Basic Concepts in Modal Logic

Prof. A.V. Ravishankar Sarma
Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

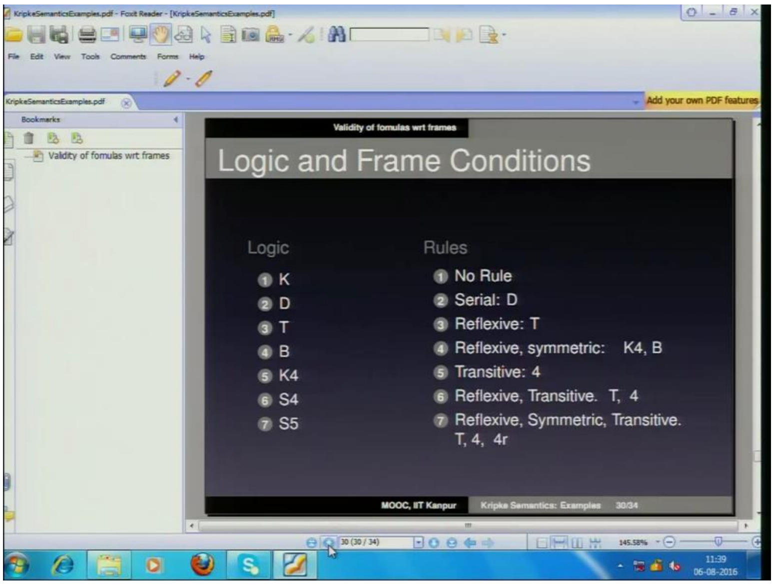
Lecture - 21
Semantic Tableaux Method - More Examples

Welcome back. In the last lecture I introduced semantic tableaux method and using semantic tableaux method, you can talk about validity of a given moral logic formula; that means, when the conclusion follows from the premises or you can say that when 2 groups of sentences are set to be consist to each other or then many other things which you can talk about using semantic tableaux method.

So, in this lecture we will be considering more examples. So, that you will get use to is very important technique that we implying in employed it as some kind of decision procedure method. So, in the last class that we have seen that, if you are using semantic tableaux method and if it is a k system if we negate the formula that in an it is necessary that p plus q implies it necessary that p implies it is necessary that q, that kind of formula if we negate the formula we do not require to impose on the constraint on the accessibility relation. That is why it is called minimal moral logical system k. In the same way in the case of d if we negate the formula it is necessary that phi implies it is possible that phi if we negate the formula in constructed tree you required that that particular I mean branch closes only when you invoke this serial property. Serial property is a for all x, there is always some kind of world which is accessible from the world w, and in that world your formula phi is true.

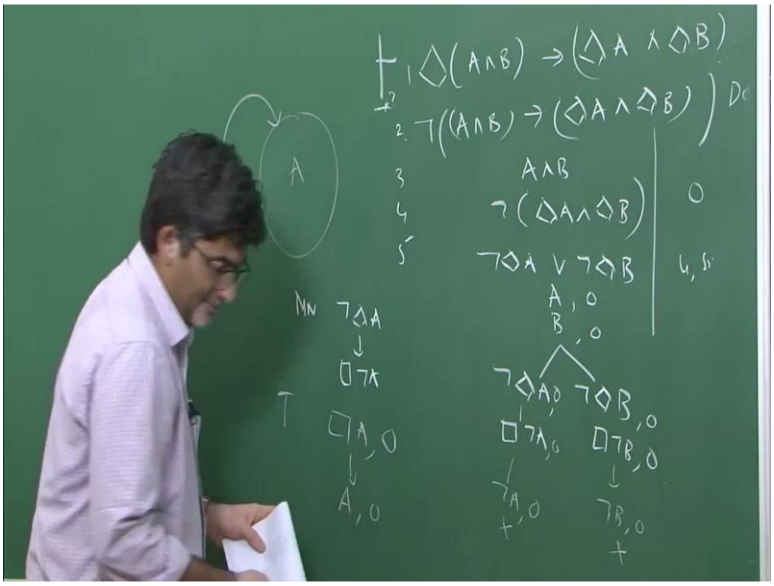
In the same way t closes only when it is reflexive; that means, negate the formula if we negate the formula, it is necessary that phi implies phi. It holds only when reflexive relation holds. So, like this if it is S5 formula which is in S5, it closes only when it is symmetric reflexive and transitive. The same kind of things that we talked about in terms of relational structures can be done by using semantic tableaux method.

