Basic Concepts in Modal Logic
Prof. A.V. Ravishankar Sarma
Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

Lecture – 25
Conditional logic: S, C1, C2 and Conclusion

Welcome back. In the last lecture we have introduced conditional logic C and then this C should take should take care of Ceteris paribus clauses as well. The idea here is that when you are analyzing conditional sentence A plus B, it is understood in a sense that A together Ceteris paribus clauses needs to a conclusion and other conclusion B. Where B is considered to be consequent and A considered to be antecedent together with this Ceteris paribus clauses.

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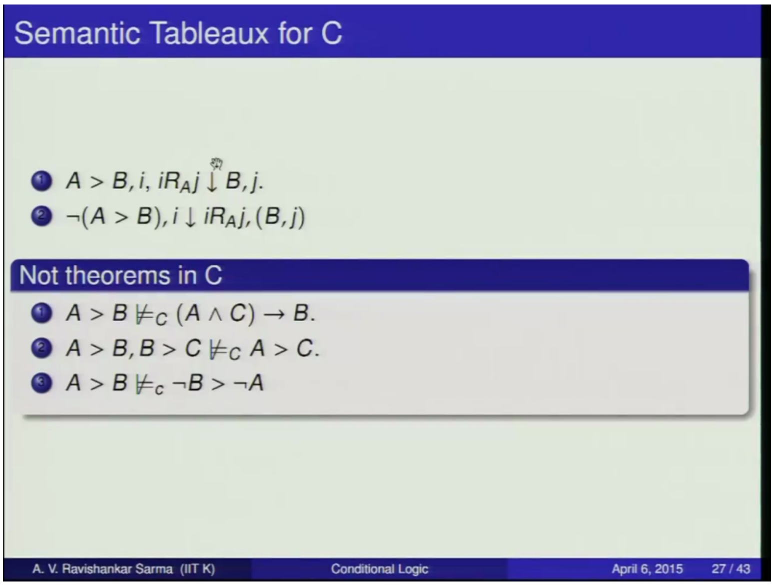

So, now the idea here is that, the main idea here is that, in all antecedent permissible worlds, that is the conclusion is true or not is the one which we are going to see. So, to begin with we are trying to talk about the accessibility relation. Particularly in the case of conditional logic is quite different from the one which we have used in the modal logic. Here accessibility relation is with respect to the antecedent of your conditional. There are 2 things which we have seen in the last class. That is faw. Faw is considered to be a kind of selection kind of function, which picks up nearest the possible world with respect to the antecedent the nearest possible world usually it will be non-empty.

So, now if we keep on imposing some kind of restriction and the accessibility relation, then we will be moving away from C, and of course, we will be talking about C plus in that context. Just like in the case of normal propositional modal logic we started with k, and we started imposing more constraints on the accessibility relation. For example, if r is considered to be reflexive if 2 worlds are related in such a way that x is accessible to itself, then it holds in reflexive frames. A there are some formulas which holds in reflexive frames. In the same way we invoke semantic property we have semantic frames etcetera. Just like that even in the case of conditional logics, if we begin with the minimal conditional logical system C, and then we will be imposing some restrictions on the accessibility relation r a. then we will move on to C plus and then this C and C plus has clear cut semantic tableaux method for showing the validity of the given formula, but conditionals are not easy to analyze by with a help of just C and C plus. We require some more semantics, like more and more constraints needs to be added to the accessibility relation. And that results in C 1 C 2 and s.

Unfortunately, s C 1, C 2 has no, as of now there is no appropriate semantic tableaux method for showing the validity of given formula, but although there are efforts to show that C 1 and C 2 have some kind of semantic tableaux methods which are possible.

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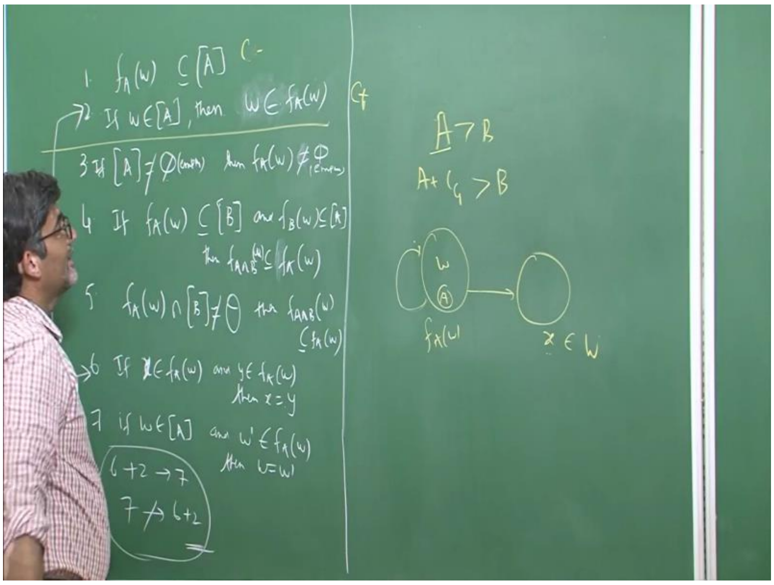

But as of now, we do not have appropriate semantic tableaux method for such kind of systems. So, to begin with in C the main condition is this thing faw, faw is like this suppose if you have a conditional A in plus B. So, this means A plus C g that is Ceteris paribus clauses leading to B. So, faw means this one this is the world that you began with w and then faw can be any other world x. So, this x has to be some setup possible worlds w.

So, usually, with the respect to given antecedent there always be at least one particular kind of world, which is accessible to that particular kind of antecedent. And this means this world is similar to this one. So, now, faws are those worlds in which a is already true. It is a subset of those worlds in which A is true. That this is the first condition which we have we do not have to impose any condition. Whereas, in the case of second condition if w itself is such that; that means, this w itself is such that your antecedent is already true there than any world with antecedent is true needs to be taken as the accessible world. So, in conditional logic what is of utmost importance is that, we have to observe the antecedent, and we need to see in which world it is true. If A is true in this actual world itself; that means, this world is accessible to itself.

