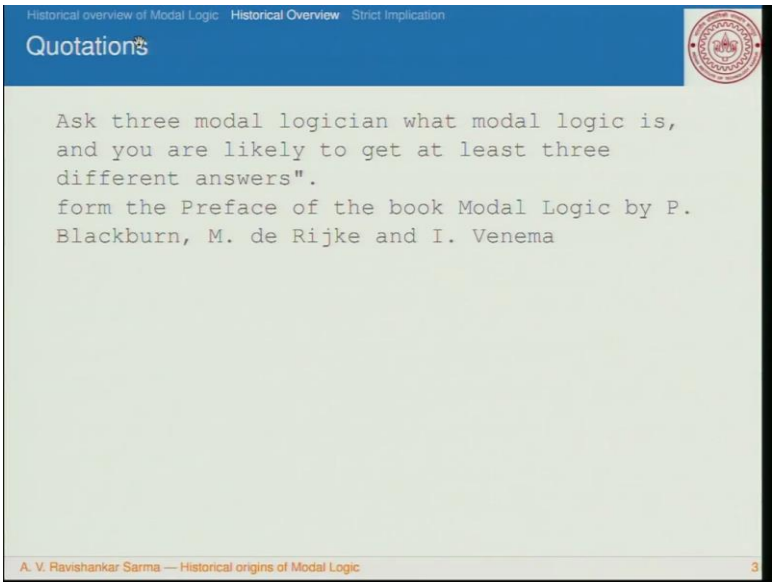

So, here it an interesting quotation by this other M. de Rijke and I. Venema their book is considered to be very popular book, anyone who takes his model logic will definitely view this book. So, very interesting book, he is of the view that the prefers he mention this thing ask three modal logician if we come across in three modal logicians ask them

what modal logical is available and your likely to get that is three different answers all are possibly true. So, this not is written in the prefers of this book, modal logic by Blackburn, Rijke and Venema this three consider to be classic book in classic book in modal logic apart from a new introduction to modal logic.

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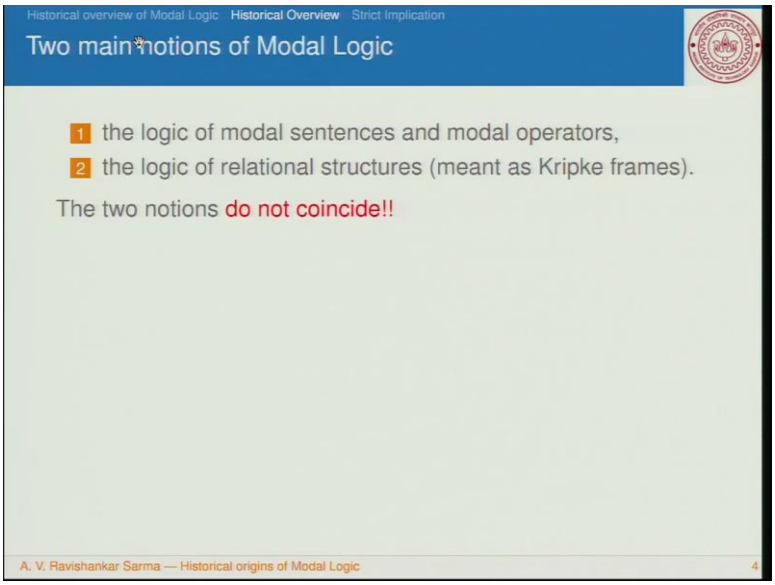

So, there are two main notions of dealing with modal logic and in the syntactical view which have a logic view which the logic we will beginning importance to the logic of modal sentences like possibility of the necessity of the etcetera. And we will be dealing with the nature and behavior of these modal operators.

So, till 1960's, modal logic was expressively studied in this addition and from 1960 onwards ever since Kripke has come up with possible world semantics it is considered to be a revolution you consider it as a revolution in modal logic people view validity etcetera. In a totally different way, there what was important is the logic of the relational structure lot of emphasis on the relational structures.

So, we its relational structures, we are going to talk about it in next week these are also call Kripke frames, but this 2 traditions clear this 2 notions of these, we do not coincide a lot of debates has to which approach is considered to be the most study quite kind of approach, these are the two notions of modal logic when in the synthetic rational and the semantic tradition.

