Basic Concepts in Modal Logic
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Lecture – 07
Limitations of Classical Logic

Welcome back. In the last few lectures we have discussed some of the basic concepts of the prepositional logic. So, why we have discussed about prepositional logic is this that modal logic is considered to be an extension of prepositional classical logic. Classical logic I mean prepositional and predicate logic, but for this course since it is considered to be an introductory course. So, we are focusing our attention on only modal prepositional logic that too normal modal prepositional logic is the one which are which is of interest to us in this course.

So, today I will be talking about some of the limitations of classical logic and we need to know about some of the limitations of classical logic before getting into the modal logic. So, there are certain things which classical logic fails to explain. So, classical logic works as long as mathematical reasoning is concerned, it works perfectly all right, but it when it comes to day to reasoning it has some issues. So, that is what we are going to discuss in this lecture. So, before I begin, let me court from a famous logician is considered to be the founder of fuzzy logic.

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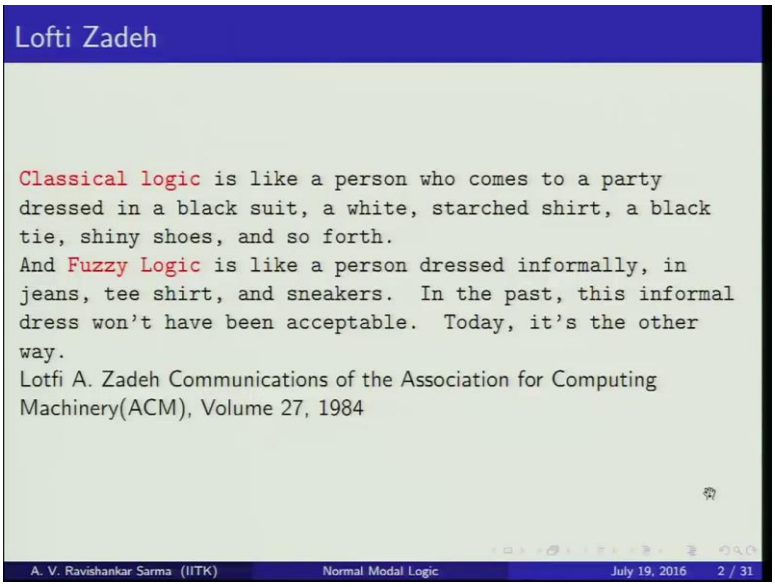

Fuzzy logic considered to be the extension of classical logic, it is also considered to be the deviant logic. So, he of the view that doing classical logic is like a person who comes to the party dressed in a black suit a white and a starched shirt, a black tie, shiny shoes and so on and forth. will be given lot of importance to the formal dress and particular deviant fuzzy logic in the same sense it applies to our context as well, non classical logic like modal logic etcetera. It is like a person that is what we will interested in it is like a person dressed in formally in jeans tee shirt and sneakers and so on and so forth. So, that is what we preferred over the formal dress in many such kind of occasions.

So, in the past that is before the development of these logic fuzzy logic etcetera, is of the view that this informal dress that is informal dress wont have been acceptable, but today it turns out to the case there it is consider to be the reality. So, classical logic obey some kinds of laws all the laws of logic like law of entities law of excluded middle law of non contradiction, it obeys and then it is also considered to be monotonic, but in our ordinary day to day reasoning is considered to be non monotonic in nature and there are some of the valid there are some fundamental laws of logic which fails in the case of classical logic.

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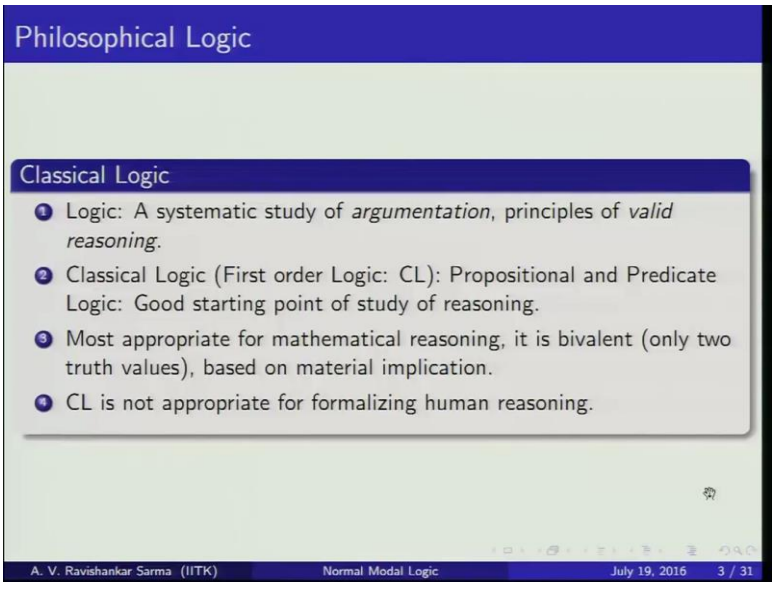