So; that means, we are imposing additional constraint on the accessibility relation, that is what reflexive property. So, now, using this if you if any conditional logical system obeys these 2 rules, it is called as C plus. If it does not require this one, it is only C. If it follows just only this one it is conditional logical system C. C is not that impressive in a sense that there are many case where the antecedent is not just true in the possible world etcetera, but it is true in the actual world itself. So, in that case you need to extent the C to C plus. Somehow let us consider some of the semantic tableaux rules for C and C plus. And we will jointly study these 2 things in somewhat detail. Just observe these 2 things. Somehow in C we have only one particular kind of rule these are like this.

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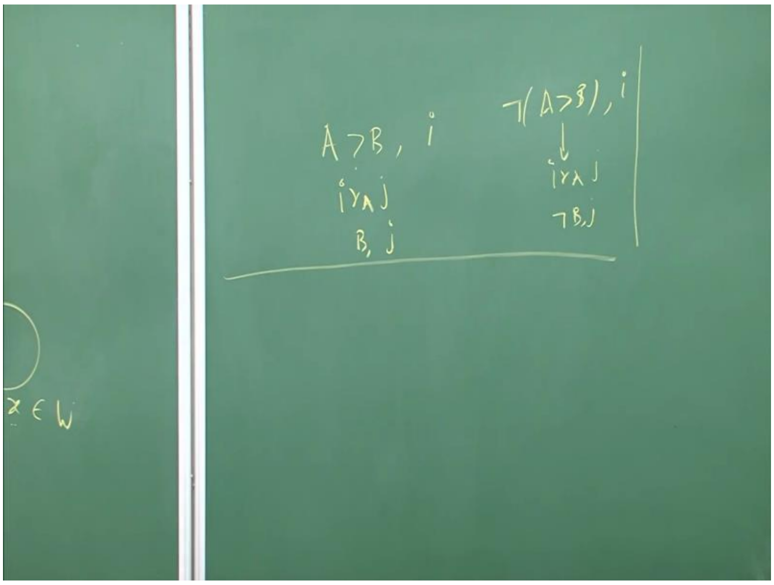

So, there are alpha and beta rules, like this A in plus B it is true in a world i. So, this is going to be true in a world i, when i r a j exists, then only I have this thing B is true in a world j. It is not telling whether A is true there or not. So, a together with Ceteris paribus clauses same has the one which you have here, but we are not talking about whether A is true in the world or not explicitly. So, this is the first rule and the second rule is a conditional sentence which is true in world i, the negation of that one is like this. You have to go to new world i r a j and in that world not B is going to be true in the world j.

So, now just by using these 2 rules, let us consider some simple examples like modus ponens and modus tollens. First we will work on modus ponens and then we will see whether it holds in C or not. Modus ponens is considered to be a good validity whereas, what all we have seen in the case of antecedent strengthen that is not that should not come as theorem in our logical system.

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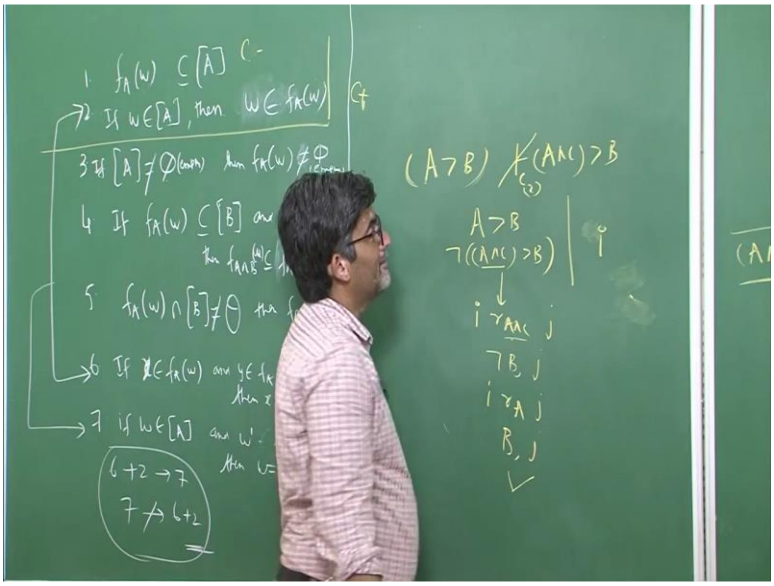

So, now let us consider a bad validity first of all, and then we will see the good validity later. A and C implies B whether this hold in C or not. So, what we are trying to do is we are applying semantic tableaux method on this one, and we are trying to check whether A and C implies B follows from A and B or not. Ideally speaking it should not follow in our system.

So, in semantic tableaux method, what you do is you start with the premises and negate the conclusion of the consequent here. A and C and B. So, now, all these things are, so, now, we are in a world 0. Let us assume that you are in the world w 0 or w or something like that. So, now, always when you are using this semantic tableaux method you should ensure that, you take care of the negation part first and then followed by that original follow A in plus B. Just like in the case of in the modal logic we have seen that always work on possibility operator move to a world and then open up this necessity operator.

So, this needs to first what is the antecedent A and C is the antecedent. That is why we have written like this; whatever is the antecedent it needs to be written here. So, now, let us assume that we are moving from i to j. So, now, we need to use this rule, i r a j where the consequent is negation of the consequent is true there. Now coming back to this one this is antecedent here is a. So, in conditional logic the one which have I am going to talk about in while from now, in all the conditional logics what the most important thing is that the antecedent of your conditional. It makes sense only when you talk about the truth of the conditional with respect to I mean the antecedent and with respect to the antecedent permissible worlds where the consequent is true or not that is the one which we are trying talk about.

