Basic Concepts in Modal Logic
Prof. A.V. Ravishankar Sarma
Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

Lecture – 06
Semantic Tableaux Method – 3: More Puzzles

Welcome back, in this short lecture we will be dealing with semantic tableaux method concerning particularly solving lady or tiger problems. This is another interesting set of problems, these puzzles are cooked up by Raymonds (Refer Time: 00:32) and you will find these puzzles in the book, lady or the tiger, that is the title of book. So, these problems are very interesting in a sense that description of this puzzle goes like this. A king wants to punish his prisoners, and then before punishing them he is giving again giving them some kind of alternatives he is a generous king.

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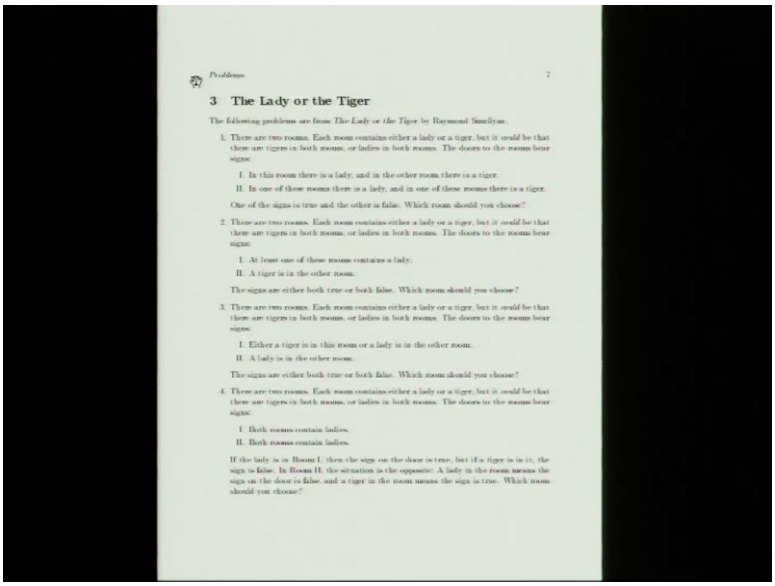

So, he says like this. Now, let us consider the first puzzle and then how we solve these puzzles by using semantic tableaux method. So, there are two rules, which he is showing it to the prisoners and I am taking about the first problem, each room contains either a tiger or the lady. So, if the prisoner finds the tiger he will be eaten, he will become food for the tiger, if he chooses the lady she can marry her and then happily live he will live is rest of his life. So, happen that the lady is also considerably beautiful.

So, the strategy of the prisoner is always to look for a door in which lady is present, nobody will be interested to choose the tiger. So, either there, is a lady or there is a tiger in these rooms, it might be the case that the king might have kept only tigers in these two rooms or ladies in the two rooms, in both the rooms. So, these are the pre conditions that we have and the doors to the rooms there he signs. So, prisoner has some kind of clues and using these clues he have to use his reasoning skills to figure out the right way, right thing for the point of view of a prisoner is to find out the door in which lady is present. In some other text books it is explained in the sense that same kind of problem tiger, if he chooses tiger he will be eaten and there is another one another door, where some kind of grand feast is waiting for the prisoner.

So, I more or less it is same thing finding a lady or having feast is he will be set free also in that case. So, now, these are the two sign boards that if that you that a prisoner sees. So, the first one is like this in this room there is a lady and other room, there is a tiger that is what is written on the first door room number one has this particular kind of thing, in the second door on the second door he have this thing in one of his rooms. There is a lady and in one of these rooms there is a tiger. So, we need to represent these two sentences in such in the language of propositional logic and then we will be able to manage this thing.

So, if the prisoner was given only these two. If he just sees the sign boards he will not be able to figure out whether which not is able to figure out the correct door he requires some more clues. So, now, then he asked the king then king is again generous, he replies like this one of the signs is considered to be true either means, whatever is written in the room number one is true or whatever is written on the room number two is true etcetera; that means, one of these signs is true, another one is false. If R1 is whatever is written on R1 is true R2 is automatically false.