This is what we have been discussing. So, far first of all logic is considered to be systematic study of argumentation and it is concerned with the principle of valid

reasoning it is not just enough that the reasoning, but we need to have valid principles of reasoning.

In classical logic I mean first order logic which includes prepositional and predicate logic are considered to be the starting point for the study of reasoning you need to note that reasoning is a very big domain, but our starting point is the classical logics, that is considered to be prepositional and predicate logics, and it is a appropriate for mathematical reasoning and it is also consider to be bivalent monotonic and it is based on a the problematic the one which we have going to talk about while from the problematic material implication and classical logic is not appropriate for formalizing human reasoning, why it is a case it is not appropriate that is what we are going to see in this lecture there are three reasons at least for why classical logic fails particularly in capturing human reasoning which is, because of these that classical logic is considered to be monotonic and in many situations we had to come up with some kind of arguments which are considered to be non monotonic in nature.

For example, all birds flies tweety is a bird and from that you immediately, come to the conclusion that tweety flies so, but you came to know from some evidential stores that tweety comes under the category of birds, but it does not fly.

So, then you need to withdraw your conclusion that you derived earlier, but classical logic will not permit us to withdraw the conclusion that you have derived earlier. So, you need to you need to be non monotonic to understand such kind of reasoning. So, which is come to non monotonic in default reasoning the second reason second reason, why we classical logic fails is this that when it comes to reasoning we reason based on incomplete information no matter how much information.

We have is always be incomplete and we need to reason under uncertainty all this cases you will not able to explain it in the case of classical logic these is another reason why we need to move aware from the classical logic the reason is this that, we need to incorporate weak predicates.

So, weak predicates are considered to be part and parcel of our logic then we need to you need to you need to have come up with another view rater way of defining the implication material implication will not work there. So, we need to come up with another kind of implication, but for us the most important reason why we will be

interested in moving away from the classical logic is this that in modal logic in particular that is that is a interest to us. So, there is no way in which you can distinguish especially in the case of the classical logic there is no way you can distinguish between something is actually the case that, we are writing it has P something which is possibly P which is we are writing it as diamond P and something which is necessarily the case that you are writing it as box p.

So, this is known which we can distinguish between these two things three 3 things. So, in classical everything is same that is users simply the write it as p. So, there is no distinction between it could have in the case might have be in the case that P or it is necessarily the case at P, we comparing if we compare it with what we what we come across in the classical logic that is simply something is actually the case that p.

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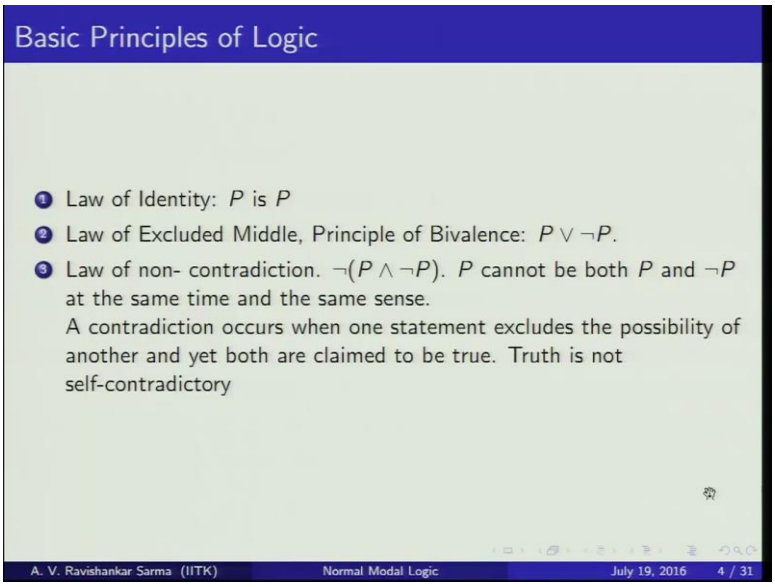

