Basic Concepts in Modal Logic
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Lecture - 08
Origin of Modal Logic: Historical Survey

Welcome back, in the last lecture we have seen some of the limitations of classical logic in much more general way. So, in this lecture will be talking about some of the examples which justify as to move away from the classical logics, which we lead has to consider either the excessive of the classical logic or the deviations of classical logic.

So, we are motivated by some of the questions like is there any difference between something is necessarily the case that p, something which is possible that p and something which actual case that p. Were the first question that will be asking our self and then any scope for sentences like this sentence is false, are is any scope for way predicate such as suppose if you say that ram is tall. How do we know the truth value of that particular kind of sentence? So, that particular kind of sentence which involves the way predicate tall. So, which we predicates always come with some kind of boundaries, were you cannot decided whether something is the case and something is not the case it is not so, easy like a in the case of a (Refer Time: 01:30) and criticism mortal.

So, we can clearly draw a line between mortal and non mortal, but in the case of were predicates which always fall in the border line cases which are neither true the friendly neither not true that only not false also. So, how do we incorporate those kinds of sentences, to for us the most important thing is to distinguish between possibility of p necessity of p and what is actual in case that p, which clears as to extent the classical logic with few more operators few many operators there is something is necessarily in case that p something which is possibly the case that p.

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So, here are some of the models sentences which classical logic fails to make any such kind of distinctions. So, these lugs move for than will be emphasizing sentence with some kind of phrases like necessity, it is necessary that 2 plus 2 is equal to 4 it is absolutely false, that 2 plus 2 equal to 5 etcetera. So, now, if take this example if it is musingly it is necessarily the case that 2 is the smallest prime member, then 2 is the smallest prime number. If it is like p is p plus p is the case and that is comfortably necessity always operates the whole condition. So, p is that p is the condition here and necessity of it is over the whole conditional.

Now, in the second example it is known that Ravi is richer than Ramesh, which is considered to be a wake kind of predicate to the movement to talk a about rich. The next question is that comes about how rich is, rich too have to too etcetera all this persons are adjusted here, it depends upon the contents; it changes from culture background also. So, then Ravi is richer than Ramesh. So, in this case I know that p and that p follows and third case it is morally obligatory that you love you neighbor a then you have to love your neighbor.

So, same kind of example that we have seen in the case of 2 that is kp implies p. If kp is represented as necessity of p, necessity of p implies p same kind of formula can run out to be false another domain were it involves some kind of moral reasoning etcetera. So, you after follow the topic rules, that does not imply that to actually follow the topic rules.

For example those who are in the case of emergency, the ambulance for example, there we violating that particular kind of rule and that have to follow any traffic rules; that means, you have to follow the traffic rules that only imply that you actually follow the traffic rule. So, this always the case where p is true and q is false. So, we raise the main problem with respect to why we are moving again from the cluster, why we are doing a model logic then several reasons one such reason is the there is a dissatisfaction of we will see that this implication.

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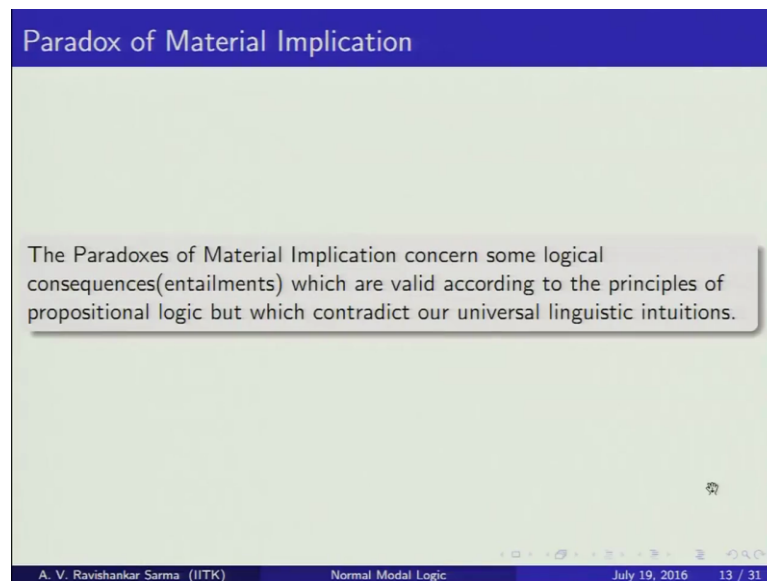
So, this leads to paradox of material implication. The paradox of material implication generally, paradox constructively a valid kind of argument something which is not acceptable to us; you have valid argument but your conclusion not acceptable to you.

