Basic Concepts in Modal Logic

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Lecture – 05
Semantic Tableaux Method: Some Puzzles

In the last few lectures we discussed about the Basic Concepts of Prepositional Logic, where we mentioned few things some of the important concepts such as what is considered to be validity, satisfiability, logical consequence, etcetera in a very superficial manner. In this lecture we will be talking about one of the important decision procedure methods that is employed in prepositional logic, so that is Semantic Tableaux Method.

So, we will talk about semantic tableaux method in somewhat greater the details and we will be dealing with some examples how to use this particular kind of method. Then we will make use of this method in solving some of the important puzzles such as knights and knave puzzles, lady art, tiger puzzles; both the puzzles are constructed by a Raymond Polin in his famous book; the first book is 'what is the name of the book', the titled of the book itself is 'what is the name of the book' and the second one is lady art tiger and some other sub title is there. Both books are very interesting to read, lots of puzzles involved in it but we will be solving these puzzles by using semantic tableaux method.

So, we might be wondering why we need to use this semantic tableaux method especially when there are lots of other decisions procedure method such as to truth table and some other method which are available to us. So, truth able is considered to be simple straight forward etcetera, but the problem is this that when number of the variables increases then it is very difficult to monitor the entire truth table. For example if you have number of variables exceeds more than 4 or 5; for example there are 5 propositional variables p q r s etcetera then we have 2 to the power of 5, 5 entries are going to be there in the truth table.

For checking the validity; how do we check validity with the help of truth table method it is simple and straight forward method. We need to find out a row in which you have true premises and a false conclusion. Suppose if we have a argument like p p implies q and not q. So, Now we need to inspect a row in which your premises are true; that means, p p implies q is true, but not q that is false. See when you have a true premises and a false conclusion the argument is considered to be invalid, but the problem there is that it is pretty straight forward simple etcetera easy to use but the number of the variables increases things would be very difficult. For example; 2 to the power of 6 entries, it is very difficult to monitor that particular that kind of thing.

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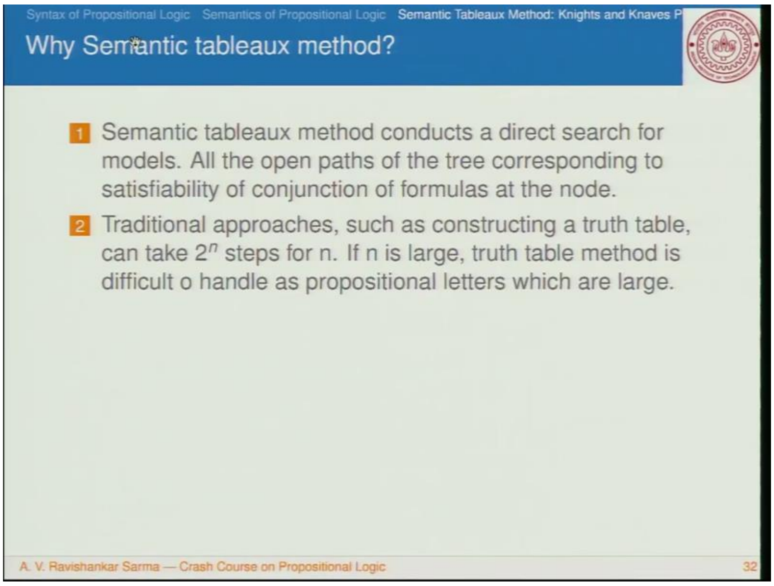

So, semantic tableaux method conducts a kind of direct search for the models. One of the important thing we need to note is this that all the open paths of a tree corresponding to satisfiability of conjunction of formulas which are occurring at the node. So, what essentially you do here is that for checking the validity of a given arguments what you do is you deny the formula and then you try to see whether all the branches closes or not if all the branches closes denial of the conclusion is denial of the formula not x is consider to be unsatisfiable, because all the branches closes.

Then unsatisfiability of not is guarantee is us that x is valid. And for satisfiability you need to take a given formula or a set of formulas and you construct a tree diagram for these things and then if any branch is open; that means, under that particular kind of interpretation whatever values that you have in that particular kind of path, that particular thing is considered to be satisfiable. Our two group of sentences for example, well formed formulas and you will come to know whether they are consistent or not by listing out one of another and construct a tree diagram and see whether all the branches open or not. Let this all the branches open.

And that particular path tells us when a given formula is set to be consistence, under what interpretation or a given formula is satisfiable or true. The traditional approaches such as constructing a truth table can make 2 to the power of n steps, where n is sufficiently large then if it is difficult for us to monitor the entire truth table. So, it is difficult to handle prepositional letters particularly when n is considered to be large. So, there are few definitions that we need to note before entering into the examples.

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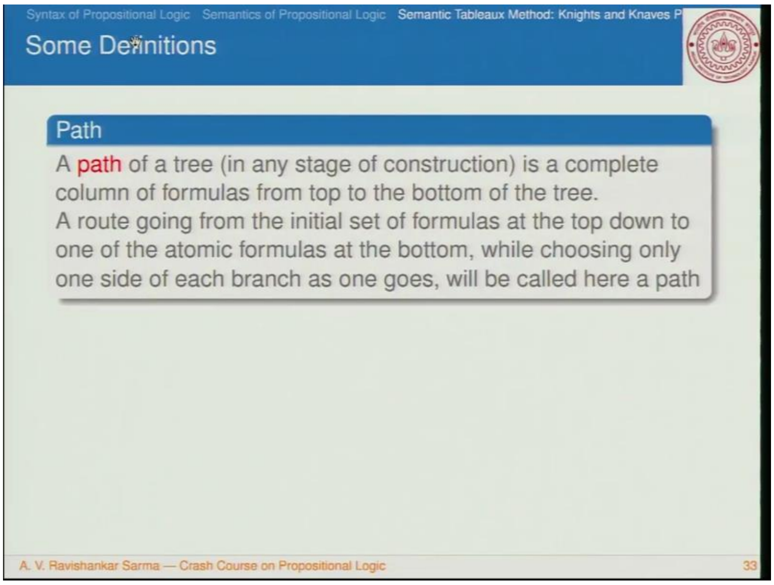

In a tree diagram there is something called as Path. So, we need to imagine an upside down kind of tree. A tree has trunk, the base and then you have branches. After constructing the tree the idea here is this that you need to end of with atomic prepositions. So, unless until you end of with atomic preposition like p s q r etcetera you will be using the semantic tableaux rules again and again to generate this atomic sentences you will end only when you end up with atomic prepositions.

So, there are few definitions that we will be using it here. First a path; a path of a tree is considered to be a complete column of formulas from top to bottom of the tree. So, the given well formed formulas are sitting at the node and then it ends of with particular kind of atomic formula and the whole thing is considered to be a path or it is considered to be route going from the initial set of well formed formulas from top to bottom till you reach atomic prepositions.