So, now this is still in a world i, we move from although we move from j, but here the antecedent think worlds with respect to this one where not B is true is with respect to A and c, but here it is with respect to A only. So, now, it is with respect to j. So, now, we will be tempted to the movement you see literal and negation will be tempted to close the branch, but you are not supposed to close the branch. Why because A and C worlds the worlds which are related to A and C are different from the worlds which with respect to a.

For example, if you say sugar in the coffee and kerosene in the coffee, that is A and C this world is totally different from the worlds in which there is only kerosene in the coffee or just sugar in the coffee. A and C worlds are always different from a world. So, this should not be there is no way in which it should close the branch. So, that is why this branch remains open. So, that is why this formula A and C implies B does not hold here. So, now, what about simple formulas like a modus ponens?

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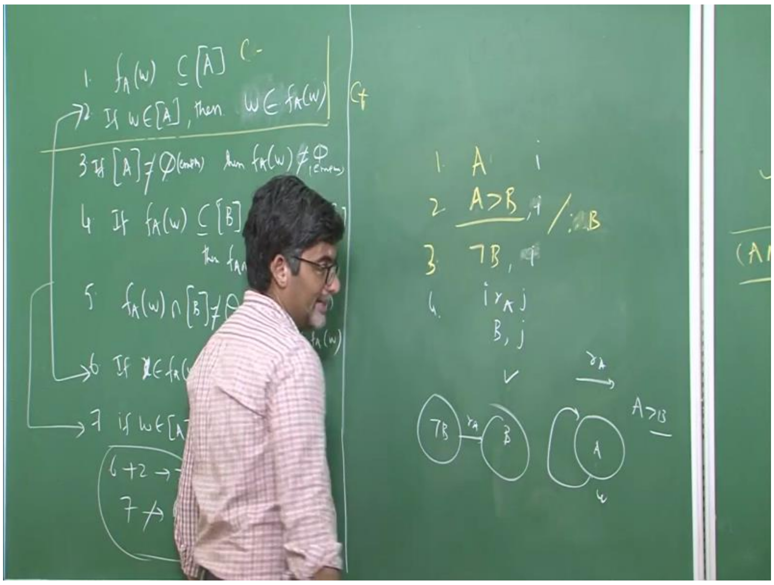

So, modus ponens tells us that A implies B whether gives to B. If you have a conditional A implies B and A is also true that is why B is false.

So, now as usually you begin with negation of the conclusion, that is B. Now we start constructing semantic tableaux semantic tree for this one based on these 2 simple rules. And this is going to be like this. So, all these things are true in a world i r j whatever it is you will take care in this consideration. So, now, there is no negation of anything. So, now, this A implies B is nothing, but i r a j, and in that world j your B has be to true. So, now, this is going to be open formula because not B is true in a world i, and B is true in a world j. So, now, if you write it like this, this is r a and then the open branch will give us the counter example.

So, now it will be like A C. There are 2 worlds i is related to j with respect to the antecedent of your conditional and in that world j B is true and whereas, in i not B it is going to be true there. So, this will serve as your counter example. So that means, A and A implies B can be true, but B can be false this shows us the example. So, modus ponens does not hold, but we do not want modus ponens to not to be a theorem in our logical system. So, what needs to be done in C nothing is interesting because we are not taking about whether what will happen if the antecedent is true in this world itself.

So, now this needs us to view the second point that is if the world itself it says that your antecedent is already true there and that world needs to be taken into consideration that itself will serve as faw, faw is any such kind of world which is accessible to the actual world w. So, now, if that is the case then w should be treated as faw. That itself needs to be viewed; that means, if world is such that your antecedent for example, if you are evaluating condition sentence A implies B, if the world says that a is already true there and this is the world which is served as faw. So, now, following this reflexive property we need to change our semantic tableaux rules a little bit and then we come up with this thing. So, we slightly change the rules of this one. And we will talk about the same rules in different kind. To incorporate idea that those worlds in which the antecedent is already true needs to be taken into consideration.

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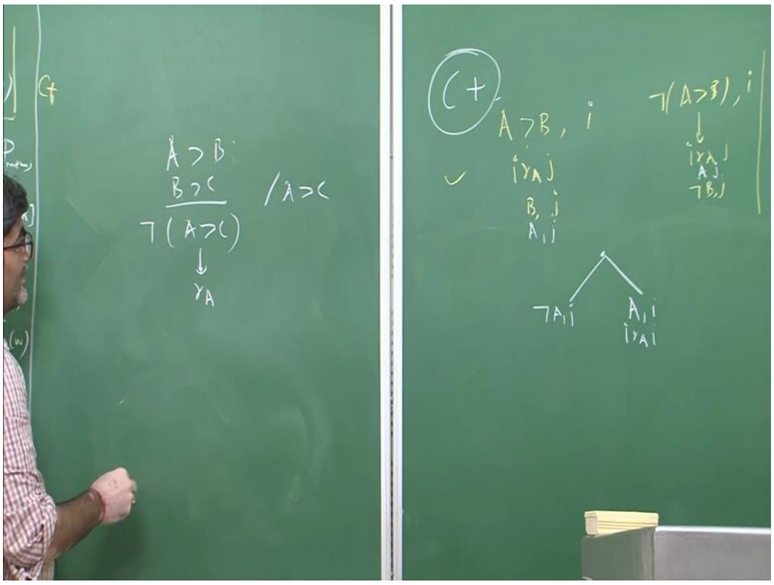

So, now, in this case A implies B as true in a world i with respect to i r a j, A is also true in that world where B is also true. So, now, we need to write like this. A is true in world j wherever A is true not B is also true. There also, there is one more rule which we need to use that is like this. So, either this has a rule it happens in this way that, either not A is true in a world i, or means A is false in a world i, like the one which we are talked about this thing, if A is true this is one which you need to take into consideration. There are only 2 case either A is false. So, that you will not take this world into consideration or if it is true then you have to take this world into consideration. That is if A is true in the world i then it has to be i r a i, where of course, A is true if suppose if you are evaluation A implies B, B will also be true there. Wherever the antecedent is true the consequent also has to be true there.