So, the paradox of material implication concern some logical consequences are entailments, which are consider valid according to the principles of logic, propositional logic here but which contradict our universal linguistic intuitions we go for against our intuitions.

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Instances of Paradox of Material Implication:

1. $p \vdash q \rightarrow p.$
2. $\neg q \models p \rightarrow q$
3. $p \rightarrow s \models (p \wedge q) \rightarrow s$
4. $(p \wedge \neg p) \rightarrow q$
5. $\models p \rightarrow (q \vee q)$
6. $\models p \rightarrow (q \rightarrow p)$
7. $p \wedge q \models p \rightarrow q$
8. $\models (p \rightarrow q) \vee (q \rightarrow p)$

So, the paradox of material implication can be stated in this way, that enough a true proposition is implied by any kind of proposition and the sentence false proposition influence anything. So, we do not want a true proposition like tautology to comparison outcome of any kind of stage, any contingent kind of proposition.

So, the first case I will not going to the details several, I will talking about when I introduce a strict implication, but there is a current dissatisfaction with the material implication that lead to the development of model logic. So, paradox material implication is define in the sense; p plus q means not p and q are it is also defined as it is not the case that p is true and q is false.

So, in the first case a, if p is true when this p is implied by q against q from the any kind of strange proposition suppose, if 2 plus 2 is equal to 4 from that we can deduce this thing if 2 is rich then 2 plus 2 is equal to 4. So, 2 plus 2 is equal to 4 should not come again outcome of he should not implied by any strange kind of proposition like q is rich p is rich etcetera because nothing, we do with this particular kind of thing in the same if sentence its proposition is false it implies anything. So, whole that is what is happening in the case of r is 2.

In the fourth case some impossibility that is from contribution p and r p you can derive any kind of a proposition a q. So, let us see how a, it arranges in the case of a classical logic.
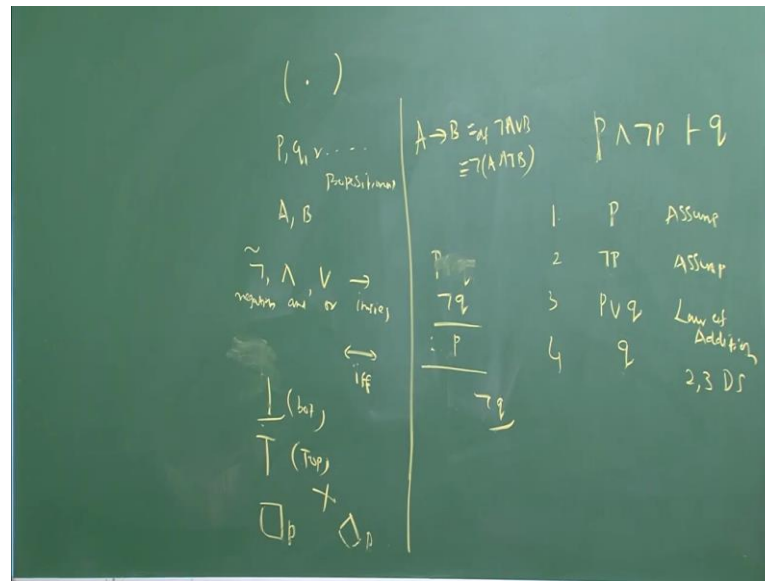
So, you have you p and not p of in the dependent reduce any kind of strange proposition see now first of use this thing p and not p assuming that the process are all true. So, these are all assumptions. Now, reason classical a logic p is already true; that means, you can simply had any other kind of proposition q. So, here the idea arises that in this proves need to ensure that each and every step is considerable true, if p is true not p is true and p are not p r q also going to be true.