So, this are the some of the basic principle of logic that all of us are aware of it law of identity P is P law of excluded middle which tells us that either P is a case and not P is a case one excuse the other possibility and sometimes this r can be used in inclusive sense and exclusive sense. If I do not mention it write up or something like this then we take it as inclusive r, otherwise based on the circumstances we need to view it as exclusive r suppose if we say fruit salad or ice cream then you are not suppose to take both of them, but you are suppose to take either salad if you take salad automatically it will restrict you to take ice cream in that sense we are using it in the exclusive r.

So, law of (Refer Time: 07:45) means one distributes the other possibility and law of contradiction is that simultaneously a statement cannot be both true and both false P and not P cannot be the case at the same time in the same sense a contradiction occurs. When one statement excludes the possibility of another and yet both are came to be true. So, sometimes truth may not be self contradictory.

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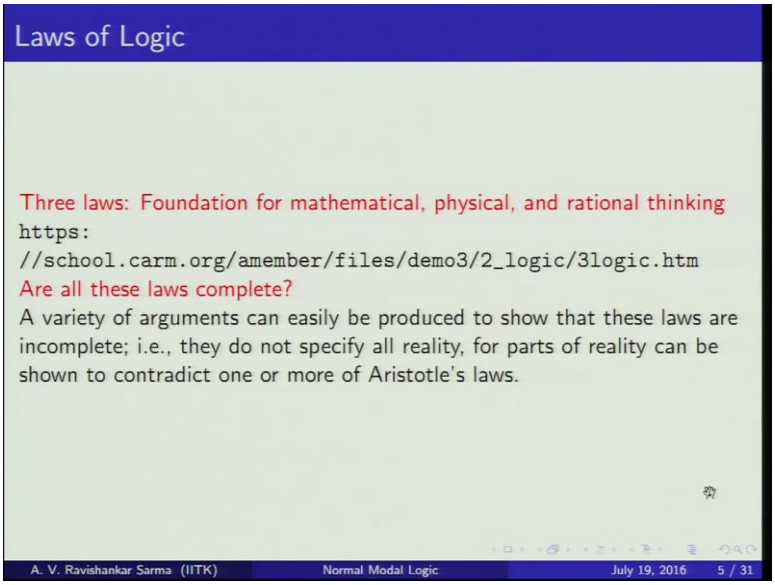

So, these now the immediate question that comes our attention is this that are all these laws complete, but situation is this that a variety of argument can easily produce to show that these laws are considered to be incomplete. So, they not specify all reality as a matter of at it refers to parts of reality parts of reality can be shown to be contradicting one another of these Aristotle laws that we have seen law of entities laws of excluded middle and law of non contradiction.

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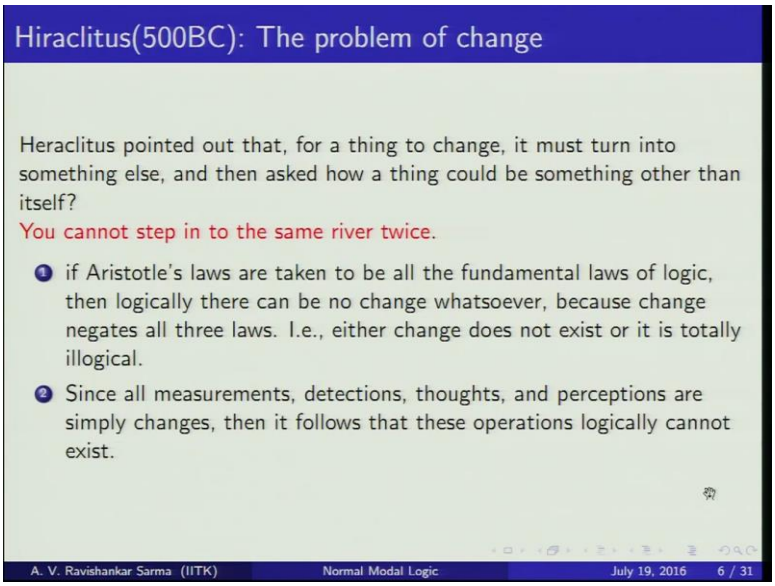

