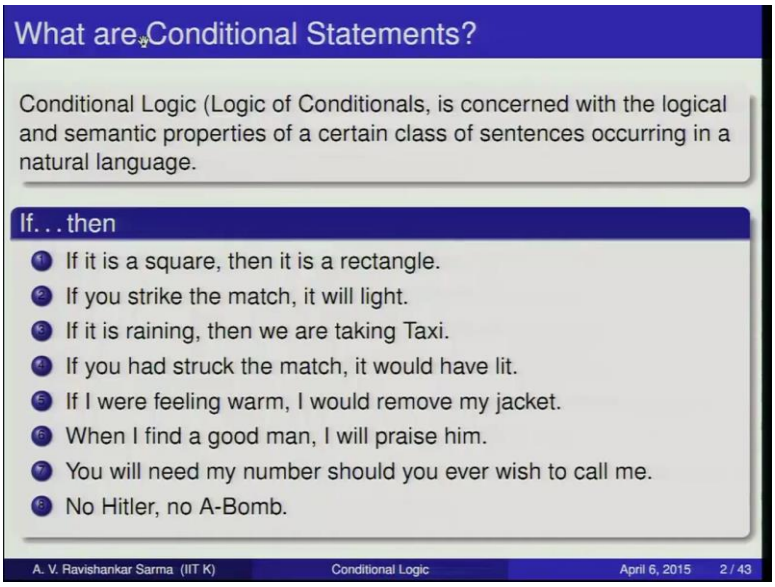
Basic Concepts in Modal Logic
Prof. A.V. Ravishankar Sarma
Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

Lecture - 23
Conditional Logic: Introduction

Welcome back. In the last few lectures we discussed about the normal modal propositional logic. Which is considered to be an extension of classical logic with 2 (Refer Time: 00:23) modalities. It is necessary that p and it is possible that p. And we have seen syntax and semantics with respective various modal logical systems. K T D F S4 S5 etcetera, and then we have seen how to show a validity of a given modal logical formula by using one of the important decision procedure methods, which has been which has occupied the central position in this course that is semantic tableaux method.

So, now we would be talking about one of the important applications of modal logic that is the analysis of conditional statements.

(Refer Slide Time: 01:05)



So, conditional statements those statements which consists of an antecedent and a consequent, and these are all of the form if p then q. Why modal logic is used in

analyzing these various kinds of the conditional sentences? Conditional logic it has become one of the important branches of the philosophical logic these days. It is widely studied by many philosophers. And it has been analysis of conditions are always been the problem right from the Greek period on wards. So, the conditional logic, logics of conditionals is concerned with logical and semantic properties of certain classes of sentences that occur in our natural language.

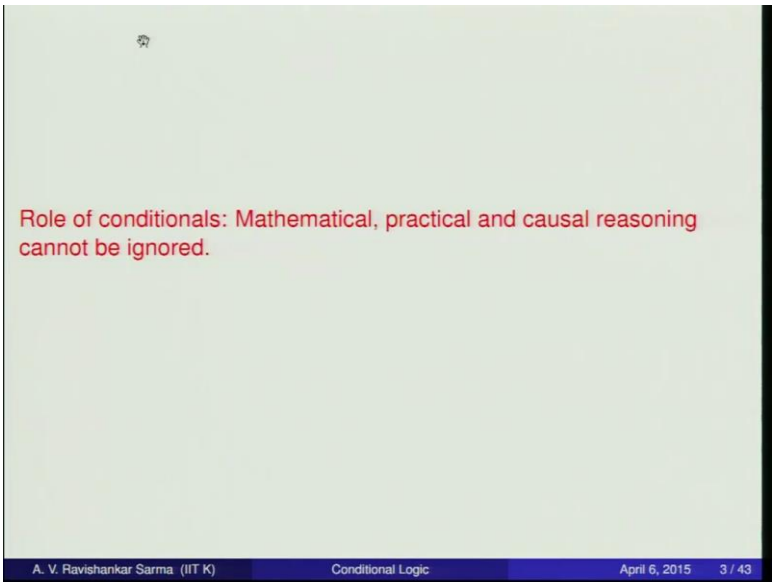
So, what are these conditional sentences? They all have a specific structure it has if p then q. Where p is considered to be the antecedent of the conditional and q is considered to be the consequent of the conditional. For example, let us begin with some examples if it is a square, then it is a rectangle it is a hypothetical kind of thing if something is the case then something is the case so, then it is a rectangle. If you strike a match, then it will light. If it is raining, then we will take a taxi. These are of they are used in some kind of sense these are called indicative conditionals. Whereas the forth one, if you had struck the match it would have lit; that means, you dint strike the match, but you know imagining a situation where you have struck the match then; obviously, it would light. And the fifth example if I was feeling warm I would remove my jacket. I am not feeling warm that is why I am not removing my jacket. In case if it is the case that I am feeling warm then I would remove my jacket.