So, now this is law of addition simply 2 and 3 it is another kind of principle valid principle which is consider to be distinctive similarly the its says like this p r q is the case and if q false and you get is p you remove this q and then, what you get is p and before that you need to note that a replays b. This is material implication this by definition it is not a or b or the same as it was not the case; a is true and b is false before all these things remain just for the sake of our understanding that we make this symbols clear. So, when you write p q r etcetera these are all proportion variables this kinds for yellow in color then is a capital of India etcetera. There all declarative sentences sometimes I used even capital letters also were a for the proposition variables.

Now, there are few symbols of time using its times for a negation half down. If you write like this it is domain and it is like the negation in front of which it is represented as tilde and what are we stands for end and this stands for r and this stands for influence this is consider material implication and this stands for if for only if. So, this is written in this

sense and there are some other symbols which are available in this course. So, these are like this stands for contradiction this feature always consider to be false which is usually represented as bot t means those which are always consider to be true. They are consider to be total it is this is consider to be top plus in modal logics we will be using 2 other symbols and these are like this stands for these are union operates which we act on only one variable proposition variable. So, this considers necessity of p and this stands for possibility of p.

So, that is the symbols data will be using and because a conviction that we used this is left parenthesis right parenthesis which is used only for the sake of our convince, now getting back to this one what we are trying to show we are is it from impossible from inconsistent kind of statement like p and not p you can derive any stage kind of the person q using loss of classical logic. So, these 2 different, 2 available, 2 and three is to q. So, now, from p and not p we have to we heaved assumes q somehow let us consider p to be it is raining and not p to be opposite of it that it is not any then there has considered q can be something like Rahul Gandhi, is the current prime minister of India. So, any kind of change a kind of a proportion you can derive from this.

So, nothing that p you can follow the same steps and then we can show that even not to be is also in the case. So, starting from p and not p you can you can derive not q also. So, this is where a para consistent, in the case of para consistent logic in particular they object to this particular kind of step. So, that is you cannot add any strange kind of proposition q to something this is already true were. So, this is the step that para consistent question a in that is your questioning soundness of the argument. So, now, argument can be valid, but one of the premises is questionable are it is wrong in your argument can be un saw. So, it is in that sense this para consistence logic will show that the argument is answer.

So, this things from impossible proportion you can derive anything and truth is implied by any strange kind of proposition and false it implicitly anything, these are the problems which we need to overcome see in order to overcome this problem C I Lewis has come up with another kind of implication and is of the view that implication that is used by Bertrand Russell write in the book principia mathematica and followed by that systems etcetera in Lewis of the view that implication that is used does not capture what exactly mean by deduction. When q is deduced by p it is not just material implication, it is

something else. So, we need to involve some intentional concepts to understand that thing how to capture it in a better way. So, there is a taro dissatisfaction with a material implication let us to do this modal logics and a sometimes may revalue logic.

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So, it is a simple straight forward example for a paradox of material implication, suppose if you say that p stands for there is no oil in my coffee, exactly opposite of that one is not p even one which was see in the slide there is no oil in my coffee that is stands for not p. So, q stands for I like it.

So, now you put in the a original premise p plus q plus p r not p plus for something then, a yourself will see that fix the sentence his false that is not p is false that is there is oil in my coffee then a form that you did use this see. So, this is what follows from not q p plus p follows. So, another example this is that if considered p as I will play football tomorrow and then q happens to be break my leg today, then it is.
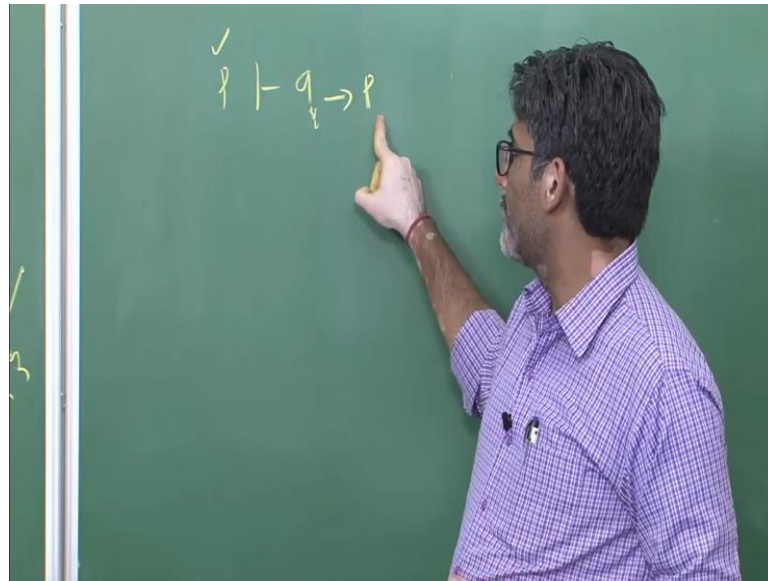
If you take the first example into consideration p q implies a p. Now p stands for this I will play football tomorrow from this you can reduce this I will play football play tomorrow and even break my leg this is consider to be counter into to us may be do will play football if we break their legs. So, p that is considered to that true I will play football tomorrow follows from any strange kind of proposition that is I break my leg today. So, when you take material implication into consideration it is just definition on that one is not p r q or it is not the case p is or q is false, it is not referring to any causal implication or the causal connection or the relevance relation between anti incident and the consequent of your at the conditional.

