Basic Concepts in Modal Logic
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Lecture – 03
Propositional Logic: Semantics

So, far we spoke about syntax of propositional logic, were we talk about some kind of conventions, then what kind of precedence how you give precedence to connectives etcetera and how do we find out the major connective in a given formula and when it is considered to be sub formula etcetera.

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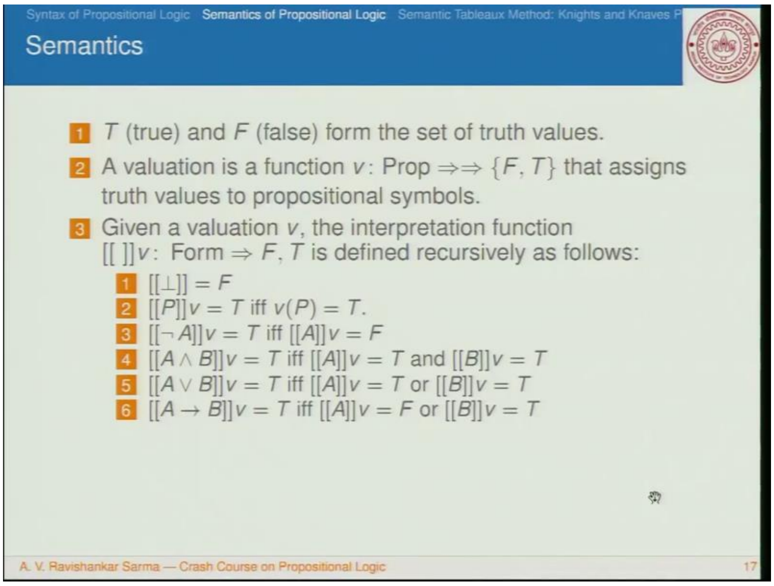

So, let us quickly talk about semantics now semantics. We talk about a formula which can be either true or false from there only two truth values. We have since we are talking about bivalent logics only two values are permissible to us either sentence has to be true or false. So, we for semantics what we require is valuation function which is which takes up any propositional formula. The variables that occur in the propositional formula like ps qs etcetera and assign some kind of value the only values that we can assign to the given propositional variables or either false or true.

So, given the valuation function v, which assign some kind of values to all the propositional variables and interpretation function which is write like this v which takes a formula and it generates, either gives us some kind of value f r t. For example, valuation of p and q depending upon what value p and q has it generate some kind of value for that one if both p and q are false then valuation of p and q is going to be false. So, these are the minimal things that we need to note that is valuation of a particular kind of thing contradiction is always false all contradictions are; obviously, false. A valuation function of valuation of p there is going to be true only when p the when p is consider to be true. For example, if we say New Delhi is a capital of India of course, that proposition is true suppose if you say something else Mumbai is a capital of India that sentence is false.

In the same in not a is true especially when a is consider to be false. valuation of a and b going to be true if and only if, both conjuncts have to be true a and b have to be true a are b is going to be true only when either one of this things are going to be true a then a implies b this is what is considered to be the material implication classical logic follows material implication and the definition of material implication is that a implies b is defined as not a or b or it is not the case a is true and b is false. So, based on that a implies b valuation of a implies b is true if and only if valuation of a is either false or b has to be true.