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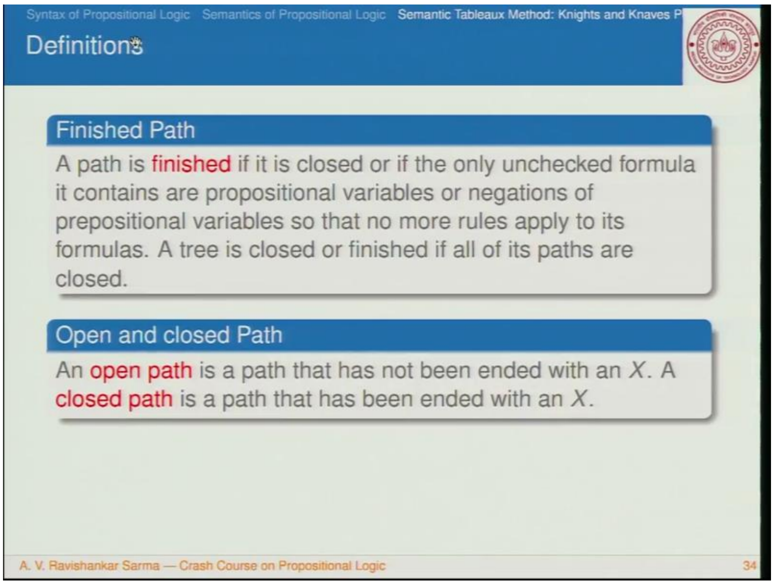

And when do you say that a path is considered to be a finished? When a literal and negations occurs a path is set to be finished, there is no way in which advance in that particular kind of path this is already some kind of contradiction. So, we have p and you have not p then obviously we need to close the path. And an open path is a path is not been ended up with we usually cross it.

So, that you know whenever you have a literal and it is negation occurs we close the branch or that path closes. So, why we need to close the paths? Because inconsistency is treated as some kind of help or on the problem here is this that if we have inconsistent kind of prepositions you can derive anything. Suppose, if you have p and not p, I can derive p we can even derive not q also where q is considered to be bit a kind of strange of preposition by simply following the rules of classical logic.

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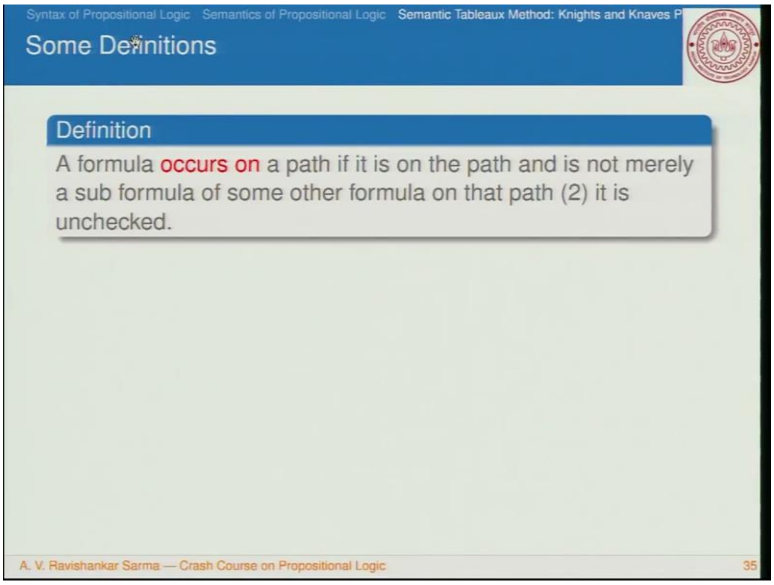

So, these are some of the definitions that we will be following the semantic tableaux method. A formula occurs on a path, if is on the path and it is merely a sub formula of some other kind of formula on that particular kind of path, where it is considered to be unchecked. So, all this things which will it will clear when I talk about some examples.

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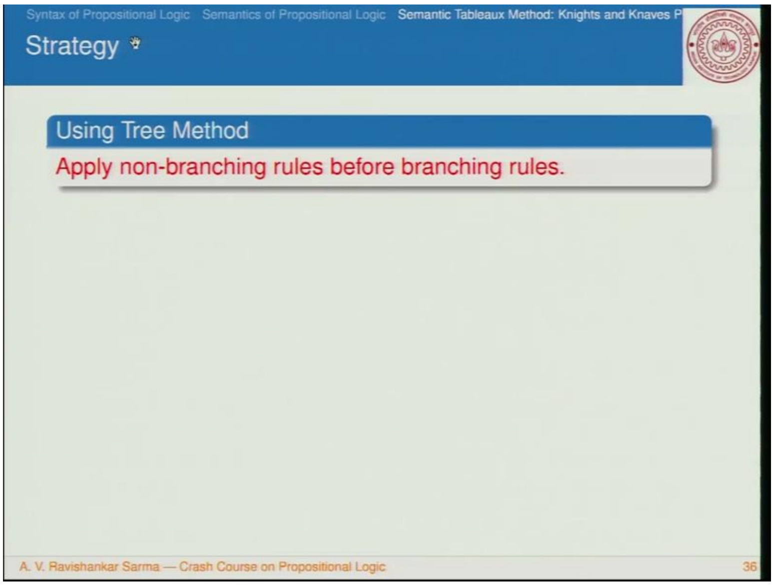

So, there are some strategies that we employ in using this semantic tableaux method. So, we need to ensure that we always use non branching rules. Non branching rules means for example, if we have not of p or q and you have p or q and you have another formulas p or q the first thing that you do is you have to adopt some kind of some bit of strategy, where when you apply the semantic tableaux rule it should not lead to a branch. The first formula that you are going to apply on is a non branching kind of rule that you need apply, before applying the branching rules. For example, p or q leads to a branch. In the same way p implies q leads to not p or q.

So, Now using this tableaux method Now we can talk about all the things that the truth table does or any other method does. There is another method called as resolution deputation method which is quiet popular in the computer science particular. So, all this methods are very important, but depending upon our convenience we use all these things are suggesting the same thing. Depending upon our convenience we will be using these method things.