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Paradox of Material Implication

$P \to Q =_{\text{Def}} \neg P \vee Q$

1. $P \models Q \to P$: I am alive to if I am dead, I am alive
2. $P \models \neg P \to Q$. I'm alive to if I'm not alive, Im famous.
3. $(P \wedge Q) \to R \models (P \to R) \wedge (Q \to R)$.

Simple examples can be like this, if p stands for I am alive and q stands for I am dead then p from is it follow I am alive today then it follows from I am alive follows the I am dead since to be counter into to us in the same way p another instance of paradox of material implication some p not p plus q is defuse. So, from I am alive stands for p and not to stands for not alive then q stands for I become famous I am not alive, but still I become I be can famous.

So, all this incidences lead as to look this looks at this material implication in a totally different way. So, the Lewis has come up with a strict implication to answer some of this questions, but unfortunately Lewis a implication also faced another problem, which is called as paradox strict implication there are other problems with respect to this material implication that is see.

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For example it was steadily things the earth is flat today is Thursday and then same way today is Thursday and then 2 plus 2 is equal to 4 is re connection between the antecedent and the consequent here. So, there for if the earth is flat then, 2 plus 2 is equal to 4 there is no connection between 2 plus 2 is equal to 4 and the earth is flat etcetera and the same way p and not p from inconsistent kind of segments you can derive anything you can derive q you can derive not q also.
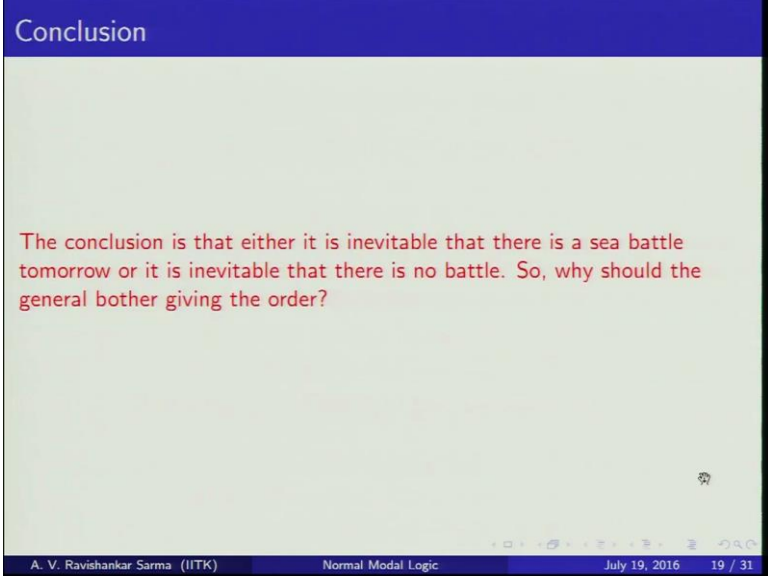
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So, all are classically true there is another problem which lead us looking to or motivate us looking into the expenses of classical logic. So, that is how to be present future contingent sentences suppose any statement which is referring to the future it is defiantly cannot be true defiantly not false. So, they are considering being contingent kind of statements sometimes true sometimes false. So, these are referred us future contingent sentences.