So, in coming ne next few minutes I will be talking about modal logic I will divide it into four important phases one we go back to Greek period were we will be talking about Aristotle modal and then we move on to of course, although medical period is considered to be most importantly then we will jump to historical sorry syntactical era from 1980 to 1959.

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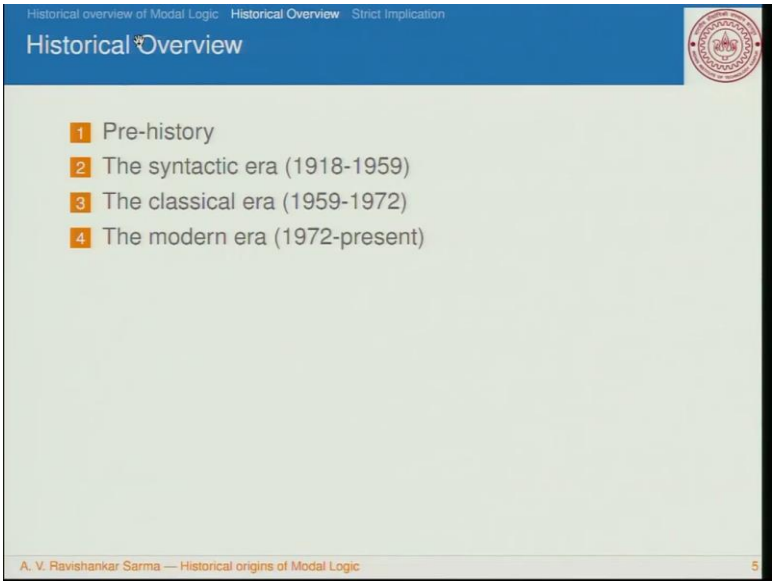

I never seen script cares and it is considered to be classical era is from 1959 to 1972 and the modern era is the one which who witness it now, lot of developments applications and modal logic particularly linguistics computer science artificial etcetera. So, you will happen find course is like modal logic for artificial intelligence modal logic for computer science modal logic for linguistics etcetera. So, all these things are the once which will you see mostly the applications of modal logic.

So, let us go back to the ancient past were in a Greek period particularly in Aristotle, Aristotle for Aristotle did talk about modal. So, it dates back to Aristotle deal interpretation.

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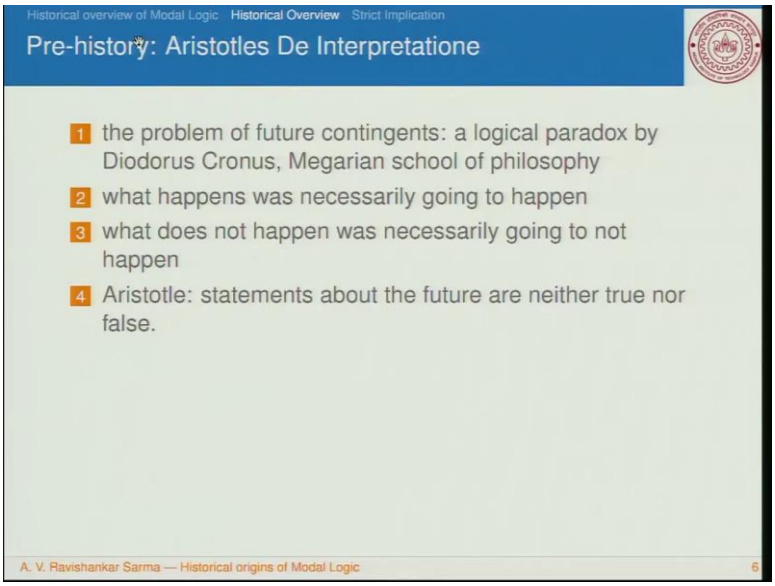

In that there is a problem of future contingent sentences. So, the future contingent sentences the problem with the future contingent sentences is that, what truth value it takes? It is this considers sort of logical paradox, which was discussed by Diodorus Cronus and Megarian school of philosophy. So, the problem here is that suppose you make the future contingent sentences true now, then what happened whatever happens was necessarily going to happen; that means, whatever is true now you go be even tomorrow and always? So, then it cannot be is not possible that was particular kind of sentence will become false.