So, these are, these 2 are different types of conditionals. Now the most fundamental the fundamental problem the philosophers trying to address is, is that is there any difference between the first 3 conditionals and second, I mean a fourth and fifth conditionals that I have, I was talking about. So, the first 3 conditionals are indicative conditionals and fourth and fifth are considered to be counterfactuals are subjunctive conditionals. Counterfactuals are considered to be one specific type of subjunctive conditionals where the antecedent is always false. Conditionals will also occur even if you do not find any if p then q kind of things. For example, in this example when I find a good man I will praise him.

There is no if and there is no then, but still it is treated as conditional sentence. So, this needs to be read as, if I find a good man then I will praise him. Sometimes you intentionally or deliberately remove if p, if and then. In the same way you if you say if

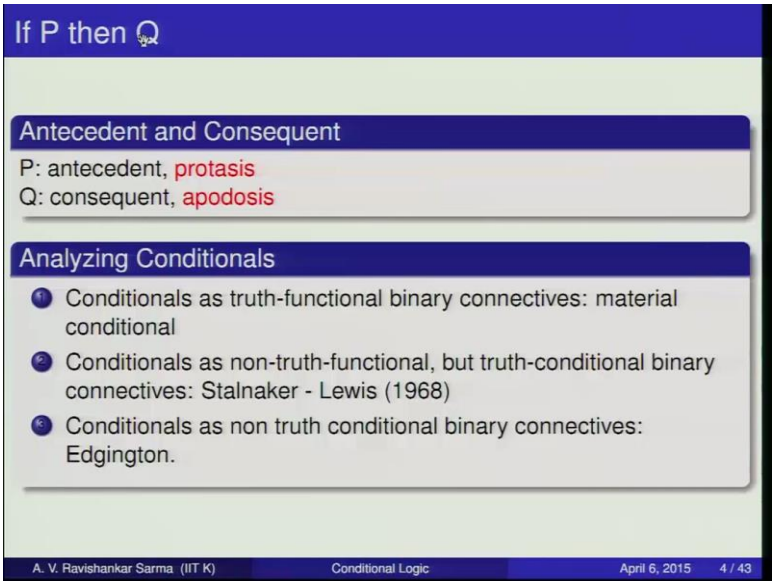
you need my number should you ever wish to call me. Or there are some shortened form of conditionals like, no Hitler, no atomic bomb. So, what is that you are trying to do is, is that you are trying to analyze the conditional statements using the knowledge that you already have. That is the possible world semantics that we have seen in the case of normal modal logic.

(Refer Slide Time: 04:24)



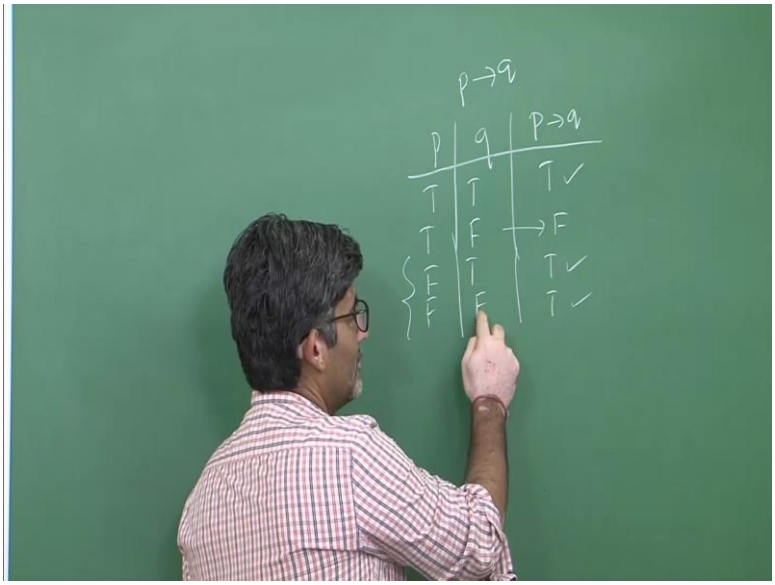
And it cannot be ignored that conditionals play an important role in mathematical discourse and practical even in casual reasoning and all these things cannot be ignored. So, usually when you talk about the conditional sentence it expresses some kind of a hypothetical situation and it expresses some kind of necessary and sufficient conditions.