So, what kind of truth value the future contingent contingencies take1 take what is of is a problematic kind of is a problem right form these period onward these period onward, but Aristotle could not come of be formal analysis of a listening Aristotle did talk about model logics particularly model syllogisms excreta. But is no proper formal analysis that are there even in the works of ca will find kind a formal kind of argument, but you all though I will stated has all this thoughts in his place. So, it is an example which is considered to be a problem, when you talked about determinism this is the example that is mentioned. So, that is like this.

A general is contemplating whether or not to give an order of order to attack or not. So, the general reason you is follow general reasons has follows. So, if I give the order then necessarily there will be sea battle tomorrow, sea battles reticent phenomena or during those days. So, if you give order for an attack every one gives that order. Of course, will necessary we, see battle tomorrow. If I and if general does not given sea battle; that means, general who is reason in this way then, necessarily there will not be any not tomorrow haven of give the order then will not be any a battle tomorrow sea battle tomorrow.

So, now I give the order again and again 2 choices a the general has either give the order I do not give the order hence for this it follows that either, it is necessary that there is a sea battle tomorrow or it is also necessary that once of no such kind of the occurs.

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So, if it is necessary that it is assume that tomorrow and if we necessary that a none of the, such kind of things occurs then the conclusion will be like this the conclusion is that even it is inevitable that there is a sea battle tomorrow or it is inevitable that there is no battle.

So, before you we knows that is it is necessarily in the case that there is a sea battle tomorrow it is also necessarily cannot be no such kind of battle which is the conclusion which; obviously, known to before even considering all those things. So, why should the general bother about a giving such kind of order. So, the problem you are not. If you take law of excluded; middle p r, r nor p it is different for us to understand this problem here. So, these are considered to be future conditions there is sea battle tomorrow.

For it is since different to the future. So, it is a contingent kind of sentences. Because it cannot defiantly with false and definitely true, not be necessarily true need not be necessarily false also, now according to the excluded middle and last excluded middle one express other possibility either p is it is or not p it is,

If see is it is exactly one of this must be true a if there is a sea battle tomorrow; obviously, the other I think is ruled there is no sea battle it is ruled, but if one is now true; that means, currently whatever, if you are making some kind of statement now that there will be sea battle tomorrow; that means, sentences is true. Now then; obviously, there must be a sea battle tomorrow it cannot happen that in a there will not be any sea battle tomorrow. If it is true now it is true tomorrow also and there cannot fail to be a sea battle tomorrow will sea battle should occur at any cost.

So, the result according to this puzzle is that nothing is possible except what actually happens some is there are no unactualized possibilities whatever happen happens twice happens whatever does not happen anyway. So, for when we say its saw faith that something happen likes that. So, will no choice here that we choice certain kinds of action etcetera.

So, now the problem here is that something is true now it has to misunderstood true the future. For example, if you say that let us say that you book your ticket and then you will stay a proposition like this I will be my native place and so on. So, time you booked your

ticket in after the exam you will be in your native place. So, now, he makes the sentence true now there is no will which you can make that sentence false. So, it is going to be necessarily true something is true it make it necessarily true if it is false; that means, if it is first true some ten thousand year ago and there will be sea battle tomorrow then its truth was just a matter of fact about the past and if the past is now unchangeable then. So, is the truth value of that past utterance enhance in the utterance of the past. So, that it is necessarily true that there will be sea battle tomorrow.

