**Basic Concepts in Modal Logic**
**Prof. A.V. Ravishankar Sarma**
**Department of Humanities and Social Sciences**
**Indian Institute of Technology, Kanpur**

**Lecture – 24**
**Conditional Logic: C, C+**

Welcome back. In the last lecture we have seen how the conditional sentence, especially when we are analyzing in terms of material implication we need to face some kind of problems. We will continue with that discussions and how material implication and strict implication are not sufficient enough then we need to come up with new connective that we are calling it as conditional connective.

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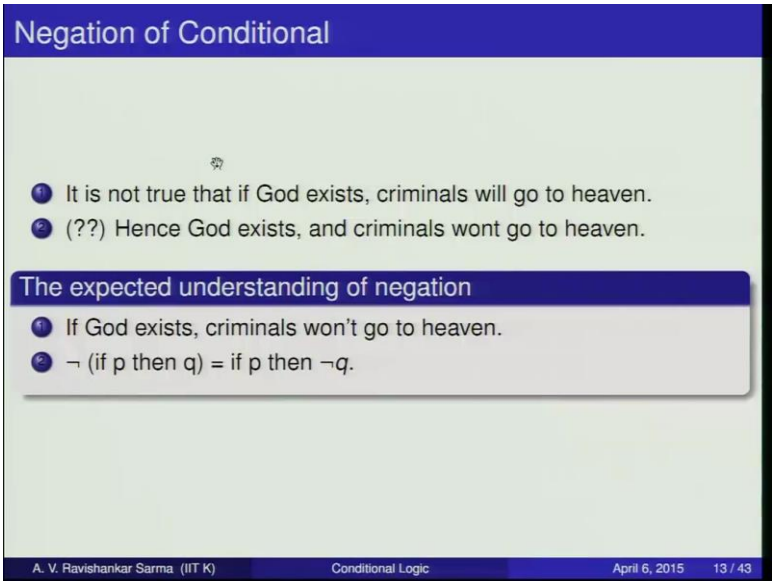


So, these are some of the examples that we have seen; one is the antecedent strengthening love counter prediction, love transitivity and anything which is related to the various instances of material paradox of the material implication. So, all this things are constructively bad violation. And whenever you come across whenever you are trying to build logical logic of conditions we need to ensure that all this bad violated is should not be part and parcel of our system.

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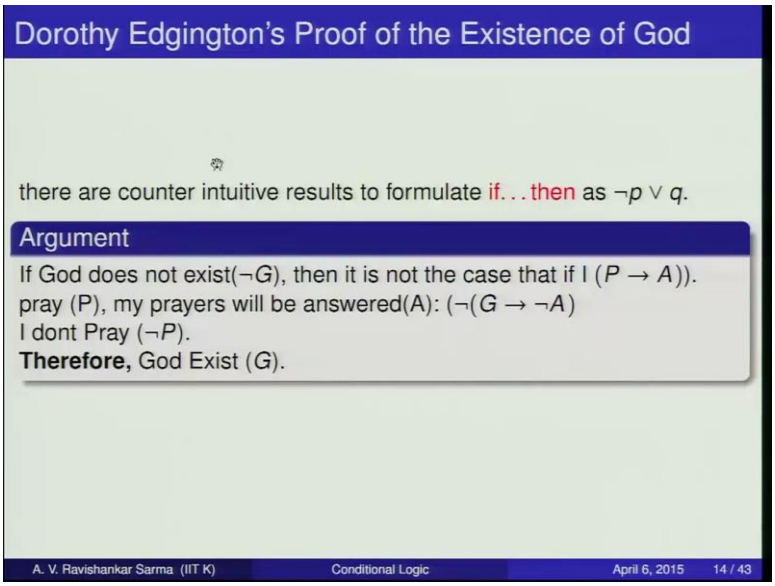


So, it is an interesting argument which is provided by Dorothy Edgington for the proof of the existence of God is the funny kind of proof it goes like this, when you define material implication A plus B as not A R B and we need to face this particular kind of problem like this.

So, if we see God does not exist then it is not the case if I pray and my prayers will be answered. If God does not exist I would not have to pray anyone and even if I pray my prayers will not be answered because God does not exist because I do not. From this it follows that God exist. This is the case even if I do not pray there no such object can like something like God exist, but still it leads us to believe that even if I do not pray, but still God exist, results.