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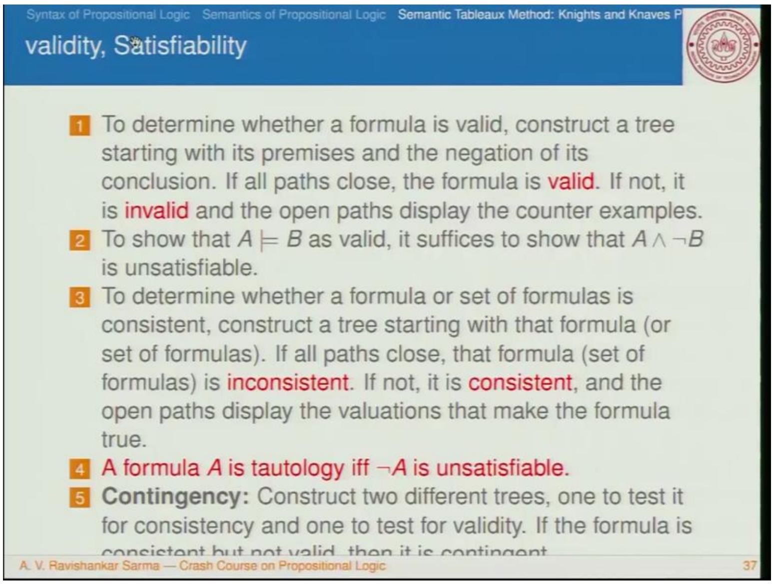

So, if you want to determine the validity of a given formula x what you do is you start the negation of the formula and see whether all the branches closes. If all the branches closes then you will say that not exceeds unsatisfiable; not exceeds unsatisfiable guarantees you that x is considered to be valid.

We want to show that two groups of statements are considered to be inconsistence to each other then you list out all the statements one after another and you start constructing the tree diagram by using the tree rules. And then you will end up with the atomic prepositions. If all the branches, if at least some branches open that tells us that are all given group of statements are set to be consistent to each other. For example, if you have p and not p if you consider it a truth table for that one all the branches closes. A formula is set to be tautology.

As I said in the last class in classical logic as we have three groups of statements; one is contradiction which is considered to be always false like 2 plus 3 is equal to 5. On the other hand we have tautologies which are always true, is this always true they are like self evident truths, axioms, theorems, etcetera all are tautology. And there are some set of other statements which are sometime true or sometimes false, they are considered to be contingent kind of statements.

So, you need to note that when we are going to discuss this modal logic one of the important problems that we will be addressing is how to represent the future contingent sentences. So, what kind of truth value that a given future contingent sentence takes. For example, if we say that I will be in my native place and so and so date, what is going to be truth value of that kind of particular thing? Classical logic fails to differentiate between something which is actually the case, something which is possibly the case and something which is necessarily the case.

That is a reason why we will be doing modal logic. And before doing modal logic which is modal logic is considered to be in extension of classical logic. That is why we are doing classical logic in somewhat kind of kind of crash course on classical logic, where we will be highlighting on some of the important thing. You should note that this is not considered to be detailed kind of study of classical logic.

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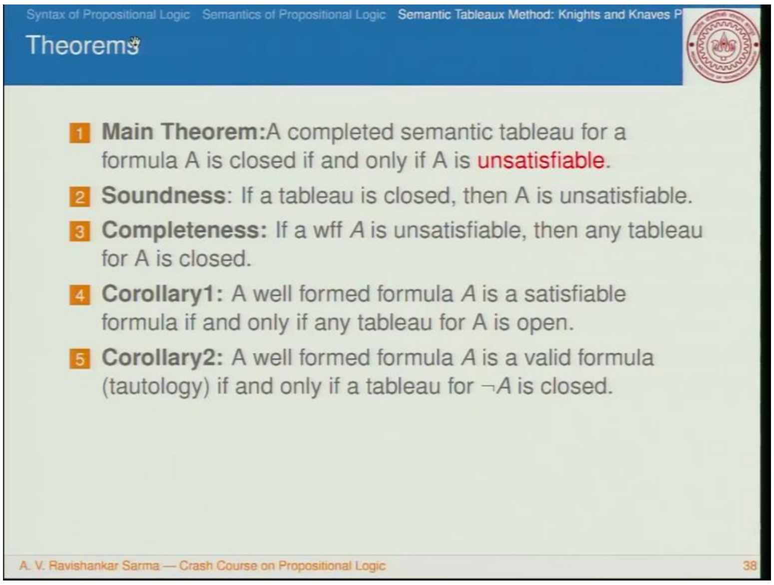

So, these are some of the things that we will be observing it in the case of semantic tableaux method. A completed semantic tableaux for a given formula A which is considered to be close if and only if A is unsatisfiable. That means, if all the branches Suppose if you start with a formula x and you take not x in to consideration in all the braches closes then x is considered to be valid. And how soundness, if a tableau is closed then obviously A is considered to be unsatisfiable, and completeness if a well formed formula is considered to be unsatisfiable then any tableau for A is set to closer. Idea here is that all the things that we proved it in the case of classical logic, all the properties such as these things can be done by using the semantic tableau method.

A well formed formula A is considered to be satisfiable formula if and only if any tableau for A is considered to be open. And corollary two is that a well formed formula is set to be A valid formula or a tautology if and only if not A, I mean you take the negation of the formula and if it get closed then it is considered to be a valid kind of argument.

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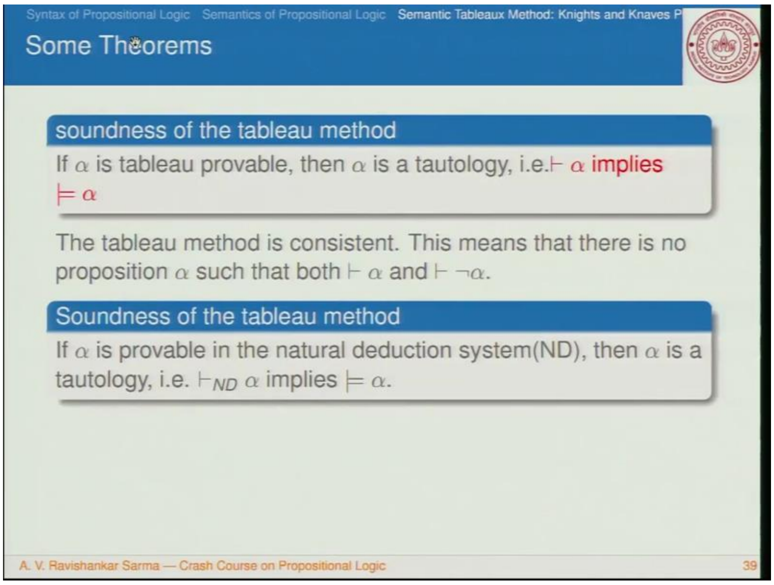

So, some of the theorems that we have is this that; if a statement alpha is tableau provable that means, if you given kind of alpha then you take the negation of that one and it leads to the branch closer. That means, it set to be provable. That means, you are saying that x is a tautology, all truth are provable. Then alpha is considered to be a tautology. That means, if alpha is tableau provable, that means you take the negation of that one it does not it leads to contradiction that means not of alpha is unsatisfiable that means, alpha is valid.

So, a tableau method is again said to be consistent and this means there is no preposition alpha such that you can derived both alpha and even not alpha also. So, soundness of all at tableau method is considered to be sound consistence etcetera. All things at you have done in the case of prepositional logic by using so many so many other theorems can be done by using semantic tableau method. The same way you can show that natural deduction method is sound consistent etcetera. Like this you can say tableau method is also said to be sound consistent etcetera.