So, what does not happen suppose you make the sentence false now, it is the case that does not happen whatever does not happen was necessarily not going to happen in way. So, things which are considerable false are necessarily is not going to happen. So, Aristotle was worried about how to preserve law of excluded middle that is p are not p and at the same time how do we incorporate this future contingent sentences.

For Aristotle statement about future contingent sentences like I will be in etcetera there are neither true nor false. You want to preserve the law of excluded middle. So, this is the one which we get it that is translated by rows in 1928 it is goes like this. So, Aristotle in the interpretation is of the view that if I thing is white.

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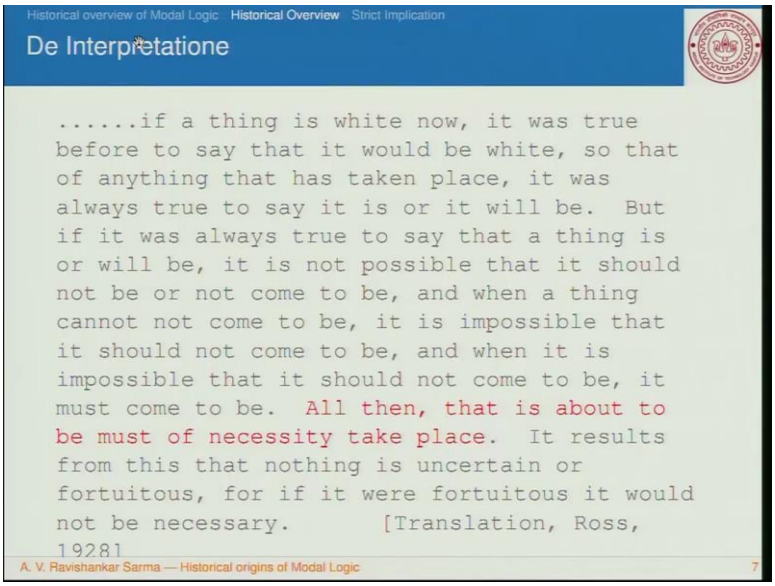

Now, it was true before to say just before some time it was true to it was true before say that it would be white. So, the anything that has taken place it was always true to say that it is or it will, but if it was always true to say that the thing is or will be it is not possible that it should not be or not come to be and when a thing cannot come to be it is impossible that it should not come to be and then it is impossible that should not come to be it must come to be.

So, then is all then that is about to be must of necessarily necessity take place. So, this is kind of whatever happens out of necessity whatever does not. Happen does not happened anyway, its results from this that nothing is considered to be uncertain or nothing is consider to be uncertain concern whatever happens out of necessity whatever does not happen also happened out of necessity anyway again this consider to be impossible. If you make the future contingent sentences true now it is necessarily true if you make the future contingent sentences false it is got to be absolutely false and this never going to happen.

Example if we say that will be a native place. Suppose if you make the sentence true; that means, you must travel and must go to your native that is no going which we can postpone your journey or something like that show will happen it might happen that to book your ticket. Of course, but you may cancel your journey or train is late train is cancelled etcetera, but you may not be able to reach your place or you may be able to

reach the place next day. So, it is not necessarily going to happen suppose if make the sentence false now suppose if you say that I will be my native place in source to that in December 21st 2016 if you say that thing then, suppose if you make the sentence is false anyway this going to cannot be the case there it is necessarily false I mean it cannot happen that you will be the travelling that your place on December twenty first it prevents us prevents you to go to your native place. So, its results for this that nothing is uncertain are fortuitous for if it were fortuitous it would be it would not be necessary what is translated in by Kimaki Ross in 1928.