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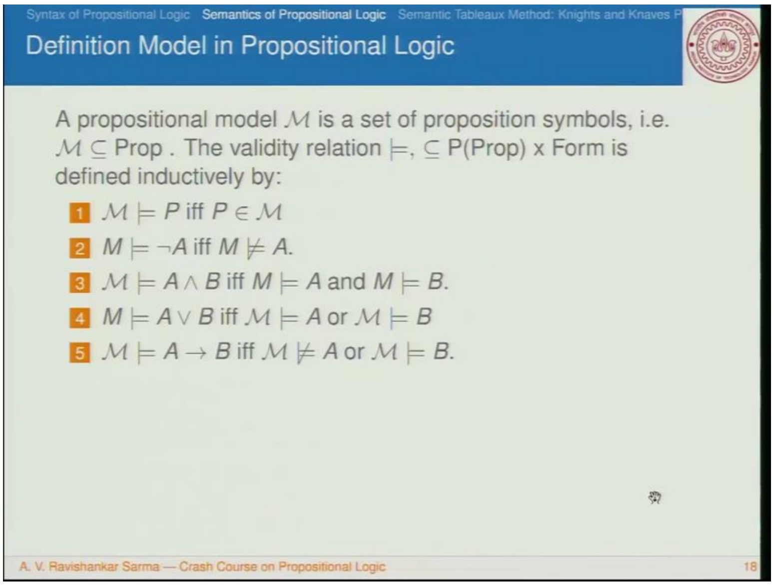

So, a propositional modal now we talking about a modal which consists of propositional symbols which belongs to some set of propositional variables and we have some kind of validity relation which we write it as double. So, there is a difference that we will be following that is like this.

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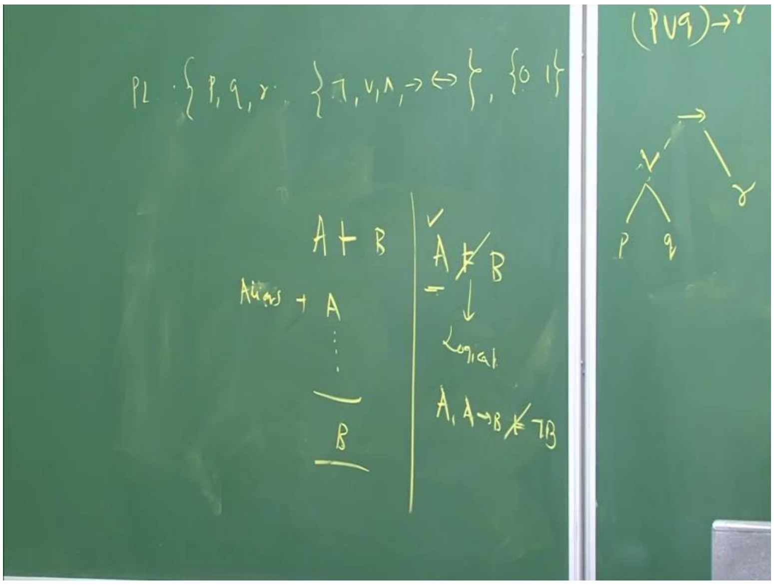

So, we make use of two symbols. So, in the syntax we talk about this particular kind of symbol single turnstile b this means from a with some kind of axioms together with a you are reducing b. So, b is reduce from a when I write a single turnstile b it is some we are trying to talk about proving something. So, this is what syntax all about is.

When I write like this a double turnstile b, this means under all interpretation where a is true b also has to be true and this symbol we discuss it get it in little bit later. So, this is also called as logical consequence a semantics consequence or we can say that b is a logical consequence of a or we can also talk about this thing in a different way that a makes b true the idea here is that, if you take it as the left hand side is our premises and this is a conclusion in a any given valued argument. If the premises are true the conclusion cannot be false if you come across situation where this is true, but b is false then b is not consider to be a logical consequence of a for example, if we have a implies b from that, if we say not b. So, under some particular kind of assignments or interpretations where the left hand side is true, but the right hand side is going to be false then this not b is not consider to be logical consequence of a implies b.

So, now in this given model m p belong to p is logical consequence of model particularly when it belongs to m and not a it is true in a model m particularly when a is not a consequence of m a should not be there in your this thing now a and b is true in a model m. When both are true and a or b is going to be true when either one of this thing are true a implies b is true and we just follow the definition of material implication that is not a r b; that means, not a means a is not a logical consequence in your given model, but b is a logical consequence of your given model.