So, now taking this into consideration, let us see whether our modus ponens hold in C plus or not. So, we want good validities to be there and all the bad validities to go away. First let us assume this thing let us consider this one A implies B, B implies C and A implies C. Whether this hold in C plus or not. So, now, you take the negation of A implies C and start constructing the tree using this C plus rule, C plus rules are this. So, now, always render the negation formula first, and this is going to be you have to go to a new world. Now first you need to write your antecedent of your conditional, this is accessible to relation with respect to the antecedent. Here the antecedent is A here the antecedent is B here of course, it is only i r a j, then in that world where A is true in a world, j not C true in the world j.

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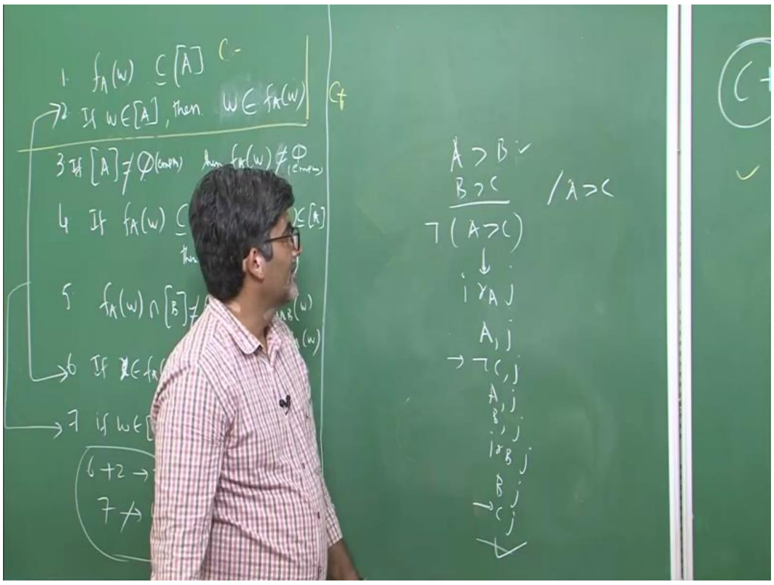

Now, this is our, now B implies C. Now i r a j is already the case; that means, in all those worlds in which the antecedent is A is there, then this will become A, in all those worlds where A equal to j, B also should be true in a world j. So, this over and then this is the one which we have. So, now, the antecedent here is B, that means, i r B j in all those worlds in which B is true in a world j, and C also has been to true. Now there is A we have a temptation, we will be tempted to close this C and C here. This not C is true in a world j with respect to r A, i r a j, but this C j results in with respect to i r B j. So, these are 2 different antecedents. There is no way in which you can close the branch here. This branch remains open and from the open branch you can construct a counter example.

So, that this means does not hold in C plus. Now consider good validity like modus ponens whether it holds in C plus or not and let us see.

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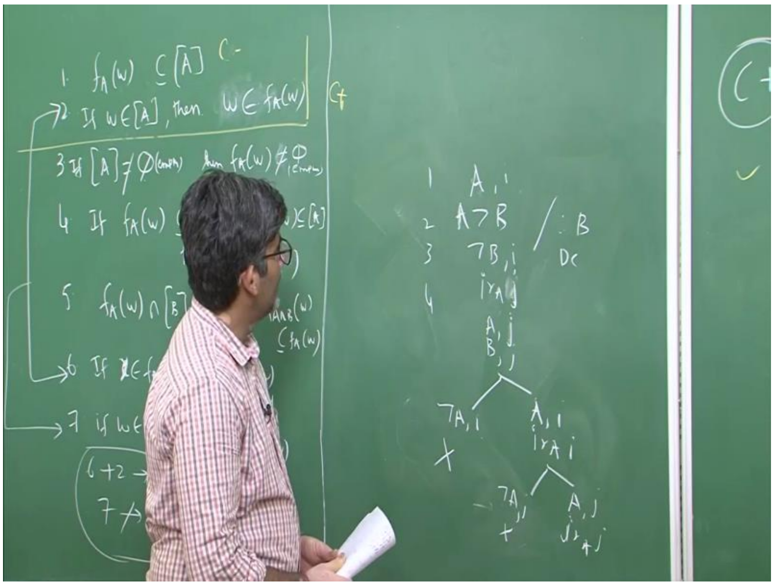

So, A implies B and we have B. So, we negate the conclusion. Here it is A this thing. B now starts constructing a tree diagram for this one. So, now, A implies B, A implies B is this thing i r a because antecedent it a. So, it is B. Now, in that wherever A is true, i r a j, now that world wherever A is true in j B also has to be true in a world j. Now this does not end here. If it is C it ends here only, you cannot cancel B j and not B i. It is true in a world i this B is true in this world and not B is true in any other world. So, this cannot be canceled.

Now, we need to use this rule. When we use this rule particularly whenever the antecedent the antecedent here is A, whenever the antecedent is true in a world i this world also needs to be taken into consideration. That is additional property that we have. So, now, we need to use this rule either that A has to be false in that world i, or if it is true in that world i it has to be i r a i. So, now, i r a i. So, not A i is true in a world i, and A is true in a world i, it closes now A is true in a world i, here B is true in a world j. B is true in a world j and j now this can be viewed in a difference sense also. So, instead of going to j, now you can even go to i itself, i r a i that is the case then it will be like this. A i and B i. So, the branch closes.

So, now let us assume that this is the case. Now A is already true in a world j once again you can apply this rule. Either not A is true in a world j or if A is true in a world j, it has be i r A, j r a j. So, now, we have not A i a i closes. So, one second, actually this should not come as, this should come as validity in our conditional logical system. So, that is what we are trying to see. So, something went on wrong here.