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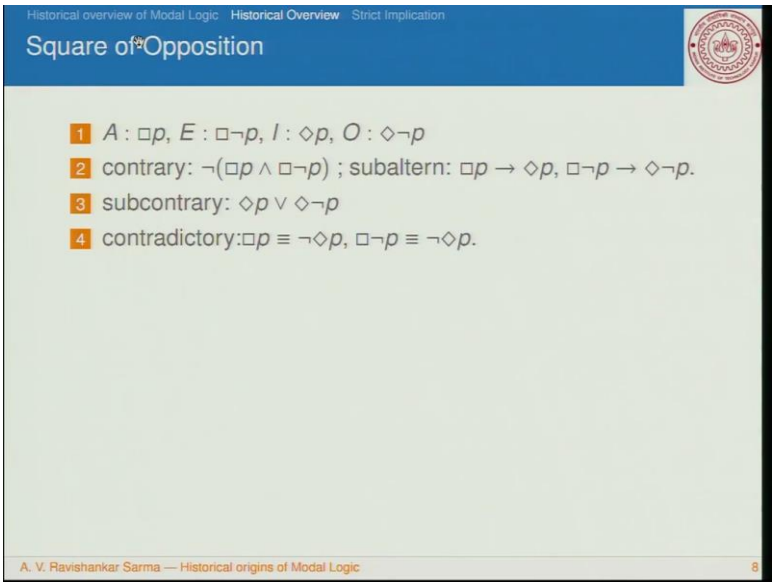

So, Aristotle also spoke about square of preposition it is going to be like this he spoke about four kinds of preposition categorical preposition. So, they are like this. So, a there are four kinds of preposition which was considered and then this categorical preposition combined together in some ways and then there will form another categorical preposition which is consider to be the conclusion of the consists of categorical prepositions to will survival premises another one which is consider to be the conclusion. So, Aristotle way of preposition is obvious to us. So, it is like this suppose.

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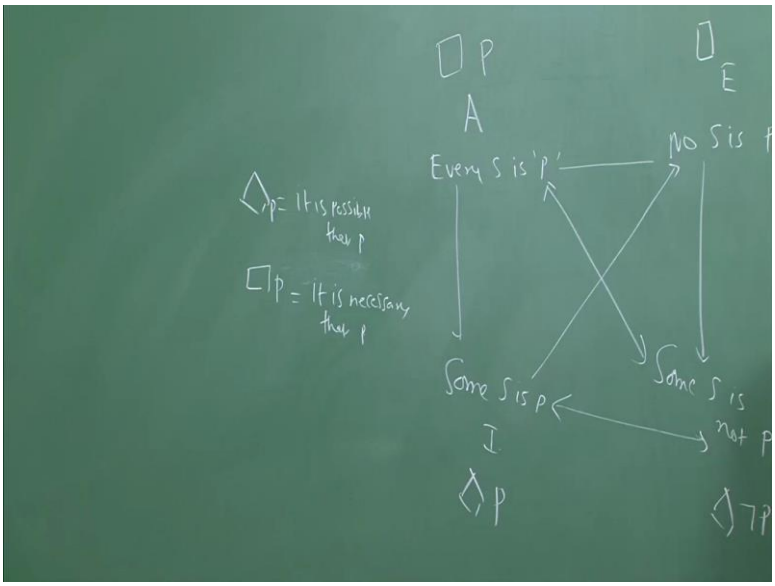

So, it is like this from the one hand you have on A preposition, which transfer every S is P first of all any sentence will have this particular kind of structure is has a subject and its dedicate and in every S P is a preposition and then, you have something called as some S is P. So, this is I preposition and then you have e preposition which state us that no S is P and some S is not P.

So, now you note that Aristotle spoke about this connection between this p s. The diagonal as well as the parts of your square, so these two or oppose to be each other; that means, these two are contradictory to each other. So, these diagonal are contradictory whereas, these two are consider to be contraries and the below one is considered to be sub contraries and something called as alters, sub alters etcetera. So, these is the famous square of opposition that you got you will learn it in any introductory course on logic. So, now, how to looking to this one for example, in this case there are various issues concerning level from every S is P, some S is P follows or not. So, for example, if you. So, other how you will concern intelligent, does not imply that there is some unicorn that unicorn to be considered to be inter it.

So, Aristotle of the view that weather is kind of existence import in the conclusion, suppose if you import some S is P from every S is P existence is imported from the premises to the conclusion. So, this is also called as existential fallacy then you will issues get to this way. Now, these diagnose are considered to be contradictory suppose of

you take both of them together it is to. Suppose if it is take this preposition again and these are p and the truth value of that one combined together is going to be false and in the case of contraries. Contraries are these one every s is p and no s is p. So, the problem is that one here both cannot be true will be this case they are considered to be contraries, but it is in down of this particular kind of square here this two both cannot be false, if it take these two to the it cannot be false together this is what is considered to be contrary and contradictory means both have to be false together or were having opposite truth one it this is having truth value P and the diagonal some S is not P will have opposite truth value.