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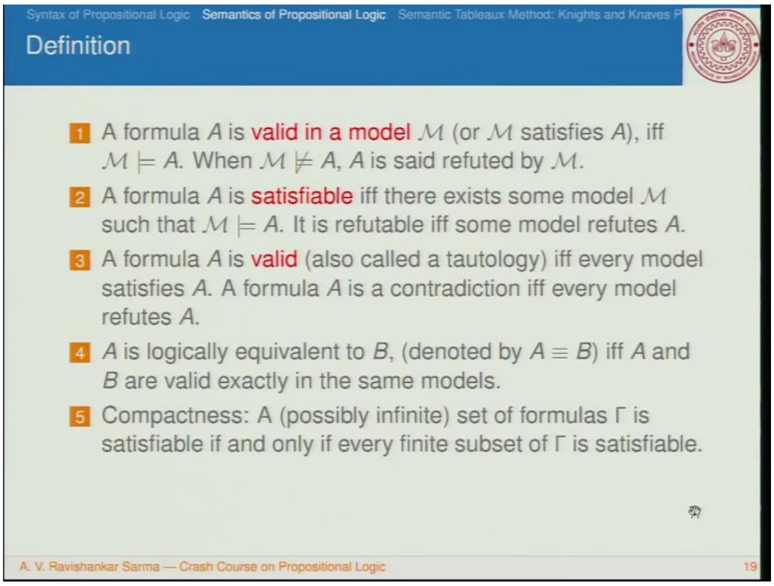

So, there are some definitions that we need to note and with this we will end this lecture. So, that these are the important definition that we need to note with respect to the semantics consequence or logical consequence. Say a formula a is considered to be valid with respect to a given model m are you also call it as a m satisfies as if and only if, a is logical consequence of b that particular kind of model m. When a is not a logical consequences of m. When a is consider to be a refuted by m refuted m means this m can make the formula a false.

So, just like now a a implies b from that b follows b is consider to be logical consequence of a a implies b were as not b is not a logical consequence of a, a implies b and there is a one more term that will be you will coming across this course that is satisfiability a formula any given formula in the proposition logic set to be satisfiable. If and only if there exist some kind of model m such that that m makes the formula true at least in one particular kind of interpretation. if a becomes true than it is set to be satisfiable and it is refutable if some model refutes this thing and some interpretation if the formula a is going to be false then, it is set to be not satisfiable and a formula is also consider to be valid.

Why we are interested in validities because all the validities are tautologies and all the true statement are consider to be valid in propositional logic a formula a is consider to be valid if and only if every model satisfies the given formula a and formula a is consider to be contradiction if and only if every model refutes given a.

So, this is the things which we discus it in either in the truth table are in the one which will be talking about in the next lecture. So, that is our semantic tableaux method using semantic tableaux method we will be talking about all these things in greater detail when do you say that two formals are consider to be logically equal into each other. That is a if and only if b or a is equal into b they exactly have the same models the means if the truth table matches then of course, they are consider to be logically equivalent to each other and not a or b or a implies b. They have the same kind of truth values so; obviously, truth values matches that is why they are consider to be logically equivalent there is a other term which have come across which is called as compactness. Compactness is like this a set of formulas in a given set of formulas like gamma is set to be satisfiable if and only if, even if you take the finite subset of such particular kind of set gamma and there also consider to be satisfiable. If that is the case you are that is consider to be compactness.

So, in the next lecture we will be talking about a particular method which will employ it in our course. So, that is semantic tableaux method with which you will be talking about these things that an this is consider to be a decision procedure method with which you will come to know when a given formula is consider to be a tautology or when two groups of statements are consistent to each other or when something follows from something that is consider logical consequence or when two formulas are set to be logically equivalent to each other. We are still talking about the basic concepts of classical logic all these things we make use of it when we discus about advance concepts in model logic. So, in the next lecture we will be talking about the semantic tableaux method.

Thank you.