(Refer Slide Time: 04:51)



So, in any given condition if p then q p is considered to be antecedent and there is a word Latin word for the antecedent that is, protasis, and q stands for consequent and there is some kind of relationship between p and q. Where p is considered to be the antecedent and q is considered to be the consequent. So, there are 3 different views in analyzing the conditional statements. I am not talking a specific kind of conditionals, but any kind of conditionals, but any conditional which begins with any conditional which has a specific form if p then q. So, there are 3 views conditionals and analyses the truth-functional binary connectives like material implications. You can express in terms of semantics of material implications where you have this thing.

(Refer Slide Time: 05:43)



If you have in material implication p implies q, these are the things that we have T F and TF. So, it is going to be false only in this case in all other cases it is going to be T.

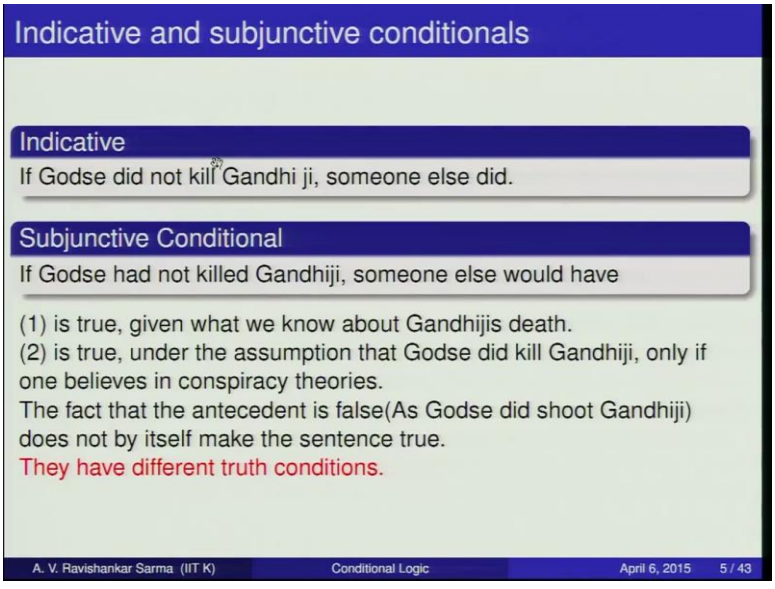
There are 3 ways of analyzing the conditional sentences, conditionals. Conditionals are treated as truth functional binary connectives; that means, you have you are analyzing conditionals in terms of if, material implication and the semantics of material implication is that whenever you have this is the antecedent and this is the consequent. The material implication is going to be false only when the antecedent is true and the consequent is false. Where as in all other cases as you are observing all the cases it is true, even if the antecedent is false.

So, now, these are the 2 cases which is of interest to us. Whenever the antecedent is false irrespective of the consequent, whether it is true or false is going to be T always that one is true. That is one way of analyzing the conditional statements if you analyze the conditional statement with respect to the material implication, there are some kind of conditional sentences like if I had dropped these chalk piece it would have fallen in the ground it is going to be true, in this case the antecedent is false I have not dropped this chalk piece it is still in my hand.

And on the other hand there is a there is going to be another counterfactual you can construct it like, if I had dropped this chalk piece it would have flown to the air or it would have turned into a donkey or cat or something like that. So, the second of condition in this is not acceptable to us where as the first conditional, right if I drop this chalk piece could have fallen on the ground seems to be a for us.

So, that is one thing it is difficult. It is not we can analyze the conditional sentences with the help of material implications the limited sense. Now the second views are that conditionals are treated as not treat functional, but I have mailed truth fun truth conditionals binary connectives and this is the approach which is followed by Stalnaken and Lewis, from a seminar paper on conditional conditionals. And David Lewis extensively worked in the area of counterfactuals. And there is a third view which we are not going to talk in this at least in this lecture. So, the conditionals are treated as non-truth conditional binary connectives, and this is the view that is taken from another logician Edington.

(Refer Slide Time: 08:19)



So, now coming, back to the issue of conditional sentences that all have a having a structure if p then q, where there is some kind of relationship between the antecedent and the consequent.