So, this is not to is the case of square of preposition, but Aristotle square of preposition can be discussed in terms of modal syllabus and this way you can represent A preposition has necessarily p and E preposition has direct necessity of not p and I preposition is possible to of p and what preposition ahs is possibility of p and what preposition has it possible that not be in the case. So, it is simply this one is for this one and then some s is p as this and then o preposition is it is possible that not p. So, if it in the beginning of this course that I will be using this no invitation. So, these kinds for it is possible that p.

Suppose if write like this suppose if you say box p into say that it is necessary that p is the case p is true. So, now, in this case it is e preposition is this same it is necessary that not p is the case; that means, not p considered to be. So, even that context same square of preposition can be expanded in terms of modal operators like this we contrary preposition is one for example, a and a that truth values of A and E cannot be true together; that means, it cannot be the case of necessity of p and necessity of not p is the case. So, likely the first one it cannot be true together if you make necessity of p and necessity of not p; that means, it is true that that is to be ruled on that you are put negation before that are and sites of in particular.

So, from necessity of p whether you can infer possibility of p and necessity of not p is to necessity of possibility of cannot p and sub contraries are there in down kind of this capacity for it sub contraries are cannot be false together.

Suppose if you take both difference to the it cannot be false together that is why it like it possibility of p or it is possibility of that not p this p is possible or p is not p is possible

one of these should be the case and contradictory preposition are differenced in a different way like this we cannot both these true values. So, p have a p is not a true or cannot be true and not p is contradictory the value f. So, this is represented in the sense necessity of p is a logical equivalent necessity of it is not possible that p are necessity of not p is logical equivalent to it is not possible that p. So, this in that contents of Aristotle which talks about that etcetera. And from a preposition and how is to I preposition etcetera there was from varies rules etcetera he spoke about it.

Now, coming back to the medieval period it is virtual to mention. Let me read from 1646 to 1760.

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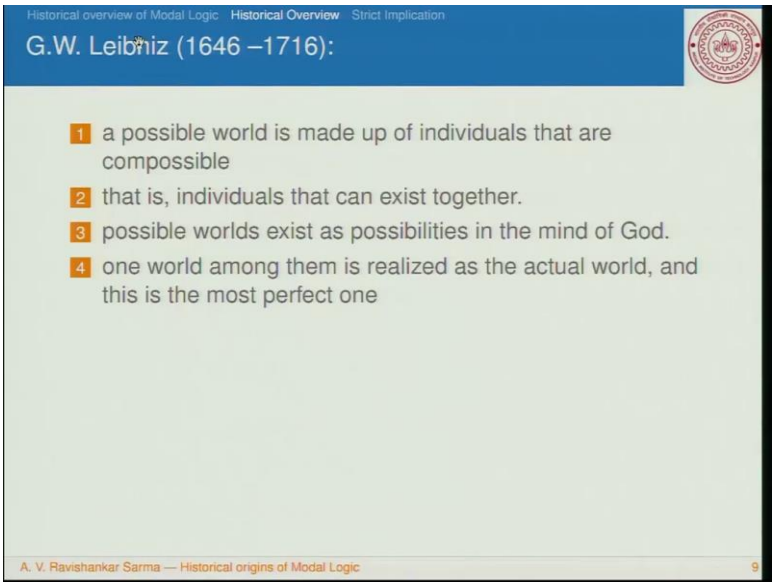

So, there many logicians was working that period like this is definitely what mentioning might be in many other important contributions, but we need to like emphasis on because there is technical notion that will be using in throughout this course the that technical notion called possible worlds it possible worlds introduction about worlds is due to is the idea of like this.

So, for lively a possible world made up for individuals that are come possible in equally possible and that is individuals that can exist together and possible worlds exist as possibilities in the mind of god and one world among them is constructed to be the best possible world and which is which we are calling it as actual kind of world and this is

considered to be the most perfect kind of one that why considered to also considered to be the best possible world.

So, the idea here is that you have all are considering possible worlds out of this possible worldS1 world is considered to be the best possible world which is designed by god. So, life is not to attribute all kinds of absolute quality is to god for that you has to cook up with these best possible worlds etcetera.