So, they say if I talk about future contingent sentence then you are still working in the classical logic for example, if you say that in a you know I will be in my native place so and so a do it. So, time suppose you make the sentence true now that is will be necessarily true and for making the sentence false it is a; obviously, false that you know you not able to go the native place he will all the possibilities that you go to your native place, but it. So, happen that he booked your ticket everything is all right, but all of the suddenly might change your plan this kinds of things are not possible as if you take this thing in to consideration.
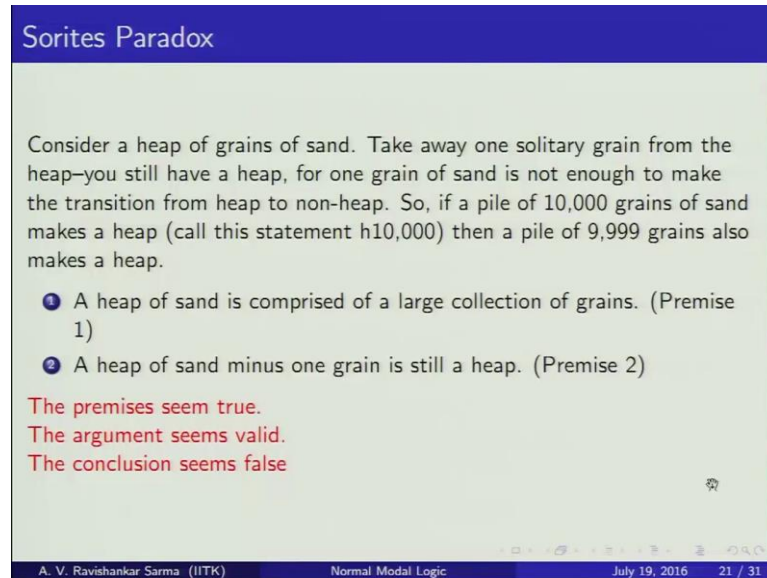
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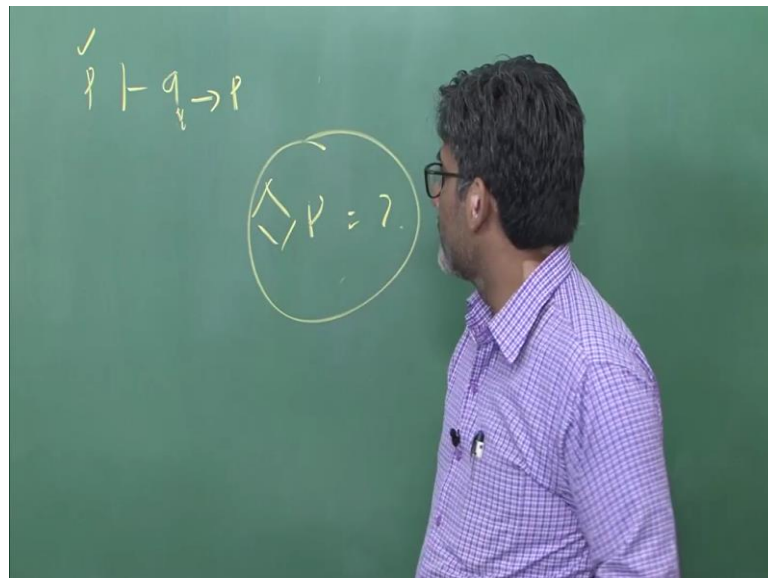
So, particularly what kind of truth value that this future contingent sentence has to take, what is going to be valid definitely possibility of p the value of possibility p cannot be one cannot be even 0, the problem is 1 and 0 is a its pushes us to extremes, one esteem is a something has necessarily true and something ruled out all the possibilities etcetera it

pushes us to another esteem. So, we do not want. So, what kind of truth value is possibility of p take there is another instance with which I will stop in this lecture another interesting paradox which arises in the case.

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Sorites Paradox

Consider a heap of grains of sand. Take away one solitary grain from the heap–you still have a heap, for one grain of sand is not enough to make the transition from heap to non-heap. So, if a pile of 10,000 grains of sand makes a heap (call this statement h10,000) then a pile of 9,999 grains also makes a heap.

1. A heap of sand is comprised of a large collection of grains. (Premise 1)
2. A heap of sand minus one grain is still a heap. (Premise 2)