So, it is an interesting problem right is there right from the period onwards, it is considered to be the problem of change. So, it is Heraclitus pointed out that for anything to change it must turn into something else, if it is not turning into something else you are not set to things are not set to change at all and he ask he ask this question how a thing could be same thing how a thing could be something other than itself, if it involves some kind of change. So, this we has come up with the same famous and this is quite popular that is this thing you cannot step into the same river twice once you step into the river the water flows and then next time when you step in it is not be the same river different flow of water you will observe it.

So, now, if Aristotle law are taken to be all fundamental laws of logic that is law of entities law of excluded middle, law of non contradiction, then logically speaking there can be no change possible what server because change negates all the three laws why it is negates all the three laws from Iraq it is point of you if something is a changing into something else this a is turning into a prime or a star or something like that. So, which is quite different from now what is the case earlier what was the case earlier so; that means, either change does not exist or it becomes totally illogical that because all over loss of logic fails. So, for example, if a changes to a star then he is not equivalent to a and the same way if we if law of entity fails law of logic will automatically fails because you can always have a are not a prime something like this where it will not law of excluded

middle will not hold and law of contradiction also fails law of non contradiction also fails.

Somehow since all measurements detections thoughts perceptions etcetera, are simply considered to be changes then it follows that these operations logically cannot exist, why because, if you want to talk about change in logical terms these fundamentals laws of logic cannot capture it and if you do talk about change laws of change if to do talk about change in terms of laws of logic it is going to be illogic.

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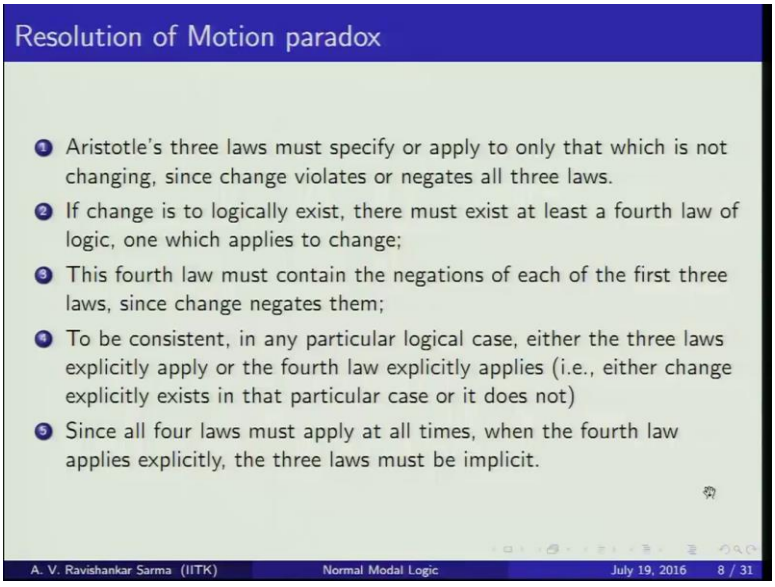

So, now the question is this that how does you reserve this particular kind of paradoxes this is also considered to be a paradoxes. So, which is called popularly known to be motion paradox whether motion is possible at all motion because some kind of change change from one position to another position? So, Aristotle three laws must specify or apply to only that which is not changing a static object since any kind of change violates all the three laws that we have seen earlier that is law of entities law of excluded middle law of non contradiction if it all change the exits logically exist and there must exists at least another kind of law which is a quiet popular these days this is considered to be fourth law of logic we need to expressively state that one if all the three laws fails, one which applies to whenever you have some kind of change which is there.

So, the fourth law must contain the negation of all the three laws since change in negates all the three laws because a is converting into a star and then a is not equal into a law of

entities is fails law of excluded middle also fails and it did not be law of non contradiction also fails so; that means, you need to have a fourth law to explain the change. So, to be consistence in any particular logical case either the three laws of entities law of excluded middle law of non contradiction explicitly apply or the fourth law explicitly applies if the fourth law explicitly applies all the three laws are automatically implicitly automatically false and if these three laws are explicitly assume to be the case then the fourth law is assume to be implicitly false.