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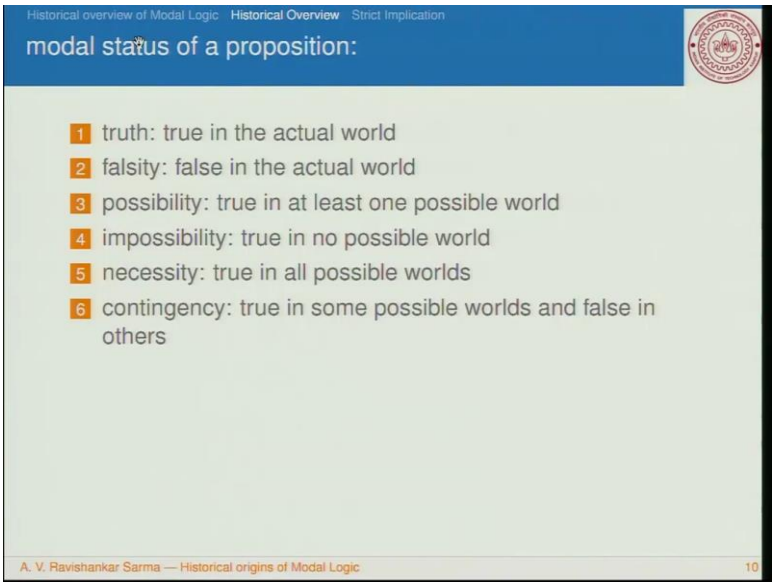

So, the modals status of a proposition is like this truth true in the actual world falsity means false in the actual world there is p cannot be etcetera and possibility mean terms it is true in at least one possible world suppose if you say that it is possible that is there should be some of one particular kind of world the situation were your sentence is false there. Suppose if it is possible that is humid outside it is also true that it is possible that it is not the case that it is the outside. So, impossibility means it is true something which is considered to be true in all possible worlds and sorry impossible in no possible world.

So, in the contexts of conditional logic particularly impossible worlds are demonstrate which we take in to consideration and some more detail all conditions are considered to be vacuously true. So, the enticer responsibility is impossible in there and the necessity means it is true in all possible worlds all the magmatic true's etcetera there are true in all possible world which you can which are access able to the actual world, on this things which I would talking about when I discussed about semantics of modal logic.

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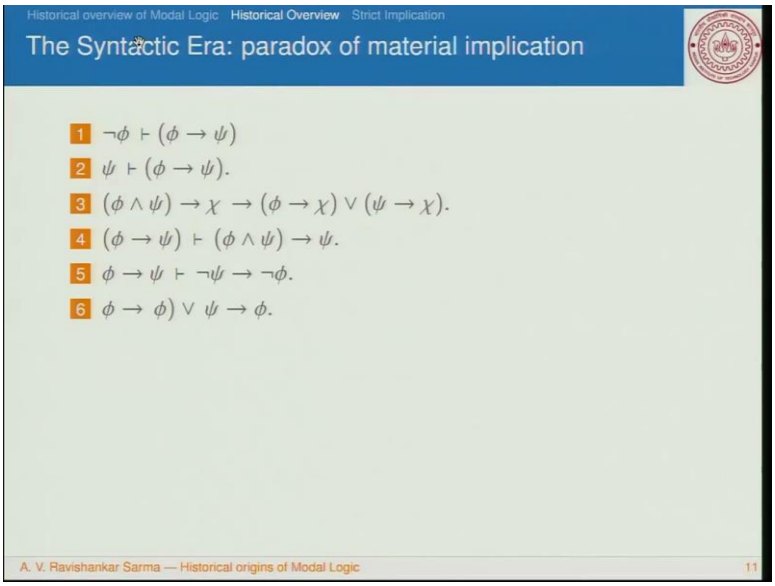

So, continuously means to in some possible world it is false in some others, in syntactic era one particular kind of that problem facing is this thing a problem the paradox of material implication. One of the way talks about this paradox material implication is this. So, form truth is implied by any kind of strange proposition, and if you start with the false proposition it leads to anything. Falsity to anything so it leads to some kind of counter institute results to substitute phi and psi with some ordinary propositions that you commonly observe in day today life. You will come across some kind of institute, counter institute results.