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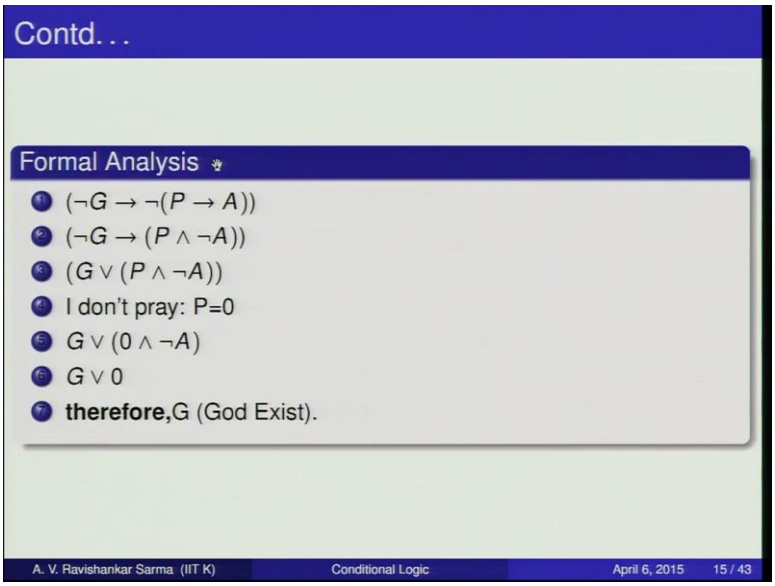


So, it goes like this if God does not exist stands for non G implies it is not the case that my prayers if I pray my prayers are answered returns not of P implies A. And then it is further simplified in to negation of G implies P and not A and this by definition x implies y is not at survive it leads to G R P and not A. And then we substitute with sentence that you know I do not pray stands for P is equal to 0 I pray stands for P is equal to 1 than it leads to 0 power 0 and not A anything which is a convention of a false propulsion is false. So, that is fine there this is the case (Refer Time: 02:48).

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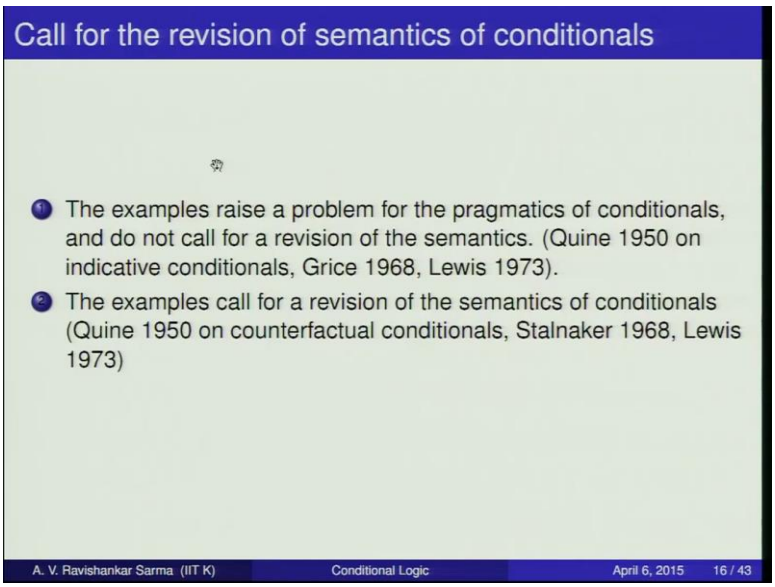

So, this seems to be little bit for us viewed kind of conclusion that we have come up with. And even if I do not pray, pray my parents are not answered, but still God exist is the case. So, this leads us to view that we need to move away from the material implication. Another example which I discussed from the last class is that whenever the antecedent of your conditional is false, especially in case of material implication irrespective of whether the consequent is true or false it is going to be true. Now, this leads us to revise the semantics of conditions.

So, the example we have discussed so far raises a problem for the pragmatics of conditionals and do not call for a revision of the do call for the revision of the semantics. So, this Quine in 1950 paper on indicative conditionals does on conversation implications on Lewis work on counterfactual all suggest is that there need to be revision in the semantics of the conditionals. So, the examples called there are two ways which you can think about, one is no revision is required but you can still define in terms of material implication and the second way is we need to have revision of the semantics of conditionals. Again Quine 1950 on counterfactual conditionals and Stalnakar 1968paper on indicative conditionals Lewis work and countrafactuals.