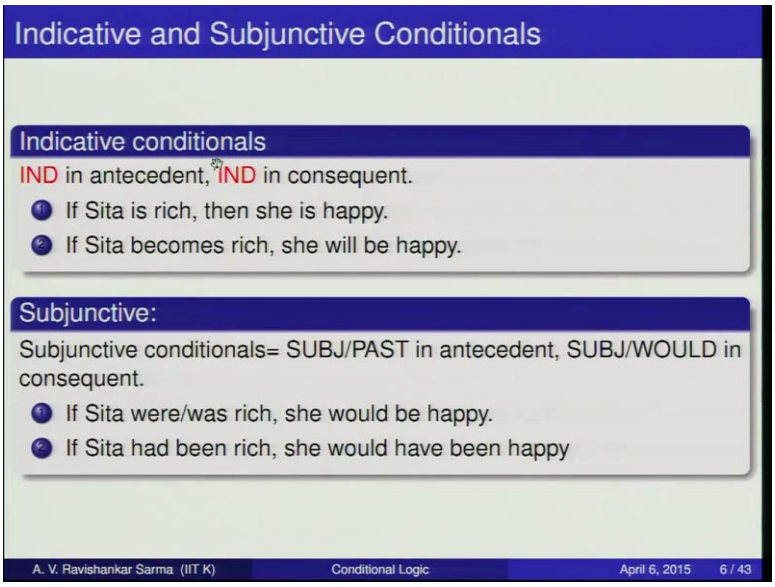
Now, these are the 2 different types of conditionals that we commonly come across in our day to day dispose. The first one an indicative conditional we call it as indicative mood present in the sentence. If Godse did not kill Gandhiji then, someone else did assuming that this is the first sentence. Now it is correspondence subjunctive conditional is being counterfactual conditionals are this. If Godse had not killed Gandhiji someone else would have. So, when you are philosophers are asked to talk about the truth conditions for these 2 conditional sentences, they were various points of view. Some philosophers are of the view that indicative and subjunctive conditions are will have different true conditionals.

So, whatever it is highly debatable kind of issue, whether they have the same kind of truth conditions or different kinds of truth conditions that well postpone it for a while, but if you observe the first sentence, statement number one a sentence is true given that some kind of historical facts about how Gandhiji's death took place. It is not difficult for us to believe or accept the first the first condition statement to be the case. Because he just follows material implications. So, in the material implication it is going to be false only when the antecedent is true and the consequent is false, then the whole condition is going to be false, in all other cases it is going to be true. It is not so easy when it comes to subjunctive conditionals are some specific kind of subjunctive conditions which are called counterfactuals, where the antecedent is always false. If Godse had not killed Gandhiji, someone else would have. So, imagining a situation where you are you are gone back then and in the past and then you are trying to analyzing the past the particular kind of condition.

So, now the second sentence is going to be true under various other kind of assumptions, and these assumptions are those that, Godse did kill did kill Gandhiji, only provided you know some kind of some theories conspiracy theories which are floating around. So, all these assumptions which are taken to condition and put it into some kind of prospective, then you will be able to analyze whether this condition holds or not. So, the fact that the near fact that, the antecedent is false as Godse did shoot Gandhiji does not by itself make this particular kind of sentence true. That means, they have different truth conditions this is the extensive kind of research which is going on in this direction, that you know how to analyze this counter factual statements. David Lewis has come up with his own view,

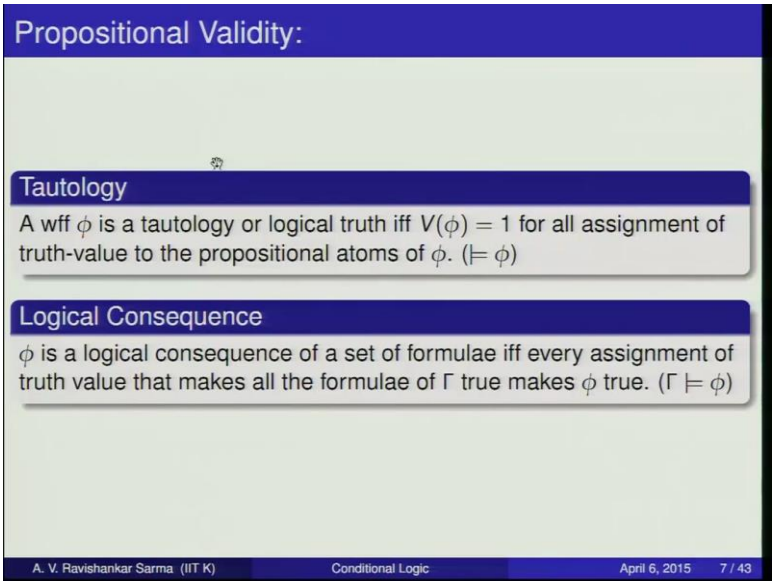
it is analyzing the counter factual using possible words semantics he views subjective conditionals as variably strict conditionals.