So, now is a prisoner job will be become, little bit simpler with this additional clue, that is he also tells which sign is correct and which sign is not correct, whenever the king tells king is so, generous enough to tell these things. So, now, which room we should choose. So, now, this problem can be represented like this.

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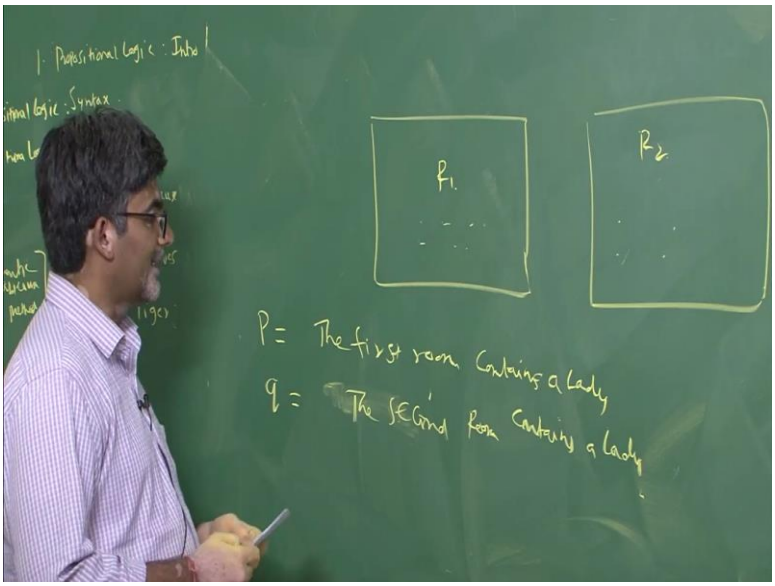

So, before solving this problem we need to first have some kind of representation. So, now, next p some kind of p stands for a propositional variable it is stands for the first room the first room contains a lady.

So, this is R1 and R2 some inscriptions are made here that is the one which you are seeing it in the slides. Now, let q stands for there is the second room contains a lady the second rooms contains a lady. So, again it follows true value logic the moment, you say that not p; that means, there is no lady in the first room and not q means, there is no lady in the second room. So, you can talk about a similar kind of thing by referring to only tiger the saying that during if p stands for in the first room contains a tiger and then q stands for second room contains a tiger. Then not p means first room does not contain a tiger not q means the tiger is not present in the second room, that way also we can represent it.

So, now let this be our representation. So, now, how to represent in this room there is a lady and the other room there is a tiger. So, this is a represented in this way.

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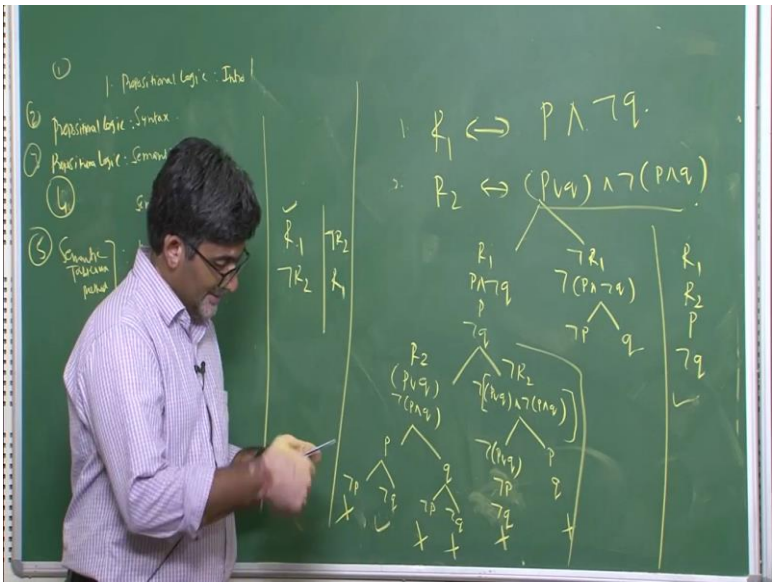

So, the first statement in his room there is a lady the other room there is a tiger is represented in this sense. So, this is what is written on R1. R1 tells us that in this room there is a lady that is p, in the other room there is a tiger. Tiger is present means lady is not there. So, in the initial description of our problem either both are ladies both are tigers is not the case that lady and tiger are there, it if it is there then tiger will eat away the lady and then there is nothing left.