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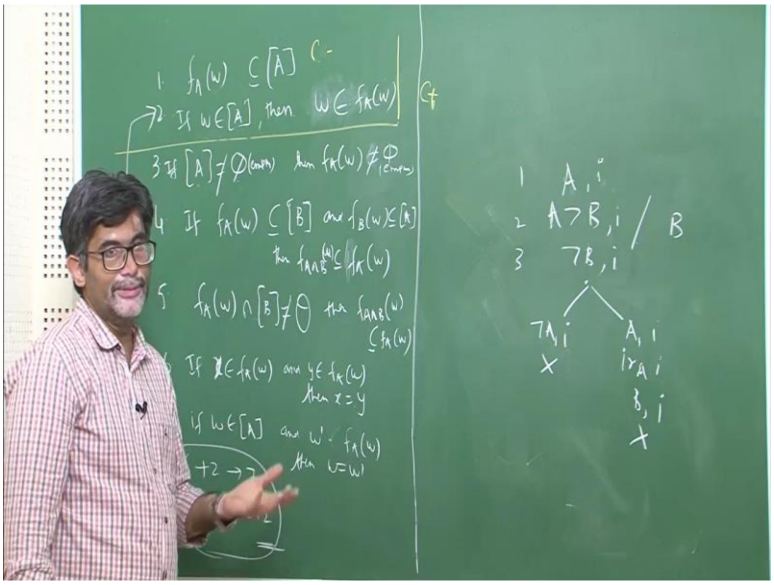

So, let just consider it again, A plus B A and then B. So, now, if you deny this formula B which is true in a world i. All this thing is true in a world formula i. So, now, A implies B is like this, i r a B, wherever A is true in a world j B also has to true in a world j. And this

5 properties are satisfied. Then it is called s system unfortunately s does not have any semantic tableaux method as of now. So, it only can be discussed in terms of spheres. I will give you 2 examples. Then I will end this lecture. So, there I will be talking about system of spheres model C 1 and C 2.

So, now these are the 2 extensions, of further extensions of s. Anything which is valid in s has to be valid is an extension of s in particular. So, that is this thing. If there are 2 worlds x and y, and they are construct to be those worlds, in which antecedent is true faw x belongs to faw, and y also belongs to faw. Then these 2 worlds have to be same. This is the view which is followed by Robert Stalnaker and this assumption is also called uniqueness assumption. So, the idea is very simple. You take any conditional into consideration.

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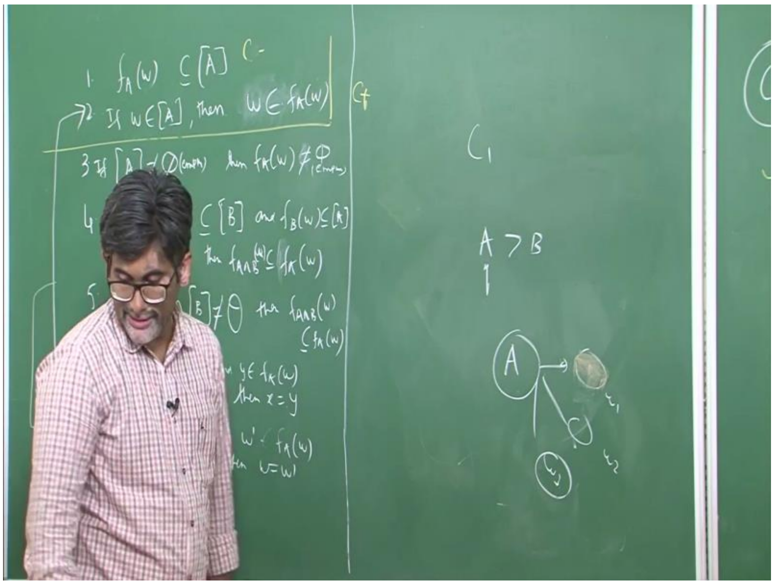

I am just giving you the idea of this thing. In C 1, it is like this any condition A implies B and with respect to the antecedent of your condition. There may be many worlds which are accessible to this one, W1 w2 w3 etcetera. So, out of all these world's w1 w2 w3 etcetera, according to Stalnaker all though all this worlds w1 w2 w3 are similar to the antecedent of your conditional, but there is one world amongst all these worlds which is considered to be unique world.

So, if you say something is unique it cannot be 2 worlds that are there. So, out of these 3 worlds, there should be one world which is unique to actual world. This is what is called as uniqueness assumption. So, let now us talk about some examples with respect to system of spheres and C1 and C2. So, in system of spheres model we do not have any clear cut semantics for that.

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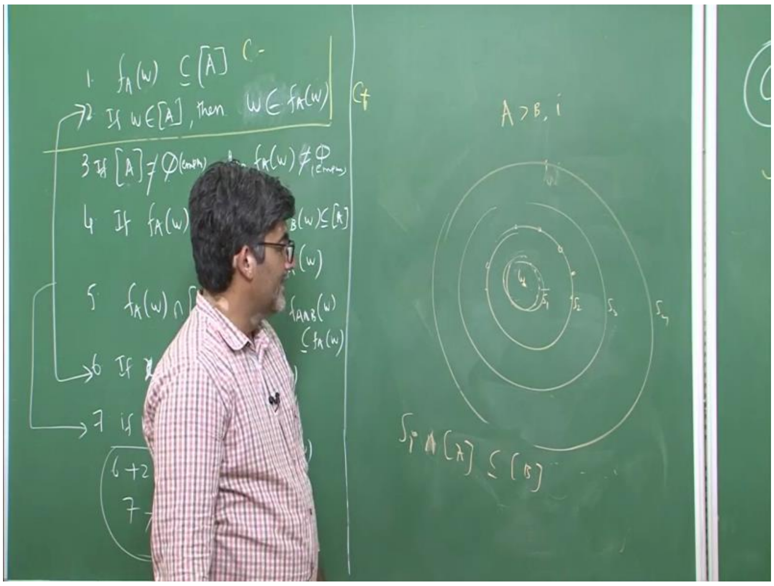

The idea on system of spheres is that, any world that you are trying to take in to considerations comes with some system of spheres. So, this goes on and on like this. So, the idea here is that, this is considered to be actual or real world. This is the world that we have w. And any world w comes up with various systems of spheres. They are all the worlds which are closer to w. It could be like you can view it in the sense that observing that you have an onion in your hand, you peel it up peel it up and all ultimately you enter in to the center of that onion. And that is considered to be w, and all the layers are considered to be your system of spheres.