The premises seem true.
The argument seems valid.
The conclusion seems false

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If we take material implication into consideration, the definition on material implication is that p implies q is nothing, but not p or q and a say must paradox and which lead to the development of a fazi logic. So, what is called as Sorites paradox which is already there in the Greeks period on words you know meaning resolution, for these kinds of the paradoxes, but still there is no appropriate kind of adequate kind of solution resolution for particular kind of paradoxes.
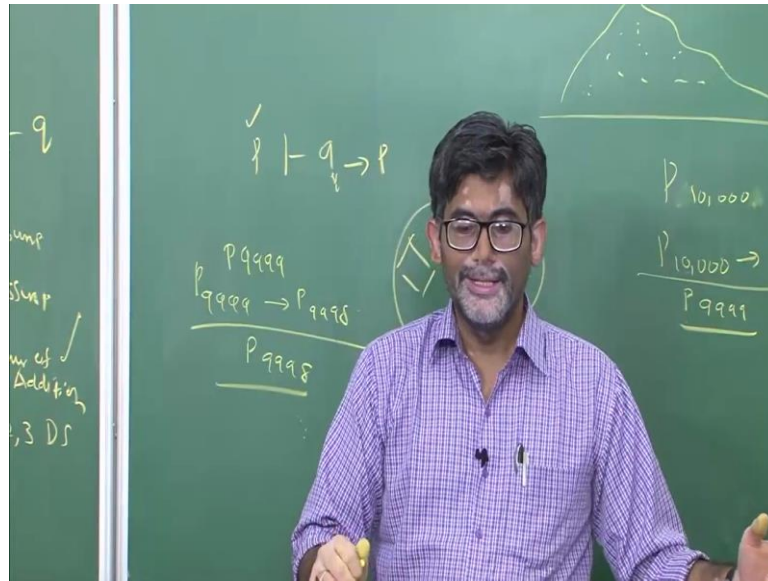
So, it goes the paradox goes this like this consider a heat of sand like this is constructive to be a heat this construct of all sands grains etcetera suppose if you take away one grain solider grain from the heat and another change into non-key which remain as v need pond and if, I for one than of sand is not enough make a transition into non heat. So, several funny examples are given in the logic text book for example, person with full heat suppose if one hair is remove and of sounds I when I go to make him bald.

So, this small minor changes in not causes a big kind of the. So, will not able to recognize these kinds of changes. So, if pile of ten thousand grains of sand makes a heap then; obviously, remove once grains from the results going to be heap. So, heap of sand is considerable of a large collection of grains that is consider to be the premise one and a heap of sand minus one grains also consider to be a heap because ten thousand consider to be heap nine thousand nine ninety nine also consider to be heap. So, the premises is to be and the arguments is to be valid because we are use no response only conclusion also problem is desire conclusion is constructed to be false in an action the argument go like this you started with let us say p ten thousand etcetera then more than that.

Now, if p ten thousand whatever it is considers being a heap. Then P9999 is also considering being heap. Now, this argument goes for non like these 2 no response he will get this things P9999 p also considering a heap. So, if P9999 consider being a heap is something is lesser than that. So, remove one grain from a P9998 is also considered to be

a heap this is; obviously, we have this P9999 etcetera again use more, you will get this thing. So, like this you apply more of points ten thousand times are n is consider to be large sorry I will write it here n is large, in an then it is thing consider to be a heap of sand like this he removed everything, but still more responsible in types ultimately you will come across the situation were you have nothing left there.

But still he considers being heap it is this goes against our intuition. So, in this lecture we discussed about some of the important examples were classical logic fails one particular problem which we have seen this is a fails to distinguish between p necessity of p and possibility of p for example, possibility of p does not imply p, but from p you can imply possibility of p and from necessity of p it leads to p or it leads to possibility of p all this kinds of infuses are not possible in classical logic.

Another problem which we have seen is the related to the problem of material implication there is a thorough dissatisfaction with respect to the material implication that lead to development of strict implication these are of interest to us when we look into model logic another important think which we have seen is this that a future contingent sentences. How to represent future contingent sentences what is what value the future contingent sentences taken is it one by 2 or zero or one etcetera a. So, definitely cannot take value one and zero because it leads us to determinism. So, if you have problems arise if the determines true them we will responsible or not.

Another thing which we was seen is a paradoxes whether there is any place for white predicates in our logics or not. So, Sorites paradox is based on the material implication and if we follow the material implication the argument consider to be valid all the valid arguments you cannot do anything away from it only you can show that this arguments are consider to be unsound.

So, how do we handle this Sorites paradox there are many valued logics its deal, with Sorites paradox a there will be deviating from the fundamental principles fundamental laws of logic that is p r not p. Because if all of you dignifies of truth then we need to all of you many truth values, these are some of the issues with respect to the classical logic which motivates, us to look into a different kind of values there extensions or deviates unfortunately in this course we will not be dealing with deviations Sorites paradox

etcetera takes us to a different turn, but will be definitely dealing with a future contingent sentences so will stop here.

Thank you.