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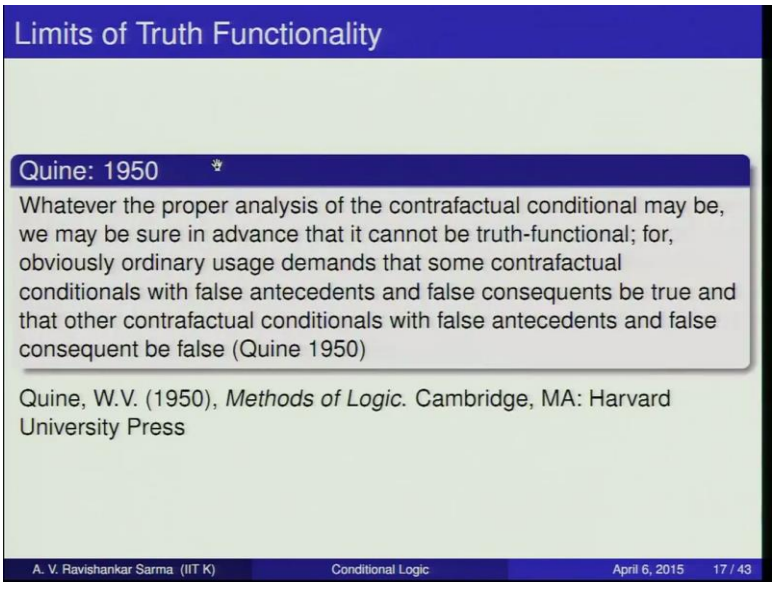


So, Quine is of the review that whatever the proper analysis of the contrafactual conditional may be he considers contrafactual conditional maybe, we maybe sure in advance that it cannot be truth functional at all. Obviously, ordinary usage demands that

some contrafactual conditionals with false antecedents and false consequents be true, and same way there some other kinds of contrafactual conditional with false antecedents and false consequent maybe false.

Now, the problem is how do you differentiate respective these two kinds of conditionals where false antecedents and false consequent leads us to believe the first set of conditionals to be true and then false antecedents and false consequent in the second case is not acceptable to us.

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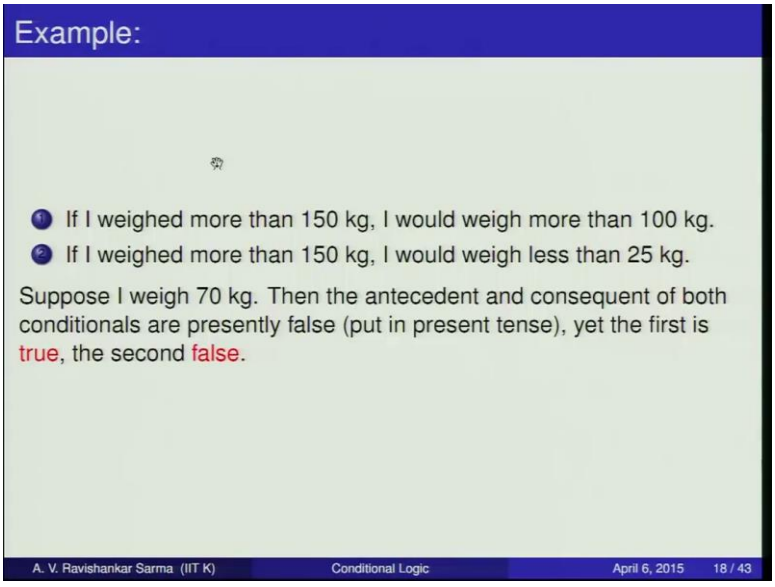


For example if you see this example it will be clear. If I weighed more than 150 kg which is false anywhere in every can more than 150 are we going to take 200 also do not matter. I would weigh more than definitely weigh more than 100 kg because 150 is greater than 100, so false antecedents and false consequent especially when your actual weight is 70 kg. So, if your actual weight is 70 kg if you weigh more than 150 is false I would weigh more than 100 is also false. False antecedents and false consequent, but still it is acceptable to us.

Whereas, if you come to the second one, if I weighed more than 150 kg than I will weigh less than 25 kg. So, both are considerably false in the sense that 25 kg is less than 70 kg in that sense it is false and 70 is less than 150 that is also false. Now, the problem is that antecedents and consequent of both the conditional are considered to be false. I will put it in the present sense like now at this moment. At the first sentence interestingly, the

first sentence is all the one which is acceptable to us, whereas if you are actual weight is 70 you will not be able to accept 25 kg is your weight. It seems to be more or less will lead us to be except the first conditional whereas when it comes to a second condition it is not going to be acceptable we are not going to accept the second conditionals.