So; that means, the worlds that fall under this inner sphere are more similar to w then those worlds in which it falls on this one. So, why we need to abstract this and view it in this way. Because any given world any antecedent comes up with many possibilities all these things can be grouped in the form of a spheres. And you have to visualize this in this way, that those worlds which fall under the inner most sphere inner most sphere is this one they are more closer to this one then and those worlds which fall under next inner most sphere, at least let us say S1 S2 S3 S4 etcetera. So, now, this is like s1 is the subset of S2 subset of S3 etcetera. Then it goes on and on and then ultimately it goes it will become w.

So, now, the idea here is that, if you want to say that a formula A implies B is true in a world i. What does it mean to say that A implies B is true in a world i with respect to system of spheres i? It is simply like this first you need to see the antecedent and with respect to the antecedent, together with that there is a smallest intersection smallest sphere where A is considered to be true. This is smallest spear in that smallest spear your A is true, in those worlds your B has to be true. Then; obviously, the conditional holds otherwise it does not hold how to talk about these things with respect some kind of examples like this.

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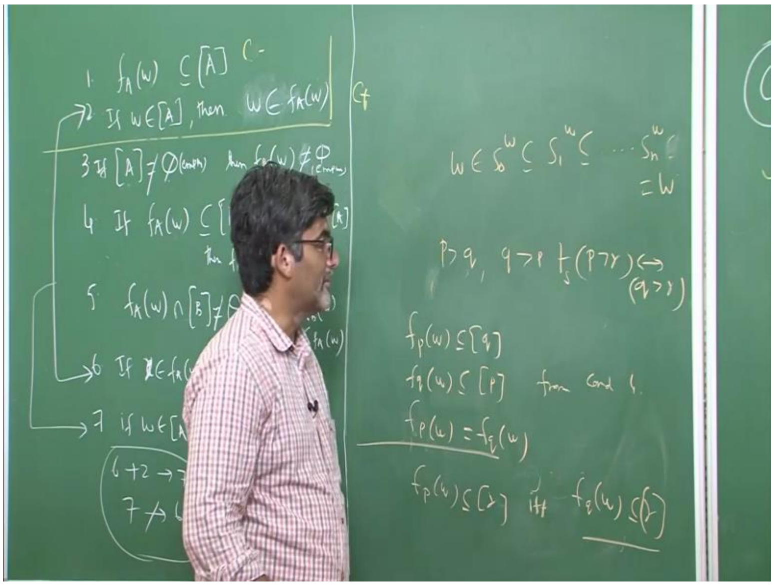

So, now in this case a world w, which comes up with like this S0 w, the inner most sphere and in a second inner most sphere is this one it goes on and on and ultimately it become s n w, and this is equal to set of worlds w, that is what is the case here.

So, now let us consider 2 or 3 examples, and with which you will see how to solve some of the problems with respect to this. So, these examples are like this. Let us assume that let us try to solve this problem, p implies q and q implies p whether this follows or not in s, p implies r, if and only if q implies r. So, you have to note that, when you have when it does not hold then you can construct a counter example. So, whether it is valid in you have to show that, you have to show that it is valid in s. So, now, what do you mean by saying that p implies q, is true it is nothing, but f p w are those in subset of those world in which your consequent is true. And q implies p, q implies p means f q w are those world in which p is true.

So, now by conditional 4, if faw are those worlds in which B is true, f B are those world in which a is true, then this not a guess f faw is same as f B w. So, these 2 worlds have to be the same thing. So, this means, from condition 4, it follows that f p w is same as f q w. Hence now we come back to the consequent. In all those worlds in which p are true, p implies r means f p w is the subset of those worlds in which r is true. So, that is why we need to write like this. If and if you write it like this are those worlds which f q w are those worlds in which r is the case.

So, it the case that, it has to happen both sides, f p w is subset of those worlds in which r is true; if then f q w is subset of those worlds in which r is true, and it should be other way not also. Since f p w f q w is same, and then you can even write it the converse is also going to be true. That is the reason why this is the formula is going to be valid in system of spheres modal s. So, now, let us show that this is not the given conditional is not valid in s.

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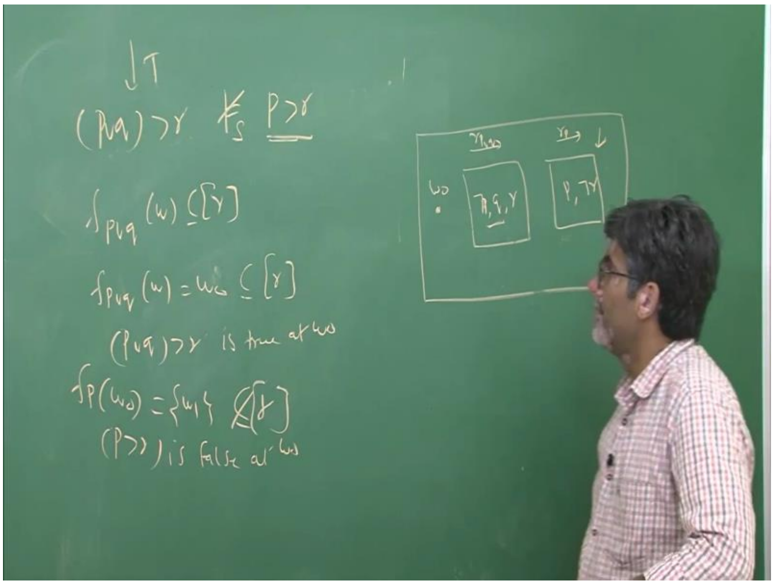