So, soundness of the tableau method how you do it? It is like this if alpha is provable in the natural deduction system is this the another kind of system where you prove certain theorems by simply following some kind of valid principles of logic like more despondence, more tollens, etcetera conducting (Refer Time: 14:10) etcetera. If something is provable in the natural deduction method in that way it can very well be done in the case of by using semantic tableaux method also. So that is what it is saying.

If alpha is naturally deduction provable can be also implies that alpha is considered to be tautology. Say anything which is provable has to be true, all the true statements are that considered to be provable. So, in that sense prepositional logics are considered to be complete. Before going into the details of something let us consider some examples where how we can use this semantic tableaux method we will be doing with some examples.

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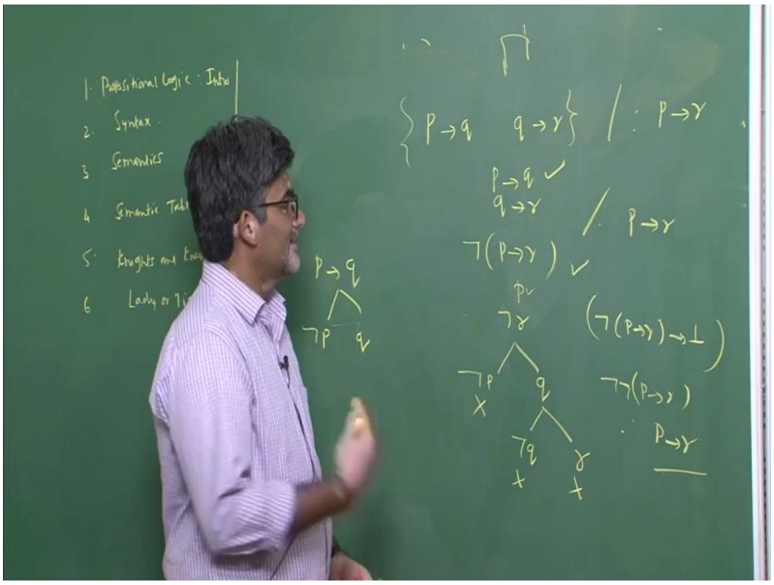

So, let us start with simple example. That is Suppose if we have some set of formulas like this just write it here. So, assuming that if we have some formulas like this, Now we you want to check whether p implies r follows from these two formulas are not. So, these are set of formulas that we have it is considered to be gamma from that whether p implies r follows or not. So, Now we are on to test whether p implies r is consequence of these two things.

So, first thing which you do is you list out all the things premises first and then separated by that you have a conclusion like this p implies r. Now the first step that you do in the case of semantic tableaux method is this that you deny the conclusion. Ideally speaking nothing is this that x is considered to be your conclusion, Now you are denying the conclusion if the denial of the conclusion leads to contradiction. So, when it leads to contradiction if you deny it on constructed tree diagram all the branches that occur in the tree diagram will close. Then it should not be not x, but it should be not of not of x. So, that is nothing, but x.

So, this is semantic tableaux method is dependent on this particular kind of principles. Not x implies a contradiction (Refer Time: 16:35) stands for contradiction. And, that means it should not be not x, but it should be not of x that is x. Now a using alpha and beta rules we will be expanding this thing. While discussing about the semantic tableaux method we have seen that always use formula which leads non branching rules are the once which you use it first and then followed by that the branching rules. This leads to non branching kind of thing is p and not r. Why because, not of x implies y is x and not y so. So, Now this is checked you do not have to check it again and again so that is why you put check mark on this, so coming back to any other formula.

So, Now if you simplify this thing it is not p or q. Why it is not p or q? The semantic the tableau rules for this one are tree diagram for p implies q is not p and q or q. Now this is checked that is why we are put tick mark here Now q implies r. Now every time when you apply this tableau rules you need to see whether a literal and negations occurs in the branch. So, Now here you have p here and you have not p here this branch closes here itself you do not have to expand it. And inconsistence kind of formula need not have to extend further, so this closes here.

So, Now this particular kind of information this is what is left here this needs to be written under whatever path which is open here though open branch. Now, this is not q and r. Now, if you observe it here, this branch closes here because of this thing q and not r occurs here, so that is why this branch closes. And r and not r occurs here that is why this branch closes. That means, what we have shown is this that denial of the conclusion that is not of p implies r leads to the branch closer, so that is why not of p implies are leads to contradiction. That means, it should not be in not of p implies r it should be not of not of p implies r not of p implies r is nothing but p implies r.

You might ask what is the thing which you get from this thing? So, semantic tableaux method is some kind of decision procedure method this is the simple transitivity property p implies q q implies r and p implies r. This can be proved in sense that you derive the conclusion it leads to contradiction. That means, it should not be not of p implies r it should be p implies r.

So this is the way you check the validity of given formula. Validity in a sense here it is consequence of this. So, you can check validity of formulas like this, I will you few examples and then I will stop this thing.

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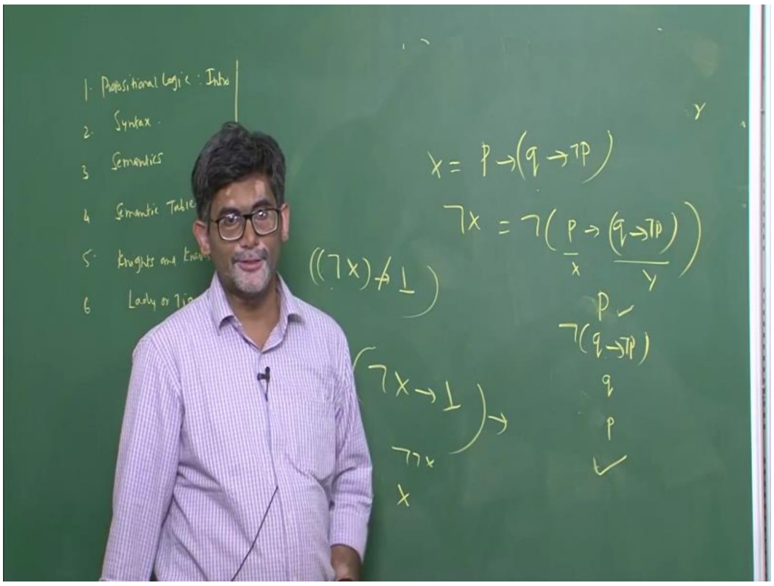

So, Suppose if you have a complex formula like this. Now we need you want to prove whether this is a valid formula or not. So, when this formula is going to be a valid formula especially when it is considered to be tautology. Now we are going to check whether tautology of a given formula. So the first thing that you do is if assuming that it is x and you take not x into considering not of p implies q implies p, the first thing that you do. So, the entire thing is x and this is why then it is going to be not of x implies y beta rule is this thing x and not y; both x and not y have to be true for making this not of x implies y true.