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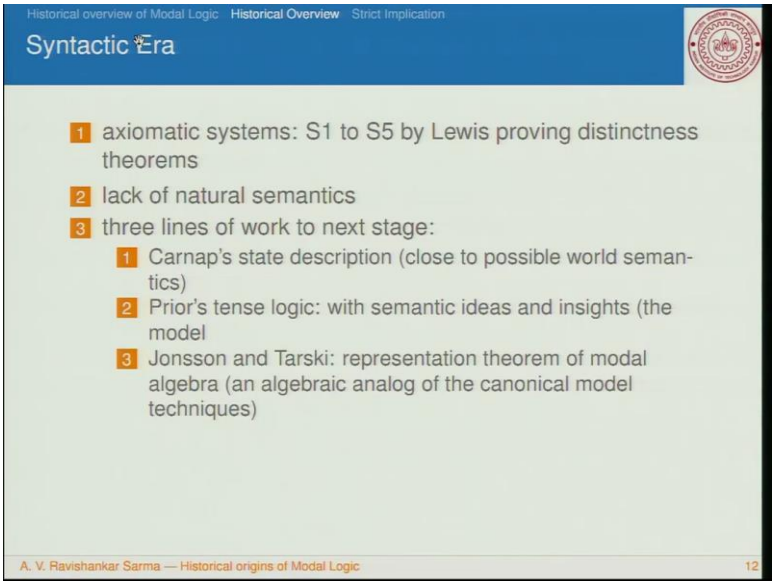

In the syntactic era derive axiomatic systems some S1 to S5 was developed by C I Lewis proving distinctness of these theorems etcetera. But not the important problem of this is syntactic era is that in 1960 is conclude in this tradition. It really lacks natural syntactic and three lines of work.

And three lines of work to the next stage are like this Carnap's state description, close to possible world semantics possible world semantics, possible world semantics got some kind of inspiration from Carnap's state description and Priors tense logic will also develop around in the same time, but unfortunately, less influence of temporal logic and the modal logic for a long time. So, priors tense logic with semantic ideas and insights and in Jonsson and Tarski representation theorem etcetera are consider to be the other possibilities.

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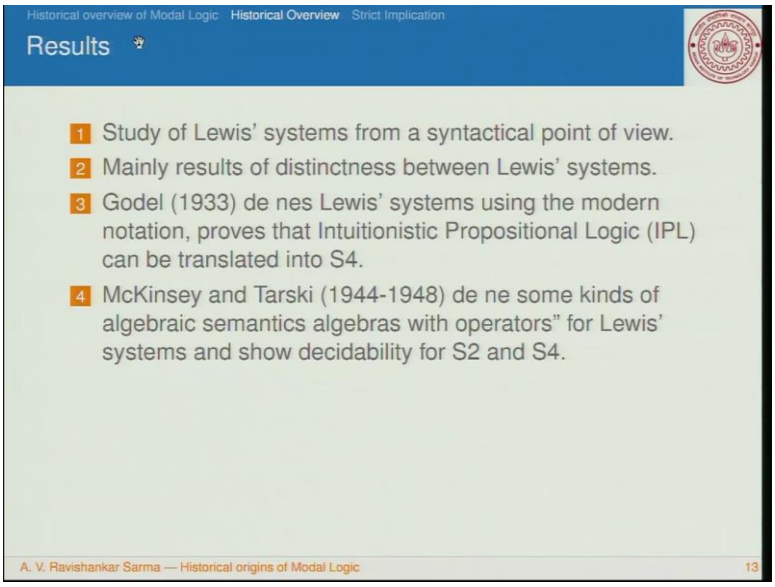

So, if you can classify this syntactic of era the main results are like this there was several dissatisfaction with the material implication which led to the development of the strict implications, and it may be the results of distinctness of Lewis systems S1 to S5 and Godels extended this thing define this new system is the modal location is that intuitionistic propositional logic IPL can m be translated into the modal logical system which we were going to talk about little later that is S4.

Mckinesy and Tarski defined some kind of algebraic semantics algebras with operators for Lewis systems and show the decidability of S2 and S4 the ideal it is the Lewis some

of it was thorough dissatisfaction with the material implication that led to development of its implication and in the process of understanding this implication Lewis come of with 5 different modal logical system, from S1 to S5, the first to system he consider non normal modal logical system and its S4 and S5 are the once which are widely studied and these are considerably normal modal logical systems.