So, either the change explicitly exists in that particular case or in the case or it does not since all four laws for supply it all times when, the fourth law applies it explicitly it is automatically assume that the three laws must be implicit. So, from this what we get is this that laws these three are considered to be the fundamental basic laws of logic law of entities law of excluded middle then law of non contradiction and based on that we have this classical logic that is propositional and predicate logic. So, now, how do we explain motion etcetera dynamic features like motion etcetera by using these three laws of logic?

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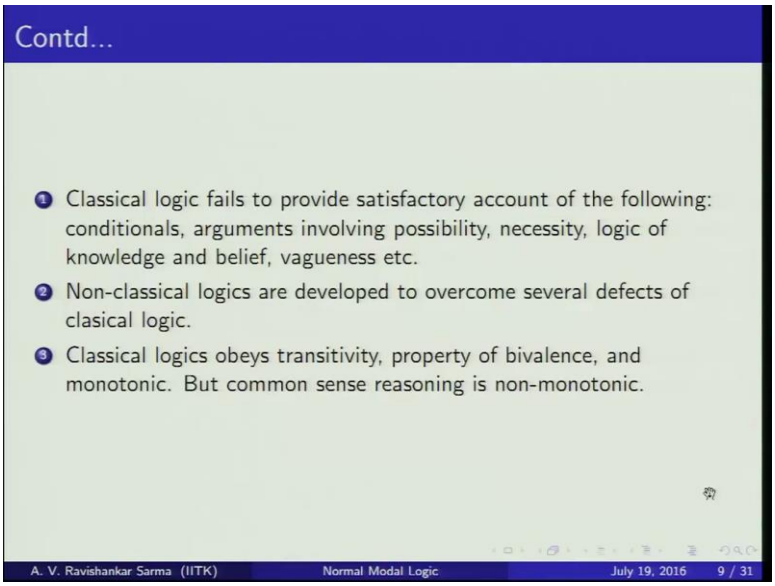

So, classical logic fails to provide a satisfactory account the following things apart from the change etcetera, these are the things at there are of interest to us it fails to account for proper analysis of conditions there are many different kind of conditionals which cannot be defined just by means of material implication like counter factual arguments subject to conditional etcetera our count our conditions with a feature and condition in which has

an ancient and it is different to a future a future continent a sentence. So, these are problematic kind of things.

So, arguments involving possibility because in classical logic there is no distinction between possibility of P and p, we always make this distinction that in our, we do not know what is happening in outside in this from this room. So, we always we can say that you know it is possible is humid outside or it is possible raining outside. So, it is possible then in outside does not implies it is actually raining there, but if you take a classical logic into consideration it is possible that P the same as it is actually in the case that P when it comes to necessity possibility actual it is no distinction in classical logic.

So, we need to make such kind of distinction and when it comes to logic of knowledge and belief we have separate kind of we have different kinds of modal logic to take care of this logic of knowledge and belief these are called a pessimistic logic and when it comes to vagueness the classical logic completely fails vagueness is not is not a part and parceal of classical logic. But if you want to live vagueness etcetera the then classical logic has no answer it has answers, but it may not be appropriate if you follow just two valid logic because with any kind of vagueness involves a kind of boundary lines in a suppose, if we take to examples of classical logic it is clear that there will be considering only in the cases like motel non motel you can clear a draw a line between, what is considered to be motel what is considered to be non motel like donkeys cats etcetera.

You can easily draw a line what is considered to be donkey what is considered to be non donkey, but in many situations in day to reasoning it cannot be in the case are in draw a clear line between what is a case and what is not the case when it comes to any kind of vague predicate like rich poor all these kind of vague predicates boldness etcetera. For example, if you take simple example of rich somebody who is having billions of rupees its considered to be automatically, it considered to be rich a person who is considered to bankrupt or beggar he is not having any money or any bank or anywhere else he is considered to be poor, but there are some other kinds of people who fall under this particular it definitely not rich definitely poor. So, they can be called upper middle class middle class lower middle class etcetera all these things are gradations of any kind of truth.