In that sense, so p as it is and not of q implies p is the one which you this way you list it one after another. Now, this again you apply beta rule on this one it is going to be q and not p. So, Now you have p and not p here this closes. So, what did we show. So negation of this given formula leads to contradiction. That means, it should not be not x but it should be not of not of x so that is why it is x. In our case x is the original formula p implies q implies p.

So, you can play with formula by putting not p here then you see that the branch is going to remain open. For example, if you put not p here then it is going to be like this, then this will become p Now, so this branch remains open. There is no way in which you can close the branch. That means negation of x that is one which you started with does not lead to contradiction. This does not give guarantee that x is considered to be valid. Simple other example that I have I will be talking about where it can shown a group of sentences are set to be consistent or satisfiable with this I will start.

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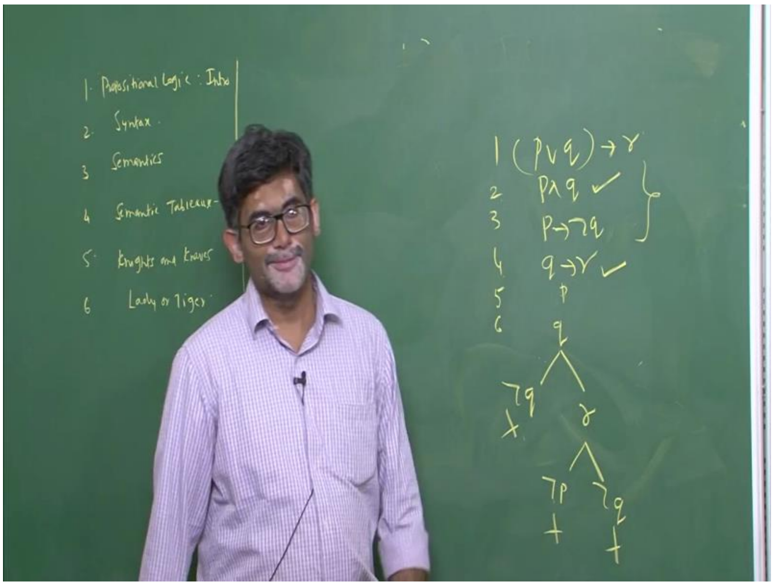

So, let us say you have some sentences like p implies q not q are something like q implies r; 1 2 3 4 then it will bit complex statement that you take. Let us say you have these four sentences Now we want to show whether these sentences are considered to be consistent or not. So, they are set to be consistent only in even after applying this three rules you under with the atomic prepositions then ultimately there should be at least one open path that shows us these are set to consistence.

So, consistency is considered to be the most important thing in a sense that you cannot talk about two things like this is you know it is raining and it is not raining, it is a inconsistent kind of statement. So, Now again first strategy that use is that always use non branching rules the one which does not lead to branch is the one which you need to take into consideration first. So, this is this one. So, p implies q we simplified has p q. Now, this is checked so that is why you have to put tick mark there, Now any formula that you can take into consideration. Now, I am handling this one not p and not q. So, this is from three you can write it justification like this.

Now, each time when you apply this rule you need to see whether literal and it is negation occurs in any path, open path like this, had a branch. So, actually this is like upside down kind of tree. All this formula should be there here, but we have shifted according to our convenience. Now this closes here because p and not p occurs here and even q and no q this also closes here. There are two things observation that you can make here. Suppose if you did not use these two things initially non branching rule is the not the one which you apply then what happens is this that the proof will become little bit lengthier.

So, always in our it is our strategy is this that always apply non branching rule first and then so it comes only through our practice. That for example, in the same kind of example, Suppose we have not instead of going into this one I went to this formula then what happens is this. So, Now the proof will become little bit lengthy. Now, I am opening this formula instead of this, so it is going to be not q implies r. So, q and no q this closes. Now this is check here.

Now, coming back to this one there is no way which you can close the branch. Now you apply this p implies not p implies not q. So, Now q and not q closes and p and not p closes. The idea here is that your proofs can become lengthier if you follow this one first then followed by that you take this into consideration. So, when do you say that a given proof to be considered to be an effective proof, an effective proof is a one which ends in finite steps in finite interval of time. So that is considered to be an efficient kind of prove unnecessarily they should not some excessive kind of information.

The one which have showed it in the last thing that is considered to be in the efficient prove because it involves less no of steps. So, I will stop here than in the next lecture we will be talking about how we can use this semantic tableaux method to solve knights and knaves puzzles that is the one which will be taking into consideration in the next class.

Welcome back in this lecture we will be seeing how we can use this semantic tableaux method in solving some of the important puzzles they are called knights and knaves puzzles which are constructed by (Refer Time: 26:20). You will find this puzzles in the book what is the name of the book, what is the title of the book in that book you will find all this puzzles. And the second set of puzzles that we will be dealing with the next is the lady art tiger kind of puzzles.

So, using prepositional logic particularly the semantic tableaux method you will be able to figure out who is knight and who is knave. These puzzles are interesting in a sense that the description of the puzzle goes like this. There is an island and in that island there only two kinds of inhabitants. So, these two out of these two inhabitants. There are some kinds of inhabitants who always speak truths they are considered to be knights. There other kinds of inhabitants who always tells lies, that means they always tell false things. For example, if you ask a knave is 2 plus 2 is equals to 4 he will say no, if you ask knight is 2 plus 2 is equals to 4 he will say yes. So, knights tells truths and knaves tell only false things.

So, it is beautifully it can be described into two valued logic where some inhabitants always talks true and other inhabitants set of inhabitants talk lies. Now, let us consider some problems. So, Now you went such a kind of island.

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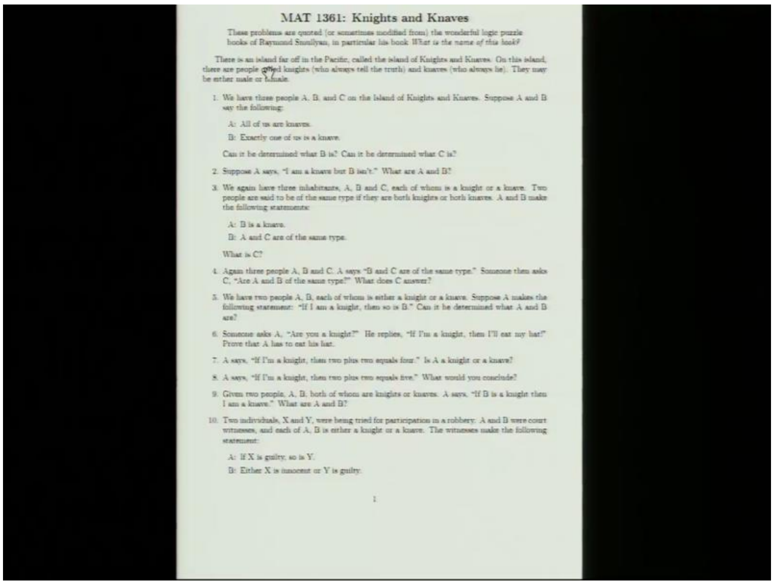

MAT 1361: Knights and Knaves

These problems are quoted (or sometimes modified from) the wonderful logic puzzle books of Raymond Smullyan, in particular his book *What is the name of this book?*

There is an island far off in the Pacific, called the island of Knights and Knaves. On this island, there are people called knights (who always tell the truth) and knaves (who always lie). They may be either male or female.