So, how do you analysis these two things, how do these things reflect in our semantics and you what is our (Refer Time: 06:51) when is n is work on count countrafactual conditions.

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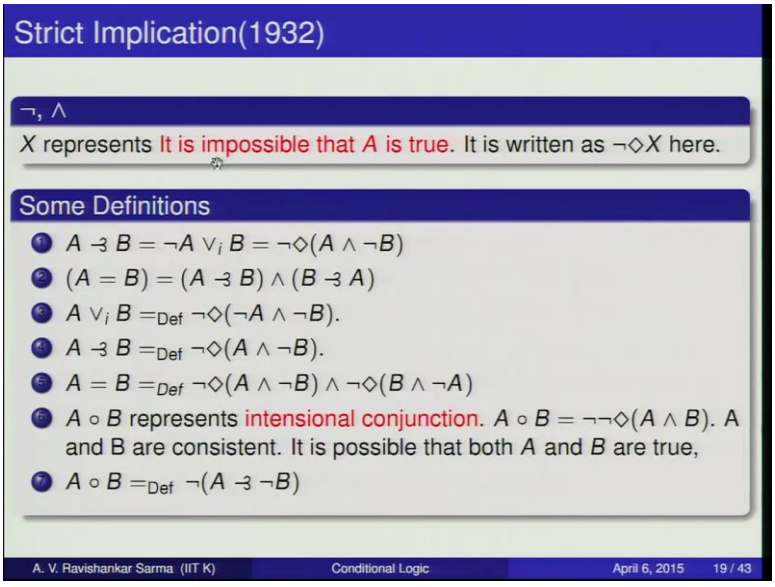


So, the process in order to fix the problem of material implication C A Lewis has come up with a strict implication. This is the one which you are extensively studied when we discussed about normal model propositional logic.

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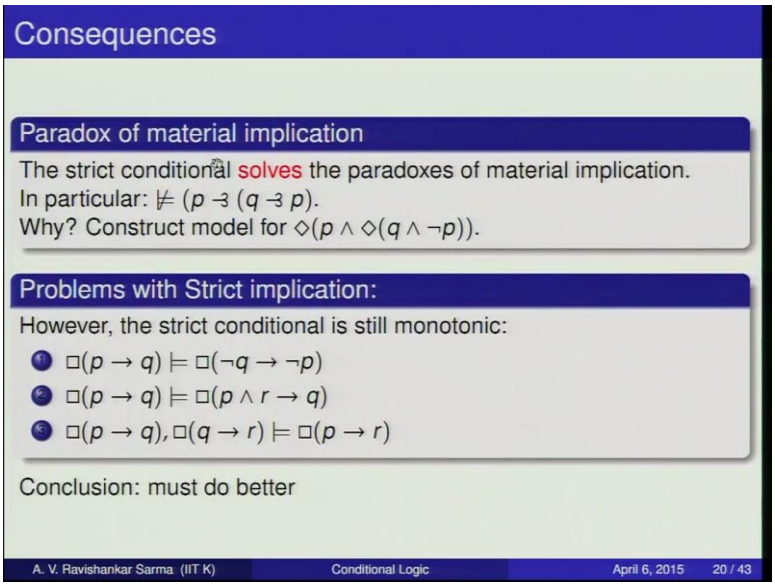


But unfortunately the problem is that even strict implication of course solves the problem of paradoxes of material implication especially when we take a implication as strict implication. But where when it comes to strict implications also leads to some other kinds of bad validities like distinct constructive monotonic, but due to the reasoning or reasoning with respect to conditionals is considered to be non monotonic in nature.

And the other thing is that strict implication also suffers from paradox of strict implication. And the antecedents is considered to be strict any kind of strange that one instances of paradox of strict implication is that from impossibility you can derive anything. If you start with p n naught p you derive anything. And then q strict implication follows from any kind of strange kind of proposition. There also it remains the paradoxes of strict implication remains the same as in the case of material implication.