So, that is this thing p r q r, which is not valid in this thing p implies r. So, now, why is the case that this is not valid in system of spheres models? When it is said that it is not valid in s of course, you have to show that your premises are true and the conclusion is false. So, now, you can have a counter example for this one. This means that p r q implies r means f p r q w, are those subset in which r is true. That is the first one.

So, this f p r q w, it has to be at least one world w 0. And this w 0 is subset of those word in which r is true. Hence p r q is true at w 0, but you can always have another world f p, I will now coming back to the conclusion. So, what you need to show is that in all the rest worlds where p r q is true, r is also is true, but in those worlds in which p is true r has to be false. So, now, f p w is 0, it can be any other world w 1 which belong to the set of worlds capital w, and we can show that it is not does not belong to set of worlds in which r is true. Hence p implies r is false at w 0. So, that leads us to this gives us a counter examples for this one. So, how do we show with respect to system of sphere? So, now, what we are trying to do is that, we are not drawing system of sphere as such, but in a it is a 2-dimension picture. So, that is why we are drawing squares. So, how do we show that a counter examples for this one or there should be only one world which is unique to the given antecedent of your conditional and if you evoke this particular kind of property now coming back to the same example.

Now, in the nearest worlds where p r q is true, r is considered to be true this this world. So, any way this is false, there other thing is that in the nearest world where p r q is true, it can be taken this this world also you can take this world also into consideration, but this will not give you any result here, because the idea here is that if you have 2 premises you need to establish the false conclusion. Sometime it may happen that you have 2 situations where your antecedent is going to be true. Then in that case if you are using Stalnakers approach then you need to take only one world into consideration. So, just I will give you one example that you will understand this thing in a better way. Why Stalnaker wants this uniqueness assumptions. So, with this I will close a thing and then rest all the things considered to be extension of this thing one thing we need to note here. So, that is this one. So, if you take 2, 2 is this thing every world is such that your antecedent is already true then that world needs to be taken into consideration as your faw. And this 2 together with this 6 leads to 7, but 7 does not lead to 6 plus 2.

So, that makes C 1 distinct from C 2. So, David Lewis drops this assumption of uniqueness assumption. Then he talks about this different view that you do not require this uniqueness assumption, uniqueness assumption, cannot be established in some of the conditional.

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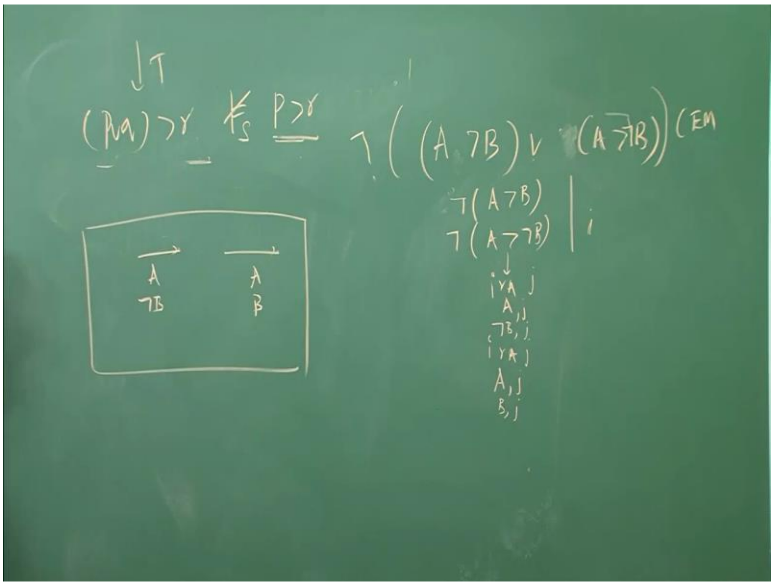

Let us talk about one particular example, where it holds in Lewis it does not hold in Lewis, but it holds in Stalnakers approach. So, that is this one A implies r, it is not the case that A implies B. So, usually Stalnaker in general he assumes that the negation of the conditional A implies B is same as by definition it is same as A implies negation B. So, what Stalnaker what is to preserve this particular kind of this is also called conditional excluded, middle conditional excluded middle. So, when you say that p to not p, it is considered to be that law of excluded middle, but it happens with respect to the conditions. So, that is why it is conditional excluded middle. So, now, if you negate this formula, and construct a counter example for this one, it is going to be like this not of a. So, this is going to be A implies not B. So, now, this is not of A implies B and because negation of this thing is convention assumption. So, it is going to be A implies not B. So, it is all true in our world i.

So, if we use C plus rules semantic tableaux rules, it is going to be like this in all those worlds, it is antecedent is A only in both the things i r a j. And in that world j whenever A is true in a world j not B it has be to true in a world j. Now once again if you use this one it is like i r a j, another set of rules, where A is true in a world j, again it is respected to same antecedent. Not, not be that is B is true in a world j now from this you can construct the counter example. And then I will talk about the difference between David Lewis approach and the Stalnaker approach. In Stalnaker approach we take this as validity in his logical system, whereas David Lewis constructs it is not going to be valid. There are some justification which are given of course, I am not going to the details of it, but. So, here is a counter example for this one. For David Lewis it is like this. So, there are 2 ways which are possible here in one world A and not B is true and in world A B is true.