1. We have three people A, B, and C on the Island of Knights and Knaves. Suppose A and B say the following:

   A: All of us are knaves.

   B: Exactly one of us is a knave.

   Can it be determined what B is? Can it be determined what C is?

2. Suppose A says, "I am a knave but B isn't." What are A and B?

3. We again have three inhabitants, A, B and C, each of whom is a knight or a knave. Two people are said to be of the same type if they are both knights or both knaves. A and B make the following statements:

   A: B is a knave.

   B: A and C are of the same type.

   What is C?

4. Again three people A, B and C. A says "B and C are of the same type." Someone then asks C, "Are A and B of the same type?" What does C answer?

5. We have two people A, B, each of whom is either a knight or a knave. Suppose A makes the following statement: "If I am a knight, then so is B." Can it be determined what A and B are?

6. Someone asks A, "Are you a knight?" He replies, "If I'm a knight, then I'll eat my hat!" Prove that A has to eat his hat.

7. A says, "If I'm a knight, then two plus two equals four." Is A a knight or a knave?

8. A says, "If I'm a knight, then two plus two equals five." What would you conclude?

9. Given two people, A, B, both of whom are knights or knaves. A says, "If B is a knight then I am a knave." What are A and B?

10. Two individuals, X and Y, were being tried for participation in a robbery. A and B were court witnesses, and each of A, B is either a knight or a knave. The witnesses make the following statement:

    A: If X is guilty, so is Y.

    B: Either X is innocent or Y is guilty.

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Now, these are the some of the set of problems that you are seeing here. Let us consider the second problem so that is like this. Suppose if A says I am a knave and B is not, so you came across the A B, so Now A is discussing A saying this thing. A says I am A knaves, but B is not.

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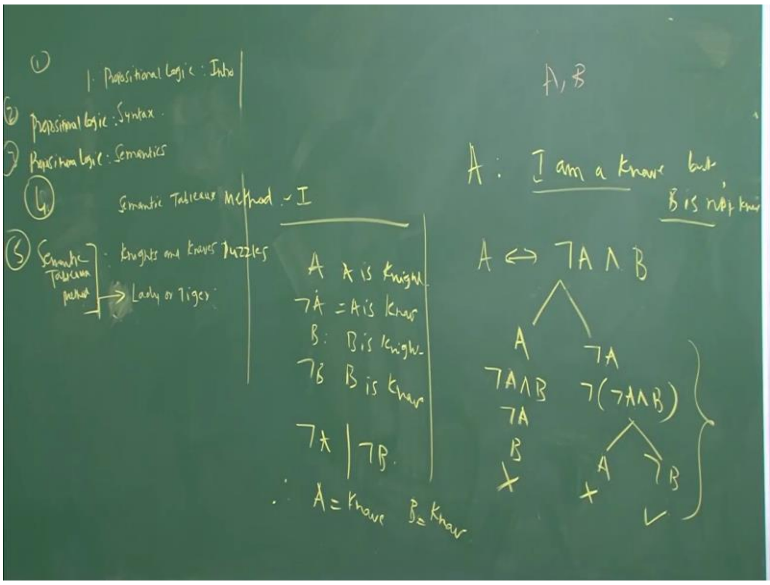

So, what are A and B. This puzzle can be described like this. So, A is saying I am a knave, but B is not. So, this is the only statement that is available to us. Now, we need to figure out what are A and B. Now, there is some all these puzzles can be solved by means of some kind of representation. You need to represent these sentences with p q s etcetera. Suppose, if I write simply A, that means A is set to be knight. And write not A that means A is knave.

Now, there is one more thing which you read to take into consideration here, A saying this particular kind of statement. So, what it means to say A says let us say x, so how do you represented it in classical logic. You need to take two things to consideration here, if A says something let us say he says 2 plus 2 is equal to 4 and that needs to be a says that 2 plus 2 is equal to 4. And the other thing is this that if 2 plus 2 is equal to 4 is the case and that has to be addressed told by only A, non other than A. So, that makes this thing A by conditional kind of thing

Now it is in that sense we write this particular kind of statement like this. So, whenever you have a says something it is going to be by implication. Suppose if I say that 2 plus 2 is equal to 4 then A implies 2 plus 2 is equals to 4 is the case and 2 plus 2 is equals to 4 is implied by this thing that is said by only A, non other than A. So, Now how do we represent this sentence I am a knave. So, here a is talking about it that is why you have to write like this not A, but is usually translated as end and B is not. So, if you write just B it means B is knight not B; B is knave, but here we have B is not knave. B is not a knave means it is not the case that not B. That means, B has to be a knight. So, it has to be like this

So, Now what we have done here is that we have translated the English language sentence into appropriately into the language of preposition logic. Now, by using semantic tableaux method we are trying to figure out what is A and what is B. So, Now this is like a simple rule which is like this A if and only if B means either both have to be true or both have to be false. If you understand one example you can understand all other examples as well.

Now this is it represented like this not A and B and on the other hand we have negation of this and negation of this also not A and B. Ultimately what we are trying to do is this that under what condition this formula is going to be satisfiable. When this is going to be satisfiable, especially when you have when you constructed a true tree diagram ultimately after exhausting all the tree rules etcetera, now you under with atomic prepositions. That means, you end of with the atomic prepositions you will not applies the rules.

In the semantic tableaux method ultimately the tree diagram ends with atomic prepositions. So, Now this is written like this. Now, you see here is A and not A is branch closes, so you cannot further Now in this case. So, not of A and B. So, this is using (Refer Time: 32:40) law we can say like this not A and not B. Not A is simply A. So, Now you have A here and not A here this closes. Now the only thing which is left is this one you have not A and you have not B. In our original representation, so this is our initial representations A means A is knight, not A means A is knave. This means not knight means knave only. So, in the same way B means B is knight, not B means B is knave.

So, the answer that we got is this that open branch is the one which you need to inspect. So, open branch tells us when this formula is going to be true. Now, this is the not A and B not A and not B. So, not A means A is a knave and not B means B is knave, so that is why the answer here is A is knave and B is also considered to knave. So, then only it satisfies this particular kind of formula or it satisfies a statement that if A says that I am a knave, but B is not a knave then this is going to be the solution So, let us considered some other interesting problems. For example, in the first problem what need to (Refer Time: 34:18) says all of us are knaves. All of us are knaves can be represented as not A and not B and not C etcetera, that tells us that all are knaves

The second statement B is saying that exactly one of us is a knave. I am talking about the first example in this thing. So, exactly one of us is a knave means there are three possibilities. Suppose if you take A is a knave you should ensure that B and C are knights. So, this needs be written like this I am not going to solve this problem because it takes lot of boots (Refer Time: 34:57) So, I am just representing this formula and later you can apply this semantic tableaux rules and then we will be able to come up with an answer.