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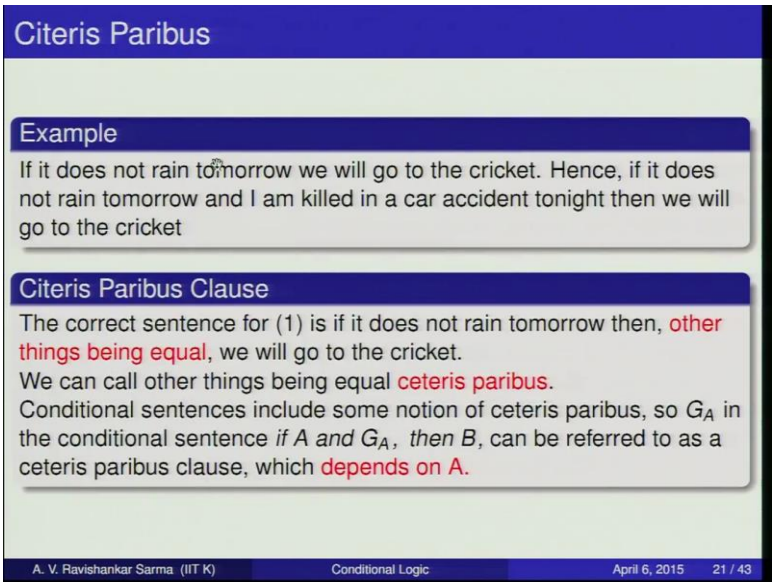


So, this is the story of with respective material and strict implication both are having problems now we take may come up with another conditional connective. So, that is the one which we are trying to talk about. Now, whenever you talk about any conditional statement if p then q we need to take in to consideration ceteris paribus clauses, to give an example. An example is like this if it does not rain tomorrow we will go to the stadium to play cricket. Hence, if it does not rain tomorrow together with that there are some kinds of clauses.

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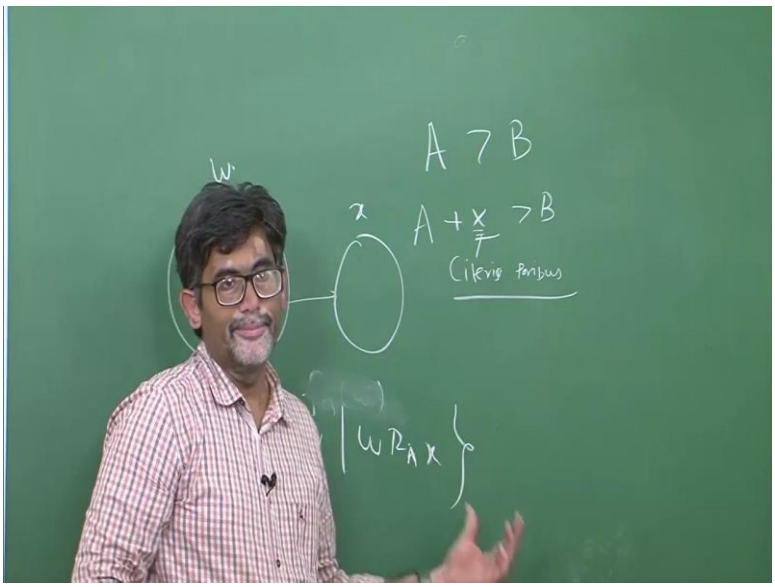

So, usually it is understood that whenever you have a conditional sentence A implies B. So, there are many ceteris paribus clause all other things being equal, so this are constructively ceteris paribus clause. A in plus B is very something A together with some kind of x leads to B. So, this x is that is called as ceteris paribus.

In this case if it does not rain tomorrow we will go to play the cricket if go to the stadium to play the circuit, hence if it does not rain tomorrow then something happen to me or met with an accident or broke my leg or something like that all other things are doing equal let me say all that thing is not happened so that is way we will go to the stadium to play cricket. Usually we do not talk about such kind of nonsense in the sense that we make although they are all implicit, but when we are analyzing the conditional statements we need to ensure that we need to make this ceteris paribus explicit.

So, what are the ceteris paribus the correct sentence for one; one is this thing if you does not end tomorrow we will go to play we will go the stadium to play cricket. If it does not rain tomorrow then all other things will be equal, like somebody did not attack you from mars we do not take this examples in to consideration. All of the sudden world does not spin from east to west rather west to east all other things are been equal are in this place then we will go to play cricket. So, we can call all other things been equal as ceteris paribus clauses.

So, conditional sentences include some notion of ceteris paribus clauses like G A together with the antecedents and that leads to the consequent B. Usually we say ceteris paribus clauses are constructively implicitly present in a conditional and it is assume to be implicit where implicitly present in the conditional sentences. So that whenever you are coming you are coming over the semantics of conditional sentence need to ensure that your semantics will take care of ceteris paribus clauses as well.