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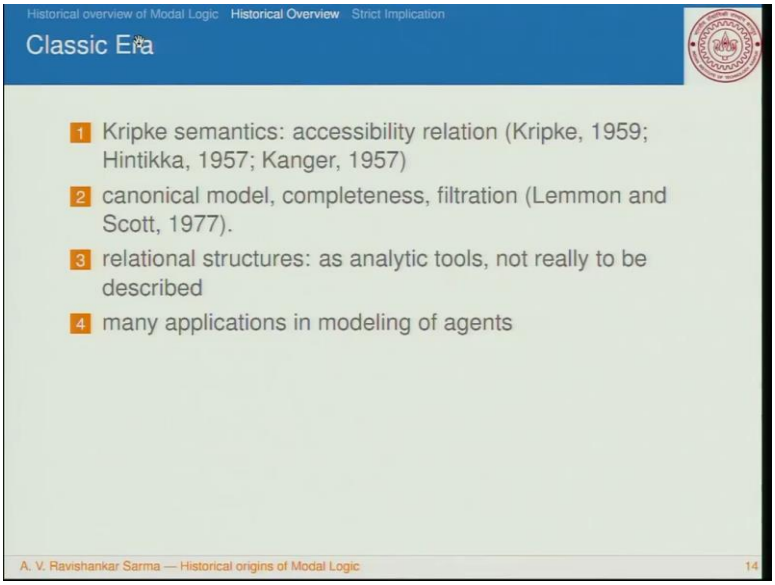

So, in that classic era, other thing which he will find it is this thing. So far we talked about pre era and syntactic era and then classic era. So, from 1960 onwards were some revolutions in the logic. So, there is Kripke semantics. So, were accessibility relation they depending upon constrains an accessibility relation, we have different modal logical systems. The much emphasis was given to this is all given to relational structure these are considered to be analytical tools that it is not really to be describe; their abstract things, many applications in modal of agents etcetera, but the once which are in the classic era.

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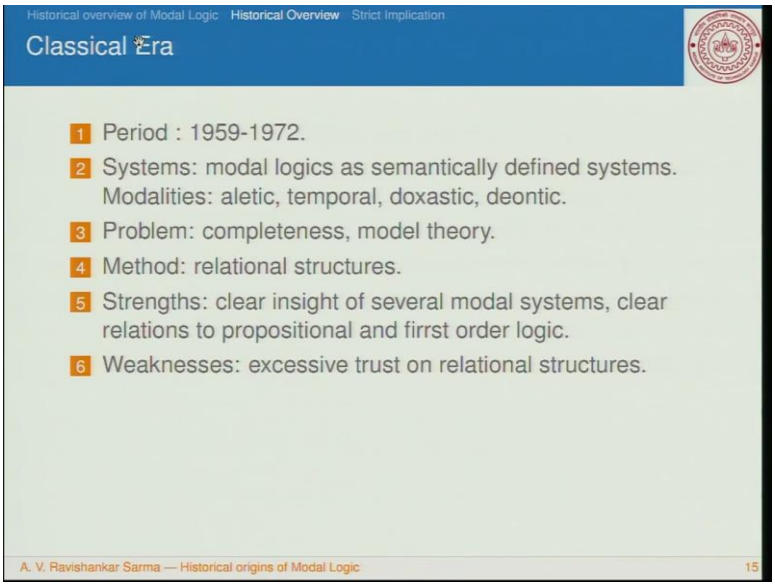

This is what you find in the classical era, it is the period from 1959 to 1972, in different modal logics semantically defined system etcetera. Modal, modalities viewed as aletic modalities, temporal, doxastic, deontioc etcetera. Then they discussed about the problem of completeness, they have come across the model theory of in the modal logics, and the method that we used was relational structures, and one of the strengths of the classical era which is that this particularly clear insight of several modal systems and there was clear relations to propositional and first order logic, but one of the weaknesses is that lot of emphasis on the relational, structures.

So, in the next class, will be dealing with exclusively about something about strict implication, which led to the development of modal logics.

Thank you.