So, p and not q, that is the first take statement and which represent this thing in this room there is a lady in the other room there is a tiger. Now the second is and the second door is what is written in one of these rooms, there is a lady in one of these rooms, there is a tiger. So, this is a usually we confuse with, confuse this statement as p and q, actually this is not the case. So, in one of these rooms there is a lady, means it rules over the possibility that there is no tiger in that room lady means no tiger and tiger means, there is no lady. So, this is a kind of exclusive or kind of thing p exclusively R q.

So, it is like either p or q is the case, but not both of them p and q so; that means, either lady has to be there or tiger has to be there, but definitely not both fix. So, both very in tiger should be there in the same room. So, in one of these rooms there is a lady in one of rooms, there is a tiger. So, which is written like this, p or q, but not both of them, this whole thing is represented as exclusive or one exclusive the possibility of other one, it is like we will go to some kind of party. Usually we see some kind of choices like either

you take salad or you take ice cream, something like that; that means, if you take some fruit salad or something like that you are not suppose to take ice cream.

So, one exclude the possibility of the other one, now, given that this is the description the other thing is that other thing which is given to us is this thing, one of the signs is true another one is false either it has to be like this or not R2 and R1. So, this gives us an indication that one of the sign board is true another one is false in this case R1 is true and not R2 is true. So, this you will use it little bit later. Now, apply semantic tableaux method on this one. So, this is going to be like this R1 p and not q is the first statement, I am writing here not of R1 sign board on, R1 is false then this is the thing not of p and not q. Somehow this is written as p and not q and this leads to branch not p not of not q means q.

If you we just see one open branch from that you can study the entire thing. So, now, this is there is no, negation does not exist here the branch is still open. So, now, we have p or q and the same formula and not of p and q this is one formula and the second one is we have to deny the whole formula and see not to p or q and not of p and q. Now, this simplifies to this one. So, p or q, p is written in the sense it is a branch which leads to this one. Now, you simplify this one it leads to this not p not q because, negation of conjunction is disjunction. So, this leads to not p not q. So, the initial description of the problem is gone. Now we are trying to handle or manipulate the symbols ultimately we are trying to our journey started with this thing, by constructing a tree diagram and ultimately tense of it with atomic propositions.

The atomic proposition tells us whether it is true or not. Now, your p here and not p here this closes. So, this branch remains opened. Open branches are the ones which you need to inspect it in greater detail. So, now, p have not p here and p here this closes q and not q closes here. We can still expand this particular kind of thing, but we have one open branch. So, that is like this with this you can find out a solution like this thing we have R1 just we need to see the open branch. Open branch is going like this R1 p not q on the way down p not q here you need to write R2 here because, R2 and this formula and not R2 leads this formula.

So, you have to this also. So, we have R1, R2. R1, R2 is there and p not q already done again p not q is there again it is repeating. So, that is why you need not have to write

anything here. So, now, this is the open branch which satisfies this particular kind of formula we need to come back to this one ah then. So, our representation is like this ah either both of them are true or at least one. One of the sign board is true another one is false. So, now, if this problem is interpreted in a different way both the rooms the sign boards are true then this is going to be the solution. But we are not getting the solution here because we need to look into ah this particular kind one of the signs is considered to be true the other one you has to be false here.

So, that is the although this is considered to be the solution for this one, but does not satisfy our initial description of the problem initial description of the problem we need to have one sign board true, another one is false and the other one is either R1 is true, R2 should be false. If R2 is false R1 should be true. So, this is this should not be taken has a solution now we need to expand this particular kind of branch. So, the ultimately we need to have R1 and not R2. So, this goes. So, now, this is not of p or q and not of not p and q is this 1 not, not p and q is p and q. So, it is written like this.