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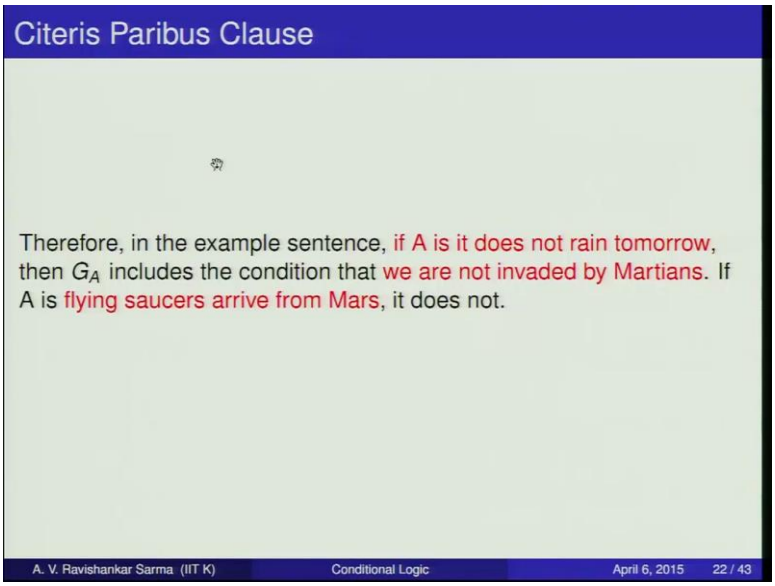


So therefore, in the example sentence if A does not play tomorrow then G A includes the condition that you are not invaded by similar things like nonsensical things like invaded by Martians are. A flying saucers arrive from mars does not (Refer Time: 11:48) but still all other things by are being equal in this conditional force. But we are not made explicit there we do not explicitly talk about ceteris paribus clause.

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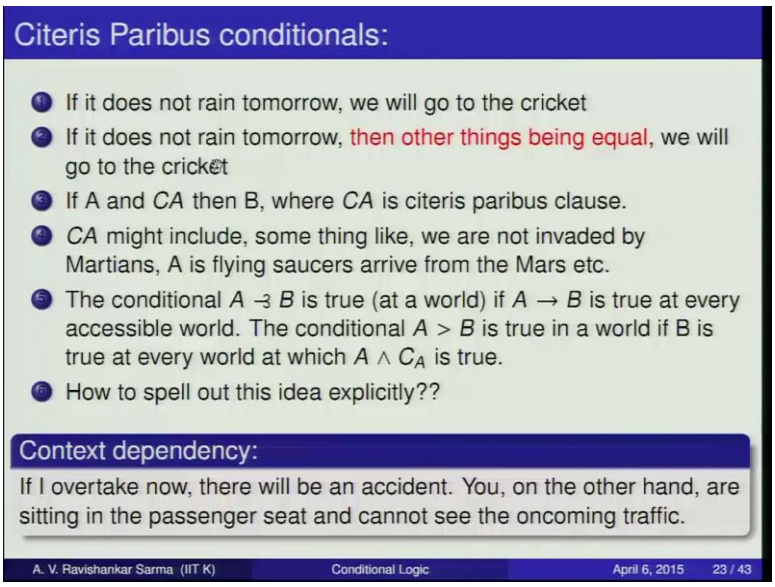


So, if it does not rain tomorrow we will go to the cricket, if it does not rain tomorrow then all other things will equal we will go to we will go to the stadium to play cricket.

That means, we been given conditional A plus B needs to be read like this A together with ceteris paribus clauses leads to B, where C A is considered to be ceteris paribus clause. When C A might include something like we are not invaded by Martians, flying saucers arrive from the mars or any other thing like there is no fiction on the earth fiction becomes zero all this things are not the case. And the earth does not spin from east to west and extra west to east extra. Our sun does not rise at all.

All these things are been equaled then we go to play cricket, there is a stadium to play cricket. But how do we spell this ceteris paribus clauses explicitly. So, we need something so that it can be reflected in as semantics. So, conditionals are not easy to is not easy to analyze the conditional sentences because everything is dependent on the context the way speaker has uttered this condition sentences all this things matters when it comes to conditional sentences. It is not just like p plus q is the truth value of p plus q solely becomes an truth value of p truth value of q it is already that is the reason we are try to analysis whenever a truth value of a compound sentence is not determined by a truth value of its individual constitutes in a second it is considered intentionally.