(Refer Slide Time: 11:39)



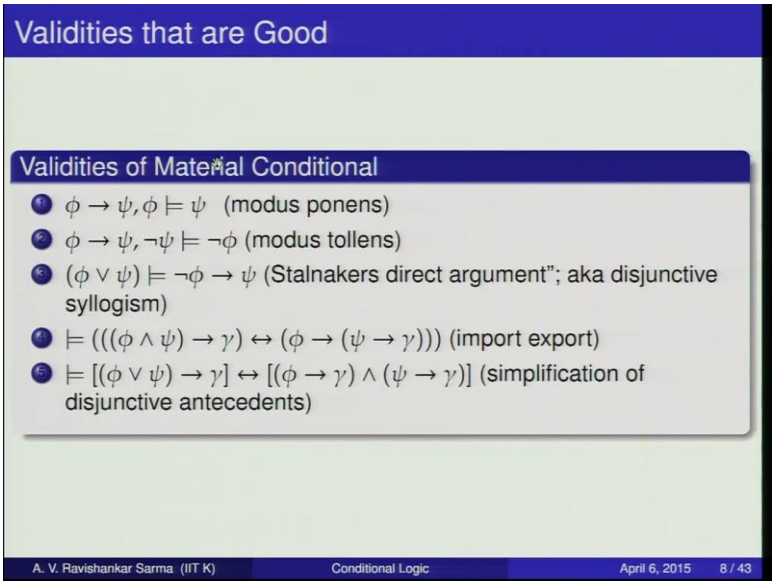
So, it is that we are going to see later, I mean that is what we are going to see in the works of David Lewis. So, what are these indicative and subjunctive conditionals? Indicative conditionals are having indicative antecedents and indicative consequents like if Sita is rich, then she is happy, if Sita becomes rich and she will be happy. There is in the case of subjunctive conditionals, we have subjunctive past in the antecedent and subjunctive would in the subjunctive oblique would in the consequent. If Sita were rich she would be happy. Sita is not rich at this moment, but if she were rich then she would be happy. Imaging a possible world in which Sita is rich, she would be happy there. If Sita had been rich she would have been happy. So, we are trying to analyze these conditional sentences using possible world semantics. So, because the conditionals can be true only if you are trying to analyze the conditional sentences, you need to sequence you need to fix the antecedent and correspondent to the antecedent, we have some possible worlds and those possible worlds whether the consequent is true or not is the one we are trying to see.

(Refer Slide Time: 12:50)



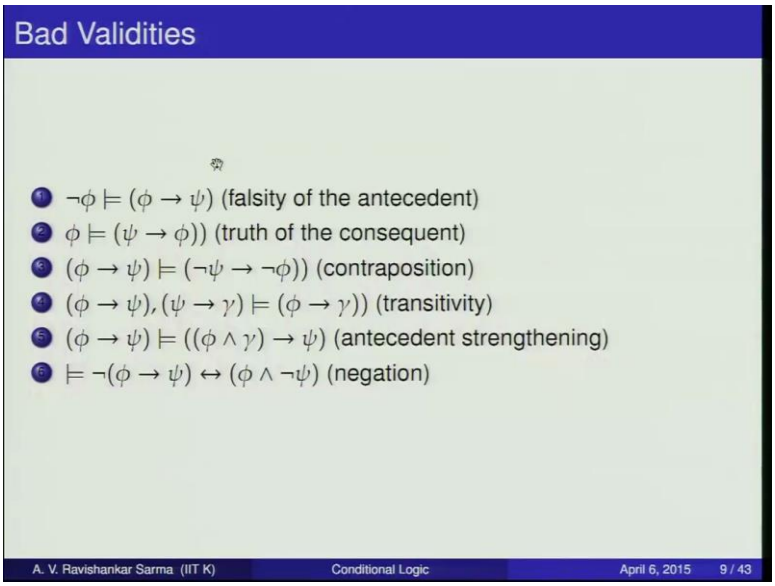
These are the some of the basic things that we need, the tautology is considered to be a logical truth. And there are all interpretations the value of phi is going to be true, for all and logical consequence is that, anything which is phi is considered to be logical consequence set of formulas if and only if, every assignment of truth value that makes all the formulas gamma true that also makes phi true; that means, gamma is going to make this phi true. Our phi is a logical consequence of gamma. Or gamma entails phi here are different ways of reading the same thing, and this is the one which the basic things which we have discussed in the beginning of this course.