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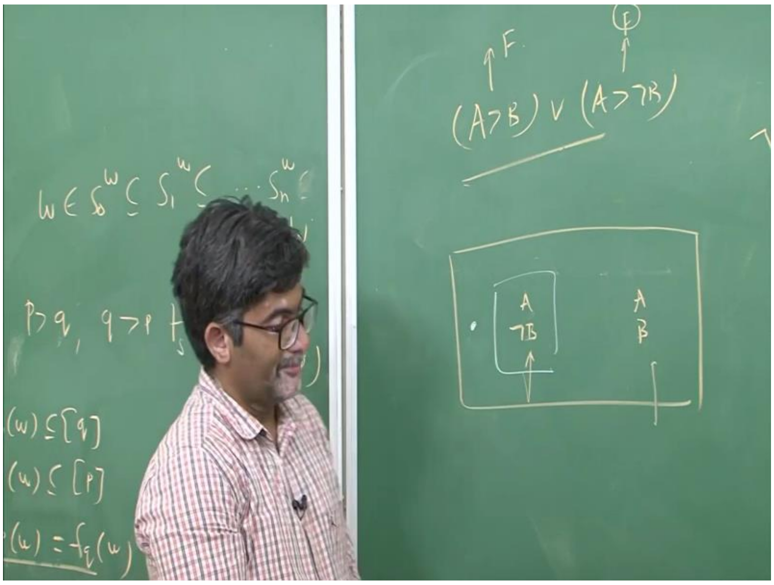

Now, coming back to this example, A implies B due to r, A implies B, r A implies not B. Now there are 2 worlds in which the antecedent is true, but for Stalnaker either you have to take this into consideration, only this world or this world consideration, but you are not suppose to take both the world into consideration. So, what makes it this conditional false in David Lewis approach is that, this thing in the nearest world where A is true B is false. So, that makes this thing false the first condition false. Now in the nearest worlds where A is true again I will take instead of this one, I will take this one again this also become false. So, the process the whole conditional is going to be false for David Lewis. For Stalnaker sense he follows this uniqueness assumption especially when there are 2 or 3 worlds which are out there, in which antecedent is true there, you have to take out only one world and that world has to be unique world either you fix this world into consideration a unique world or this one as your another world with respect to this world your validity to formula.

So, in this way there is A, there are difference between Stalnaker and Lewis. Lewis does not require this uniqueness assumption. So, he comes up with different kind of semantics in which he drops this. His construction to be in extension of s only, but the only one rule which is extra that is the 7th rule. So, with this we will end this lecture on conditional logics. It is a conditional need to be studied in greater detail. This is no way an exhaustive kind of study on the conditional logics. I just showed you on a tip of iceberg. So, we have just began with the view that whenever you are analyzing a conditional sentence, you need to ensure that you need to take care of Ceteris paribus clauses; that means, whenever you are evaluating A implies B, you need to you need to understand that it is A plus C and A together with that leads to the consequent B. So, conditionals are not straight forward and easy to analyze. Although we have best approach that is available to us seems to be either C 1 or C 2, but still C one and C 2 has it is own problems. Particularly when there are some conditionals with disjointed antecedent conjunctions etcetera as the antecedents in your conditionals.

So, to conclude in this course, we have this short codes I have, just I just began with the crash course on the classical logic, because that is going to be the starting point for this course. And then I showed some kind of limitation with respect to the classical logic particularly when it comes to the modal sentences classical logic fails to make any such kind of distinction because in classical logic possibility of p and necessity of p and p say. So, in the process you will miss out some useful inferences. And in that context we extended our classical logic with the modal logics, with 2 operators' necessity and possibility and these are called alethic modalities and we just spoke about syntax, and semantics of it. And we have highlighted on the relation structures which are also called some kind of directed graphs or kripke structures.

Then we understood that till 1960s, logicians' modal logicians were trying to only prove theorems within various kinds of axiomatic systems. Then it is it is due to kripke semantics. Things are become more simplified, but again there seems to be as I said in the history of modal logic lecture, I said that there was lot of emphasis on the relational structures. This possible world again is not straight forward and easy to follow. Some view it as marginal consistent set of sentences which are constructing to be true, but the problem remains the same thing. That is how we particularly when you are analyzing conditional sentences, how do we attend the similarity between the possible worlds. It is only intuitive kind of notion as of now, but still David Lewis approach is considered to be the best possible approach especially when it comes to the analysis of specific kind of conditional sentence like counter factual.

So, we also spoke about in the last part of this course, the fourth week we discussed about one application of modal logic. So, that is the analysis of conditional logic. In conditional logic what you need to notice is that you should fix the antecedent first. Antecedent considered to be the most important thing. So, with respect to the antecedent we are looking for some possible worlds with respect to the antecedent, by where with respect to which possible worlds your antecedent is going to be true. All the antecedent permissible worlds we need to check whether the consequent is true or not. And in that context we have we spoke about different conditional logical systems to begin with minimal condition logic C, we extended with by imposing some constraints on the accessibility relation. We talked about C plus and then we moved on to s. It has only; it does not have any fixed kind of procedure like in the case of C and C plus. Only within the intuitive kind of diagram you can establish the invalidity of given conditional, but checking the validity is again based on the 5 rules.

So, this is what went on it is only a tip of iceberg. And in the advanced courses you will be introduce with modal predicate logic, and there is another dimension which I have completely ignored. So, that is algebra interpretation etcetera, when restrict implication was introduced we have taken only one direction, that is the analysis of modal operators, but there is another direction which is towards the analysis of conditional sentences are some kind of relevant logics. And there is another direction which leads to many valued logics. So, how to interpret future continuous sentences, that is what I have discussed in one of the history one of the lectures in particular, when I was talking about the history of modal logic.

So, that is all we have in this course. And I hope you enjoyed this lectures. And definitely I will come back again with any other advance course.

Thanks.