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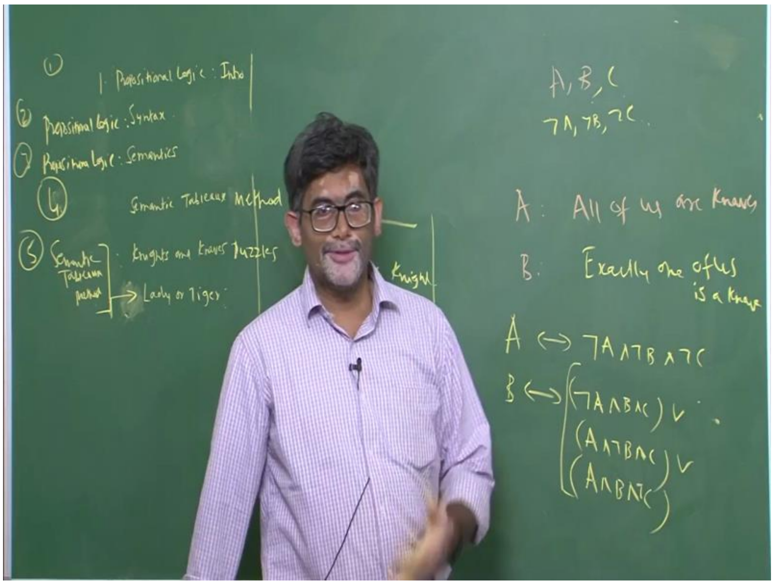

A says all of us are knaves, B is saying; what B is saying B is saying exactly one of us is a knave. So, Now the first sentence is translated like this A if and only if all of us are considered to be knaves in our representation if A B C are knights not A not B and not C are considered to be knaves. This means not A and not B and not C or considered to be knaves that means, everyone has to be knave here, that satisfies this things all of us are considered to be knaves. If at least one is not a knave satisfy this preposition that in all statement that all of us are knaves. Now B needs to be represented like this. This is considered to be little bit lengthier kind of statement exactly one of us is considered to be knaves.

That means, it goes like this; if A is a knave then you need to ensure that other two are not knaves that means, they have to be knights. This is the first preposition or that satisfies this particular kind of formula exactly one of us is a knave there are three kinds of possibilities here. Or the other possibility is this that A and not B and C, this is one kind of formula. And the other formula is A and B, but C here is the knave. So, all these things are separated by this thing. So, this entire formula is the one which you need to take into consideration. Once you apply semantic tableaux rules for this one then ultimately you need to inspect the open branch and some the open branch you can study who you will be able to understand it from the open branch.

So, in the open branch for example if you have A not B C etcetera that means, A considered to be knight not B means B is a knave and C means and C is considered to knight Let us considered some other examples I am not solving this problem, but just I am giving you some kind of hint. So, let us consider some other interesting kind of problems. Suppose, if you in the seventh one for examples a says if I am a knight then 2 plus 2 is equal to 4. That is the simple straight forward thing whether e is considered to be knight or knave, if A says this thing.

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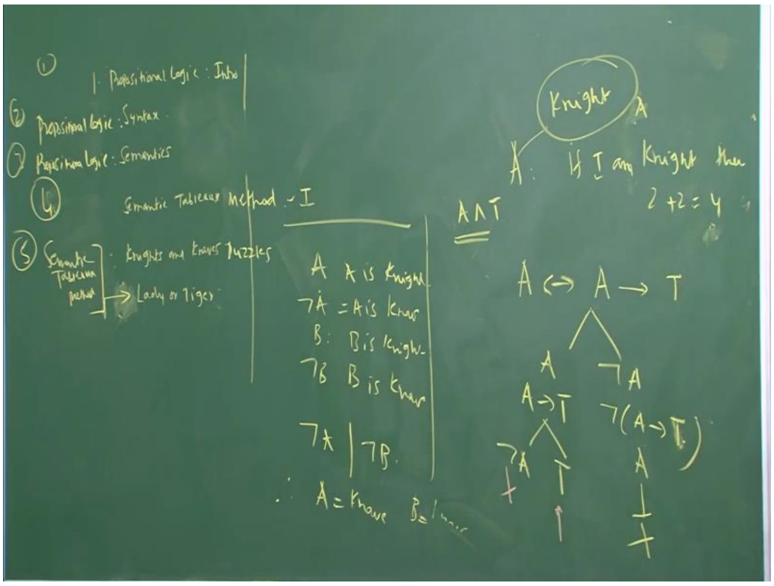

What is the thing seventh one if I am a knight then 2 plus 2 is equal to 4. So, what can be say about A Suppose if he is making this particular kind of statements we need to figure out whether he is a knight or knave. Now, usually when we 2 plus 2 is equal to 4 is always true so that is why we represent it with this letter T. T means those sentences which are always true, they are also considered to be tautologies. This sentence can be represented like this A if and only if this thing this sentence is in the form of if p then q if this is represented as A and this is represented as B.

So, if I am a knight in our original representation it is like this the knight is represented A only it leads to T. This T should not be confused with another preposition variable. There are two constant that we are talking about and we talked about in the syntax, so they are this thing T and boat; this stands for tautology, this stands for contradiction. Now, we need to find out when this is going to be satisfiable. Now, it is A and A implies T and then not A not of A implies sub T. So, this is a not A and T and then, so this leads to this one A and not of T. So, Suppose if you have 2 plus 2 is equal to 4 then the negation of that one going to be contradiction. So, this not of T is equivalent to [FL] T that is boat, so that means there is a contradiction here this branch closes here.

And Now coming back to these things. So, you have A here and you have not A here this branch also closes. Then the only branch here is this thing. We have this particular kind of solution where if A says if I am knight then 2 plus 2 is equal to 4 the solution that we are getting here is this that the open from the open branch you can find out what is what. So, A and T is the one which we have A and T here, so that means, A has to be true I mean A has to tell the truth, because A and T is going to be true only when both are considered to be true. So, this is already true it cannot be knave because if it is a knave the sentence is going to be false, so that is a reason why we can say that A is said to be a knight

So, like this we solve these puzzles Now let us enter into the tenth problem which is little bit interesting. There are two individuals x and y core drying for being tried to participation in a robbery they are considered to be the witness. So, Now there the guilt and innocence we do not knowow. So, Now we have this inhabitant. So, like A and B they are considered to be Now it is so happened there they turn out to be witness of x and y so they have to report to the judge or lawyer whatever it is. And each of A and B again is considered to be either knight or knave.