(Refer Slide Time: 13:28)



So, now when we try to analyze the conditional sentences, by using the materials implication, on there seems to be good validates like modus, ponens, modus, tollens, and etcetera. And import export simplification of disjunctive antecedent etcetera.

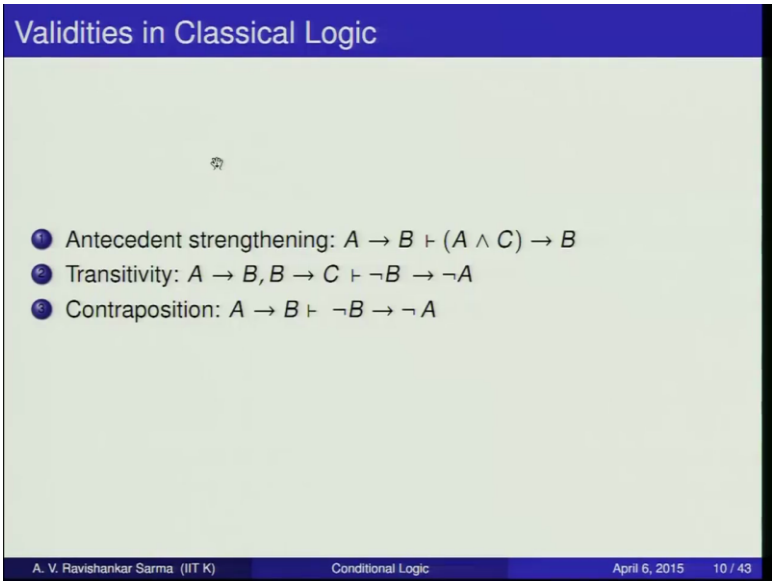
(Refer Slide Time: 13:54)



But when it this seems to be the most important thing that we require. Whereas, when it comes to some other types of inferences like that we are calling it bad validities. Bad not in the bad sense, but not in the sense of real bad, but subjunctive kind of things, these are the kind of things that leads to paradox of materials implication. Paradoxical material implication is this that truth preposition is implied by any strange kind of proposition and a false preposition implies anything.

If this is the case then all the tautology should come may come from strange kind of sentence, sorry even contingent kind of sentence which we do not want. It is undesirable kind of thing that is why we are considered to be bad validities. So, when you come up with semantics of conditional sentences, you need to ensure that good validity should be part and parcel of our logical system, like more resemblance more resistance etcetera. And bad validity is like paradox submitting implications law of contra-position, law of transitivity antecedent strength etcetera all these things should not come is theorems in our conditional logic.

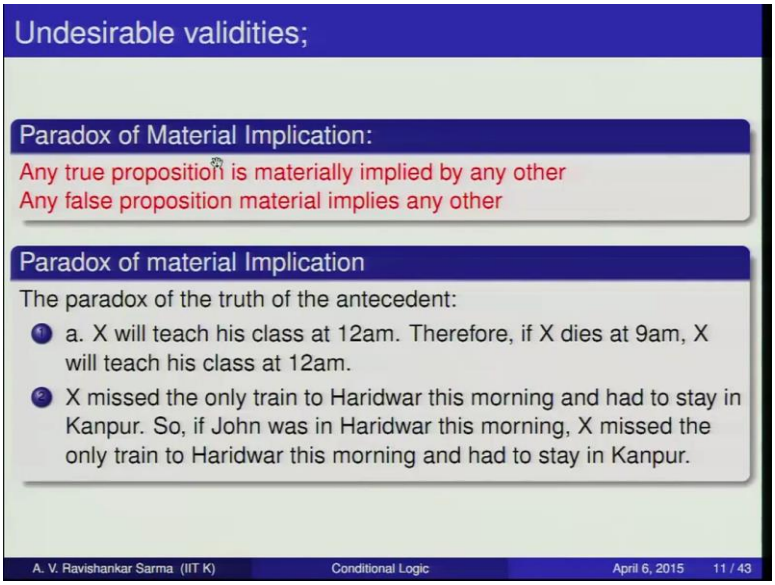
(Refer Slide Time: 15:05)



So, there are certain kinds of validities in classical logic, but when it comes to ordinary day to day conditionals, it does not make any sense. For example, if you take antecedent strengthening A in plus B in plus if you add another statement to it C. You will still have

the same kind of conclusion that is B. Suppose in the real life example if an if you take example of if there is a sugar in the coffee it should be tasty, it is tasty, and A and C stands for if there is sugar in the coffee and there is a kerosene in the coffee then it would be tasty. So, nobody will be in a position to accept. Although it is considered to be a good validity, but it is giving us counter intuitive kind of results, same is the case with transitivity and contraposition.