So, each sentences comes up with degree of truth that. So, if you want to have such particular kind of things then we need to move away from the classical logic we need to we need to give up of excluded middle and we need to allow for many values.

So, non classical logic are developed to overcome several such defects which you find it in the classical logic one such kind of thing is modal logic in the modal logic, we take care of particular kind of thing where you will be able to distinguish between possibility of possibility of P necessity of P and what is considered to be actually the case that p.

So, these of interest to us now we will be dealing with other kind of examples also in this lecture classical logic obeys transitivity a plus b b plus c and a plus c and property of bivalence it is has only two values and it is considered to be monotonic; that means, addition of new information will not you will not be addition of new information will not cause you to withdraw that conclusions that you derived earlier.

In common sense reasoning the reasoning that employ in our day to day reasoning is considered to be non monotonic in nature.

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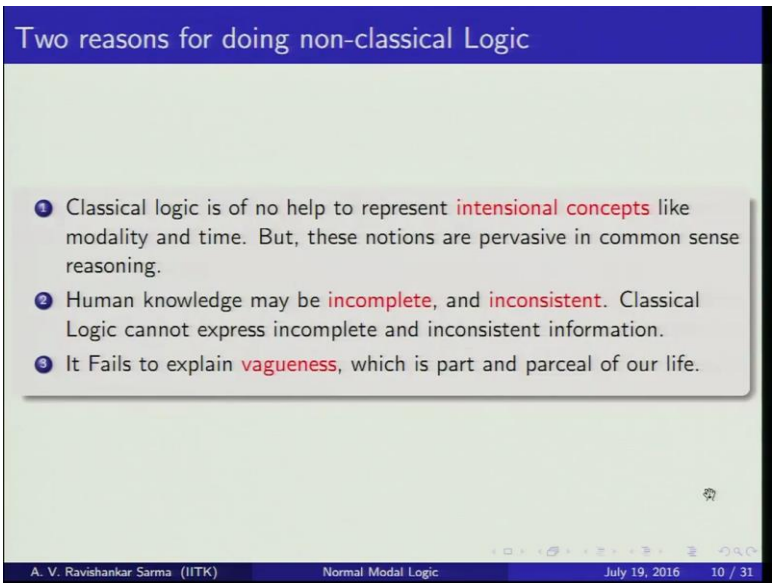

So, classical logic faces in these cases. So, the two reasons doing for the non classical logic there are many reasons of course, a classical logic is of no help to represent intentional concepts and in those concepts which involves modes of truth; that means, a sentence can be truth in different ways. For example, if you say x is happy it is different

from x is believe to happy x is thinking that he is happy x is known to be a x knows that he is happy etcetera. They are all different modes of truth in the same way whenever you are referring to temporal kind of concepts something which is always true something which is true in the past something which is going to be true in future. So, you need to have this modalities intentional concepts likes.

So, the difference between intentional or extensional which we will be talking about in the next few lectures, but in when a if the truth value of any preposition or any formula is only determined by the truth value of its constituents then it is considered to be extensional it is in that sense classical logics are considered to be extensional in nature.

For example if you have P r q, if you know the truth value of p. If you know the truth value of q you will automatically come to know the truth value of P r q and you would need to know that human knowledge also be considered to be incomplete and most of the time you have to tolerate inconsistencies of the global level and the local level certain inconsistencies at the global level you can still tolerate, but the local level you may not able to tolerate or may be other.

So, classical logic cannot express incomplete inconsistence information whether need to be more you have to invoke the non monotonic property there and it fails to explain to be vagueness which is considered to be the part and parceal of our life.