Now not q and q closes here and this is a branch which leads to sorry this leads to not p and not q then this branch also closes. Now, since this problem could be solved only when ah although we have open branch, here this is not giving as a solution here because one sign board has to be false there. So, since we did not achieve this thing now we need to go into the details of second branch. So, I will describe one problem and then I will stop it here. Now, in this case not R1, now we need to write the entire thing here this problem would be solved, we would have been solved when the king had said king probably he had said this particularly kind of thing in both signs boards are true then this is a solution.

So, if both sign boards are true R1 R2 are true then p and not q means this 1 p means lady is in the first room not q means lady is not in the second room. So, that is going to be the solution, but in the initial description of the problem it is mentioned that one of the sign boards is true another one is considered to be false. So, that is not serving our purpose. So, now, we need to get into the second one just I will close this particular kind of thing then I will expand this branch then we will see the where we will get to know the answer.

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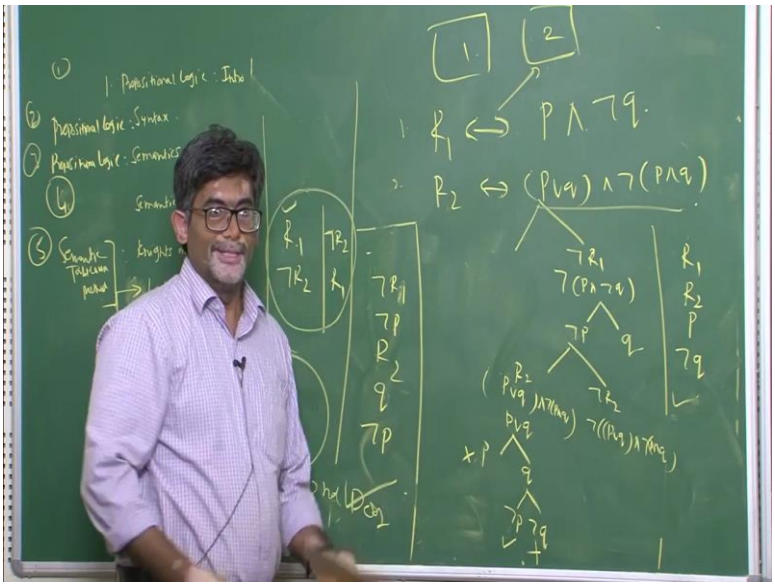

So, now this is one which we are writing R2 and the same thing p or q and not of p and q and then not R2; that mean, the sign board R2 is false and then this is negation of p or q and not of p and q.

So, now let us consider this particular kind of thing. So, this is p or q and not of. So, this is p or q p or q is simplified into this one not of p and q is simplified as not p not q not p and not q let us see whether any open branch is open or not. So, now, p and not p closes here not q all the way down here it does not close it is it remains open. So, this also remains open now q and not q closes here. So, now, here is another set of solution that we have. So, that is like this not R1 all the way down like this we have not p and R2, R2 is a 1 which we have and then p and not q this closes here itself because p and not p is there.

So, this is cannot be the solution. Now, the solution for this one is this one. So, this closes here itself. So, now, the other one is not R2 sorry not R1 and not p all the way down here R2 and then q and then not p. So, now, this satisfies our initial description for our problem. So, our the initial description of our problem, that I one of the sign boards needs to be true another sign boards needs to be false here R2, 1 sign board R2 is true and other sign board RR1 is considered to be false.

Now, this seems to be the adequate or satisfactory solution which fits into our description of our problem so; that means, here q and not p; that means, q means lady is in the

second room not p means lady is not in the first room. So; that means, the prisoner has to choose the second door. So, in the initial description of the problem these two rooms are given to him and some inscriptions are given to him. So, now the prisoner has to choose the second room rather than the first room.

So, in this way once you have a problem you have problems like this you need to represent it appropriately into the language of propositional logic and given the description of the problem then you can solve in which door lady is there or which door tiger is there. So, prisoners life would have been miserable if the king had not given clues like this thing, one of the sign board is true either both the sign boards are true or these kind of clues are not given to the prisoner. There is no way in which he can figure out the right door, the right door for him is the door in which lady is there or grand feast is waiting for him.

So, I will stop here in the next class we will be dealing with introduction to modal logic where we will be talking about the syntax semantics etcetera and then will be dealing with normal modal propositional logic. So, I stop here.

Thank you.