(Refer Slide Time: 15:49)



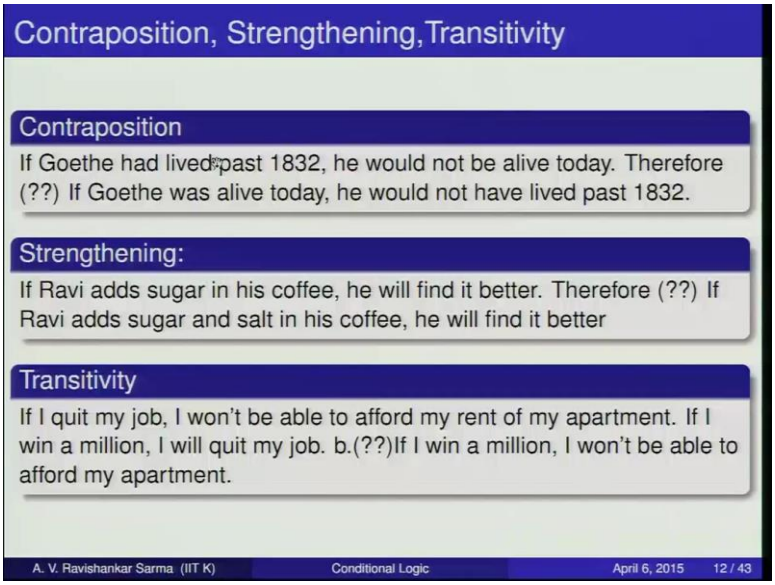
So, now the paradox of material implication is the one which CA Lewis has begun with showed dissatisfaction to this material implication and he has come up with sit implication. And you have seen how he has come up with various logical systems in order to address the problem with respective material implication.

So, he is of the view that, material implication does not capture the process of deduction. When you say that something is reduced from something it is not captured by the material implication, it should be captured by strict implication. So, there are some instances of paradox of implication, material implication it can be expressed in with the help of some kind of sentences natural language sentences. It will be like this X will teach his classes at 12 am, if the suppose if X met with the accident, then X will teach his class at 12 am. It is not possible. So, it does not make any sense to talk about p implies q

implies p; that means, p should, p should be implied by any strange kind of proposition. Strange kinds of proposition is expecting with an accident, or suppose if you say that X misses the only train to Haridwar is morning and had to stay in Kanpur. So, John was in a Haridwar this morning. So, X misses the only train to Haridwar this morning and, had to stay in Kanpur.

So, the idea here is that, we do not want strange opposition to be implied by so, a true proposition is implied by a true proposition is implied by any strange kind of proposition.

(Refer Slide Time: 17:28)