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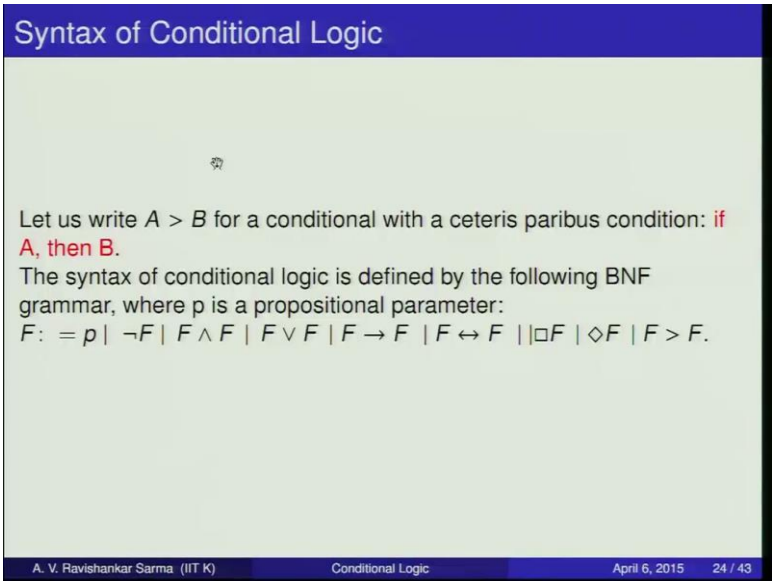


So now let us consider the syntax of the conditional logic, logic we are going to talk about that is C. To begin with C, C is more or less like analogy of the sun could be the minimum modal logic that we have seen in the case of normal proposition model logic. So that is k, k is due to crib k. Let us writer A plus B as conditional with the ceteris

paribus condition if A plus A then B. Now the syntax have conditional logic is defined in by using again (Refer Time: 14:09) normal form it is like this F is any atomic formula is well formed formula in our conditional logical system and negation of any formula is also well formed formula, anything which is combined by a conjunction, disjunction, implication by implication is also considered to be well formed formula.

Anything which begins with the unary operators, necessity, possibility, etcetera, is also going to be the case. These all nothing but the normal model proposition model. In addition to that we have a connective F implies F and reading this greater then connective as implies later than he is list to differentiate material implication and on the other hand we have strict implication.

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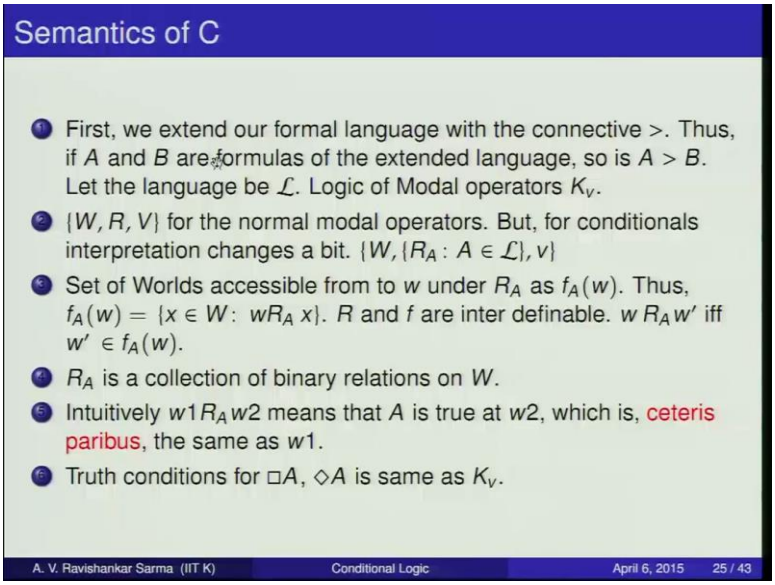


Now how this ceteris paribus clauses together with the antecedents is reflected in the semantics of all ceteris paribus clauses can be reflected in the semantics of conditional logic that we are trying to come up with. So, we need to extend our formal language with the logical connective that can be that is A implies B. And then we have kill modals w R V and for the normal modal logical operators. But for the conditionals the interpretation changes a little bit instead of W R V. Now we write it w R A, A is considered to be the antecedents of your conditional it is not just letter A, but it is considerably antecedents of conditionals if antecedents is p and q you need to write R p and q that a is an a belongs to

language a and then B is in general considered to be as usual considered to be valuation function.

So, the only thing which is different here is that the accessibility relation. Accessibility relation is always with respect to the antecedents of your conditional. That is the main thing that I need to note. There are two thing which you need to note R A and f A w this is the one which will be using in the few minutes we will be using this things. Let say we need to be clear what is R A and f A w. We still can be inter-defined to each other. F A w is considered to be set of worlds which are accessible with respect to the antecedents of your conditional so that is considered to be any other world x that belongs to set of world w say that is w accessible to R A accessible to x.