Now the witness makes the following statements. This A and B they can be either knight or it can be either knave, but this A and B make these statements from this you need to figure out whether x is guilty or y is guilty etcetera. So, Now the first statement is like this A says if x is guilty so is y. This problem can be solved like this. So, first you need to represent it appropriately into the language of this thing again you take this representation into consideration A stands for A is a knight not A means A is knave B means B is a knight etcetera.

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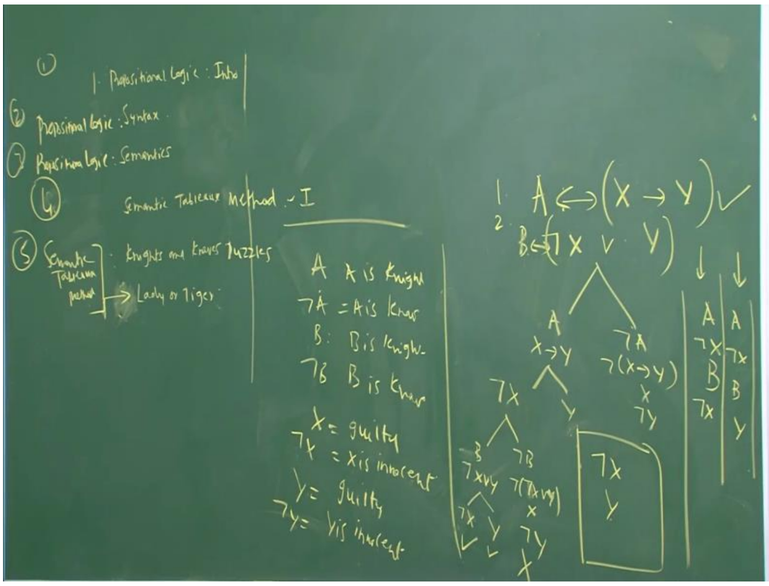

So, the first statement says that a is saying that if x is guilty then y is, this can be represented as this thing. If x is guilty implies y is also guilty, so that is the first statement. The second statement is that this thing, either x is innocent or y is guilty. So before that you need to have more representations like this, Suppose if you write x is a guilty then obviously the negation of that one is considered to be x is innocent. In the same way suppose y just we write just write y y is a guilty not y means y is innocent. So, you can consider another kind of notation. Suppose if not x is considered to be guilty for you I mean is the one which is represent it has guilty then obviously x is considered to be not x is guilty then x is considered to be innocent, opposite of that one is considered to be innocent.

Now the second statement is like this, either x is innocent or y is guilty. X is innocent, x is innocent is this one not x in our case in our representation or y is guilty means y. This is the one that they are trying to make. So, this statement is made by B. So, that is why you write it like this. Now the first statement is telling A is telling us about this thing that if x is guilty y is also guilty this is the one so as y that means, this can be translated as simply if p then q. And the second one B if and only if either x is guilty x is innocent or y is guilty is translated in this sense.

So, Now forget about the problem here, Now we manage this thing with the help of the formulas that we have. So, Now applying the tree rules for this one we are trying to figure out who is guilty and who is innocent. First I will apply on this one it is either x implies y or not A not of x implies y, so now this is expanded to this thing. X implies y means not x y and here it is x and not y. So, you check this formula and that is why you tick mark this one. So, the one second formula is the one which we have.

Ultimately what we are trying to do is this that we are trying to see when these two statements are considered to be consistent or satisfiable. Under what conditions this two are true. So, we have two things like either B not x or y other way other one is this thing not B not of not x or y. So, it is x implies y means x if and only if y means either both are true or both are false not B is the case and not of x or y. And just expand it, then you need to note that from at least if you have one open branch that gives us a solution for this one. So, this is further expanded it can be further expanded and I can be expanded to this thing not x or y is written in this sense. So, Now these this is one path this is another path. So, we need to see it whether a literal and it is negation occurs in any one of this branches. So, A x implies y not x B not x. This appears to be a open branch in the same way B not x A it is not closing this itself is considered to be an open branch. So, you can construct the whole tree to find out the solutions as well, but you know one open branch will tell us when this is going to be true.

So, Now from the open branch you construct the answers for this one like this. So, the situation here is like this A and not x B not x this is one thing. That means, if senses are if A and B are considered to be knights and x has to be guilty here that is the one that we are getting it. The other solution is this one this is not of not x is x and not of y is this one not y. So, here you have x and not x this closes here, so you do not have to go further. Now, here we have another solution where you have A not x B y. From this two things we can make out whether who is considered to be guilty and who is considered to be innocent.

So, Now we have two solutions like this which satisfies this particular kind of formula. So, from this definitely you can say that not x, not x means x is innocent. Now this is the same thing which we got it here in the second thing also. That means, not x here means x is innocent and y in our representation y stands for y is guilty. Now from this you can make out from the open branch you can see the solution here. Now, the answer for this one is this that you can expand the other branch like this you have to apply the same semantic tableaux to get to know the other things, but one open branch is (Refer Time: 49:01) you do not have to open the entire thing. Our solution is not x I am putting it in this thing and then y that means, if A says x is guilty so as y and B says that either x is innocent or y is guilty on the same sentences are given to us then obviously A not x y should be the case that means, x has to be innocent and y is considered to be guilty.

So, like this we solve most all this problems you need to note that this problems can also be solved in a straightly different way, what you need to in that particular kind of thing particularly in the book Raymond Polin’s book how we solve this problem is that first you assume something and you assumption leads to some kind of contradiction. That means, your assumption is wrong. First of all you start with A is a knight then it leads to contradiction that means, A should not be a knight he has to be a knave. There are verities of ways in which solve this problem, but we are highlighting particularly on the semantic tableaux method.

These problems are obviously very interesting if you understand how first and foremost thing you need to note is this that in all this problems first you need to represent it. Once you represent these formulas then you look for the satisfiability it leads to some satisfiability problem under what conditions the given formulas are set to be true I mean when it is set to satisfiable. Now, in the process of finding out the satisfiability of a given formula you will come up with this that from the open path you will come to know your answers.

So, I will stop here in the next class I will be talking about some examples, very interesting examples with respect to lady or tiger these are a this problems are also very interesting. And then we will see how we solve those puzzles. And another interesting important thing in which you need to note is this that these puzzles can also be solved by using truth table method.

So, all these methods can be used, but semantic tableaux method seems to be little bit easy to use particularly when the number of the preposition variable are less. So, these knights and knave puzzles you need to note that it looks very easy for us especially when A always truths B always tells lies etcetera. But what happens when you introduce another inhabitant who depending upon his will or wish he sometimes he tell truths sometimes he tells lies depending upon his (Refer Time: 51:58) interest. So, if you introduce pi into the that particular kind of inhabitants then the these problems will become little bit complex then they also it is fun solving these puzzles I will stop here.

Thank you.