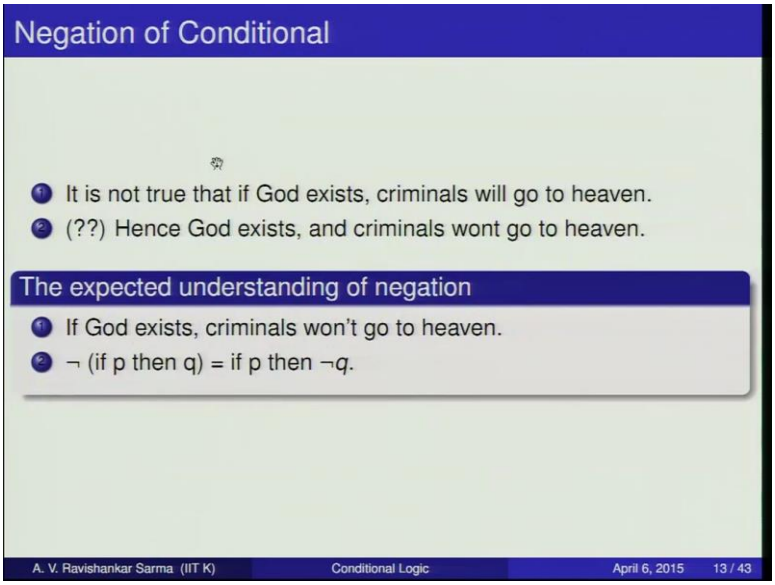


These are some of the examples of the contraposition strengthening and transitivity. For example, in the case of contraposition fails, in the case of conditional sentence when it is applied in the day to day discourse it is like this. If Goethe lived past 1832, he would have, he would not be an alive today therefore, the contraposition is a implies B implies not B implies B not a. Therefore, if Goethe was alive today he would not have lived past 1832, it is not acceptable to us.

The second example is the one which I already discussed. Now transitivity is like this, if I quit my job I would not be able to afford the rent of my apartment. I do not have any money to afford my rent of my apartment. And the second statement is if I am win a

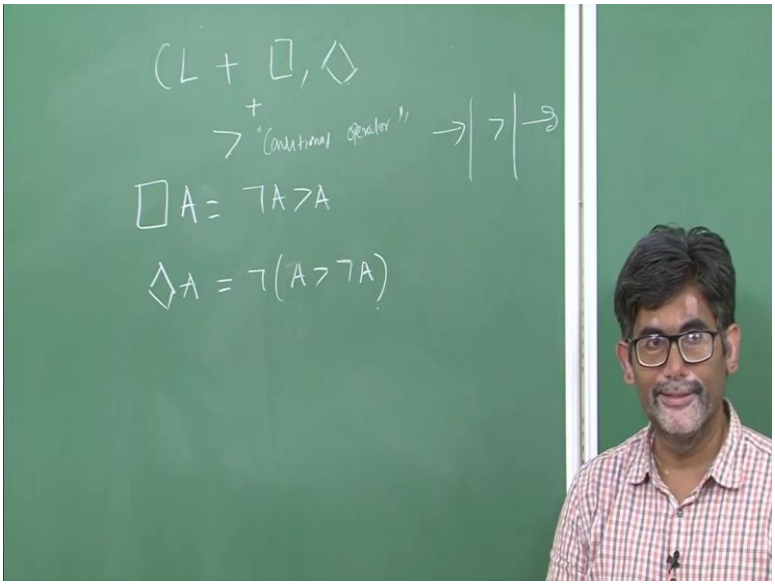
million dollars or something like that in lottery or something like that, I will quit my job I do not want to do any job. So, I will quit my job. So, it is like A implies B and B implies c so; that means, if I win the million dollars in the lottery, then I would not be able to afford my apartment. It is not possible. A implies b, B implies c, A implies c is a cake, but when it is applied we it is applied to day to day conditionals in day to day discourse, particular in natural language sentences like this, we have this counter intuitive like, if I have million even if I have millions of dollars of, in my hand, but I would not be able to afford my apartment.

(Refer Slide Time: 18:46)



In the negation of the conditional is like this. If it is not true that if God exists criminals will go to heaven. So, there is A there is A, equality there in this one. And not of A implies B is same as A implies not b. So, hence God exists and the criminals would not go to heaven. So, we are going to see in the next class, see how the material implication leads to some kind of problems and then which leads us to think about the different kind of connective, which is in between material implication. Material implication is considered to be 2 weak where as tit implication is considered to be too strong. And we need to have another kind of connective that is what we are calling conditional kind of connective.

(Refer Slide Time: 19:41)



So, basically what we are trying to do is that, we started with classical logic and we extended with 2 modal operators. These are considered to be alethic modal operators. So, now, in in addition to these things this is considered to be normal modal logic. This we talked about propositional logic which can be extended to predicate logic also. And then the extra thing that we are talking about is this conditional connective. Now we are going to study about how this conditional connective behaves, definitely it is not material implication will not work. And even the strict implication also will not work for this. So, we need to have a different kind of connective, and then we are trying to connect and we are trying to see the logic of these conditional statements.

Another important thing which you need to notice is that, necessity of any proposition can be expressed in terms of these conditional connectives like this. Not A implies A and possibility of A is expressed as, it is not the case that A implies not A. So, in the next class we are going to talk about 2, we are going to start with 2 simple modal logical systems, with respective conditionals. They are C and C plus. And then gradually we move on to S1 C1 and C2, C1 and C1 is due to David Lewis. And C2 is due to Stalnaken. There are some minor differences between C1 and C2. All this we are going to see in the next class.

Thank you.