So it is like this is a f A w means you have a started with a world w. In a f A w is any such kind of world x, so they this w is R A accessible to x. This is not it f A w, f A w is any such kind of word x such that this x belongs to a set of worlds w and then such that w or w R A x. This has to be non empty, if it is empty the new word which is which is accessible with respect to the antecedents of your world and the condition is going to be like your (Refer Time: 17:28) it happens in the case of impossible kind of antecedents. If 2 plus 3 is equal to 5, Rahul Gandhi is this Prime Minister of India for example, for to say I think 2 plus 3 is equal to 5 there is no word which is accessible to that fun.

So the antecedent is false there, obviously the conditional there is no word which is accessible to the antecedent of your conditional 2 plus 2 is equal to 4 the conditional is going to be recursively true. So, R and f are considered to be f will sound like some kind of selection function. It picks up the word the newest word which is accessible from the actual world w. So, R A and f are considered to be inter dependable it is like this, w is accessible to other world w point with respect to antecedent of your conditional if and only if is w prime has to be it belongs to whatever that f A w picks up.

The f A w picks up for example, x then w 1 belongs to x, so both ways you can define this things. Now, R A is considered collection of memory relation on w this accessible to relation with respect to the antecedents. For example, if you say 2 are accessible to each other with respect to the antecedents of your conditional w 1 R A w 2 means that is true in w 2 which is same ceteris paribus together its same as w 1. So, now the truth condition for a necessity of A, possibility of A is same as in the case of modal logic.

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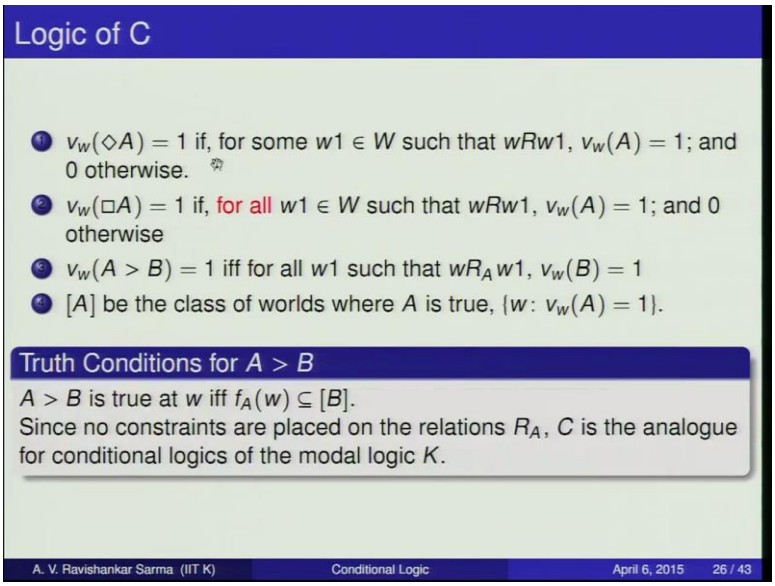


Now how do we define the semantics of the possibility operator, necessity operator and how do we know that the truth value truth condition for a given condition. Possibility of a is true with respect to the word w for any such kind of w 1 belongs to w such that w is accessible to w from and w 1 such that a is showing that world w or any such kind of w pair otherwise it is considered to be 0. And these are things which we have defined it in the case of modal logic and we do not have to go in the details of it.

Now, the conditional sentence A plus B is true in the world w that is whereas V w A greater than B is equal to 1 if and only for all such kind of w 1's where this actual world w is accessible to this w 1 your B has to be true. Now the antecedents permissible world your consequent has to be true in that case you say that a plus B is true. And another notation that we will be using that is if a written in the brackets that means, these are set to be considered to be some of the class of worlds where a is considered to be true. If you write B there and all the word should which B is considered to be true.

So, now the truth conditional for A B reflecting this ceteris paribus clauses is like this, A plus B is going to be true in the world w if and only if f A w are those worlds like x or y or any such kind of things in those worlds your B is true. Whatever is f A w has to be sub set of those worlds in which these true. In all antecedents permissible worlds you are consequent also to have be true, in that case only the given conditional holds.

Now how do we reflect, how do we talk about the semantics things in terms of how this semantics can be reflected in our main method that is semantic tableaux method? We will be talking about the semantic tableaux method with respect to conditional logical systems C and C+ in the next class.

Thank you.