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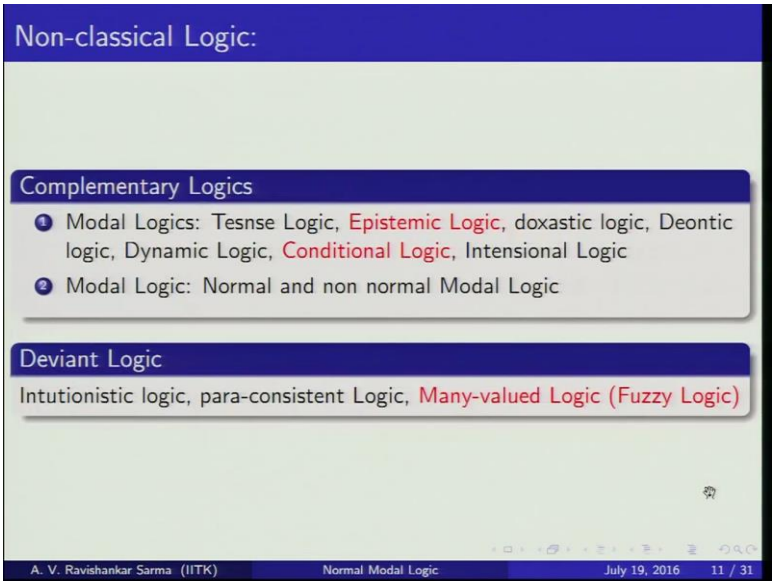

So, a non classical logic they other examples of non classical logic, but we will focusing our attention on modal logics model logic includes tense logic the logic of time and in where something which is always the case something which is sometimes it was to in the past and something which is actually true now these are different represented as some kind of modalities something in the necessary true in the context of temporal logic particular in the sense it is always true in the past present and even in the future and there some logic, doxastic logic and Deontic logic talks about logics of obligation etcetera morality issues relating to morality dynamite logic conditional logic and intentional logic. So, we will be dealing with alethic modal logic; that means modal logic which includes a possibility and necessity.

So, when we talk about necessity we are talking about logical necessity. So, this I will discuss it in greater detail when we when I talk about possible words and it is a technical notion possible word, I will be talking about it in greater detail. So, there are this is the thing which we will be during in this in this course, there is in other way in which other direction in which you can go ahead that is deviant logic. In the deviant logic what you do is this is some of the fundamental laws of logic that we have seen earlier; law of entities, law of excluded middle, law of non contradiction and situation arises in such a way you need to give up one of these fundamental laws of logic, you need to withdraw that kind of thing.

So, then you are deviant from the classical logic and these are considered to be deviant logics. So, intutionistic logic for the instancing for examples in this case suppose if you are not able to prove P and not able to prove naught P also then P are not be cannot be to there. So, when you are referring to the proof and you are not able to produce proof for P you are not able to proof for not p.

So, that is why P are not are not P does not hold there in the case of para consistence logic, it allows for some kind of inconsistencies like this thing law of non contradiction fills there, law of non contradiction states that it cannot be the case the sentence can be both true and both false, but it can have the example then many examples which you can have one example could be like this the person standing at the doors step with one step in and one step out.

So, now, if ask our self whether he is in of course, he is in he is out etcetera. So, you have more than two truth values, the sentence can be both true and both false also is not in he is in also. So, when you allow for degrees of truth of the sentence in classical logic, every sentence is considered to be either hundred percent true or everything is certain there. So, if you say all man or motels is man and is motel there is no in is in motel that is a conclusion which follows from all man or motels is in man with 90 percent with 70 percent etcetera.

So, if premises are true it leads to I mean your conclusion cannot be false, conclusions necessarily follows from the premises that is that is what is deductive reasoning is all about and classical logic is deductive prepositional and predicate transmit. So, in many valid logic you will allow many truth values and for example, if you allow from more than two truth values then you three valued logic and when you allow for many values; that means, you are invoking degrees of a truth of a sentence then it is considered to be multi valued logic and one instance of multi valid logic is fuzzy logic.

So, I will end this lecture with simple remarks that classical logic works in many cases and classical logic I mean prepositional and predicate logic is definitely a very good starting point and we need to have this starting point that is a reason why I came up with this I came with the few lectures based on a crash course on classical logic. So, we need to know where we are deviating and we need to know where we are extending that classical logic in a next the lecture I will be talking about some of the historical origins of classical logic and before that I will be talking about some examples where some examples where classical logic fails.

For examples paradox and some example relating to particularly in the distinction between necessity of P possibility P and actually the P and what interest us is some paradoxes. For example, sentences like these sentences this sentence is false if that sentence is that sentence is neither true nor false. So, how can we incorporate lire sentences into our language? So, these are the things which trigger us to move aware from the classical logic and either to extend it are deviate from the classical logic, but in our course we are only extending the classical logic by adding few more operators is are considered to be unary operators which stands for possibility and necessity.

Thank you.