(Refer Slide Time: 02:22)



So, let us consider some examples and see whether these formulas are valid or not. So, we will start with the simple formula like this one.

(Refer Slide Time: 02:38)



Imagine a situation. Suppose if you write like this. Whatever I write is consider to be a theorem. Now we are trying to check it that is why I put question mark. Suppose if you have a formula like this A and B implies, it is possible that A and it is possible that B. So, now, we want to check whether this formula if valid or not. So, in the semantic tableaux method, what you do is you negate the formula and start constructing the tree for this formula. So, that is you take the negation of the formula it is like this.

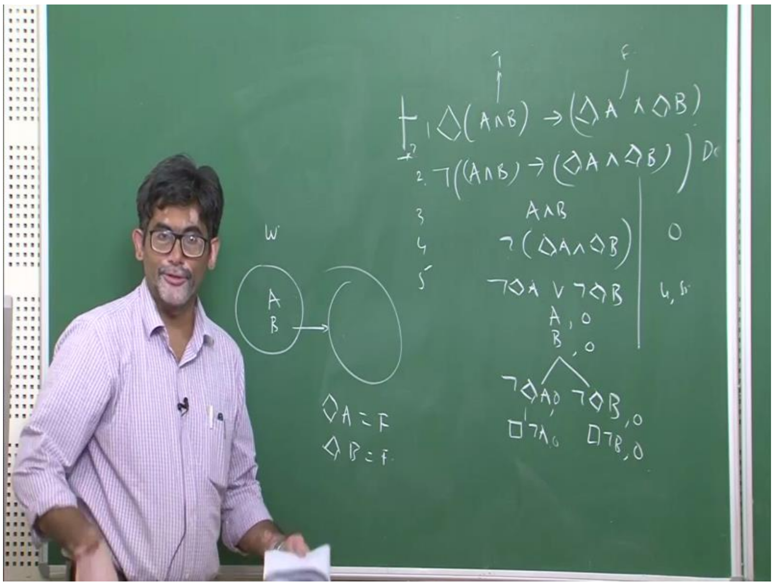
So, now this is simply x implies y. So, that is why A and B and not of possibility of A and possibility of B. So, you need to note, that suppose we need to mention here under what logical system you are checking this formula. So, if I write k here; that means, you are checking this formula in the logical system k. K does not require any constraint to be imposed on the accessibility relation. So, now, coming back to this 1, 2. So, this is denial of the well form formula, and then 3, 4 and 5. So, 4 simplifications you will get this thing negation of conjunction is this conjunction. And it is negation of possibility of A it is impossible at A and negation of possible B.

Now, this all this thing is true let us assume that they are true in a world 0 you can w or you can take u or you can take v also, now, A and B. So, same thing 3 is simplified into this thing A 0 and B 0; that means, A is true in world, B is true in world 0. So, now, negation of this one, this is it should be like this is a branch. So, that is why we write it like this negation of possibility of B. So, this is we have this modal negation rule which we need to use. So, that is whenever you have negation of possibility of A, you can simply replace it with it is necessary that not A. So, this will become this and you need to mention worlds that they are true. Otherwise it does not make any sense, true in modal logic you need to talk every formula either with respect to truth, of the formula with respect to world truth of the formula, with respect to A frame or a modal or can say modals.

So, this will become not B and 0. Now so, we are trying to understand, under what conditions this branch closes. So, now, observe A is true in a world 0, B is true in a world 0, but you have necessity of not A is true in a world 0. So, now, unless and until there is a rule with which there is a t rule, which tells us that if formula is true in a world 0, then you can straight away replace it with A 0; that means, necessity of A is 0; that means, A is true in that world itself; that means, you require some kind of reflexive relation here. So, once you apply the t rule then only this closes. So, A and B and everything is same now you can straight away replace this one with not A is 0 and not B is 0. So, this then you have A 0 and B 0, and you have literal and it is negation occurs this closes even this closes. This formula holds in t. So, now, what about k usually it is expected that you know possibility of A and B need not have to be same as possibility of A and possibility of B.

So, let us see whether a possibility of A and possibility of A leads to the other way around converse is true or not. So, in this case at least in the case of logical system k, it is going to be invalid, but there is in the case of only when you when you invoke t rule, this branch closes otherwise branches going to remain open. Open branches tell us that whether the given formula holds in something or not in k it is definitely invalid.

(Refer Slide Time: 07:23)

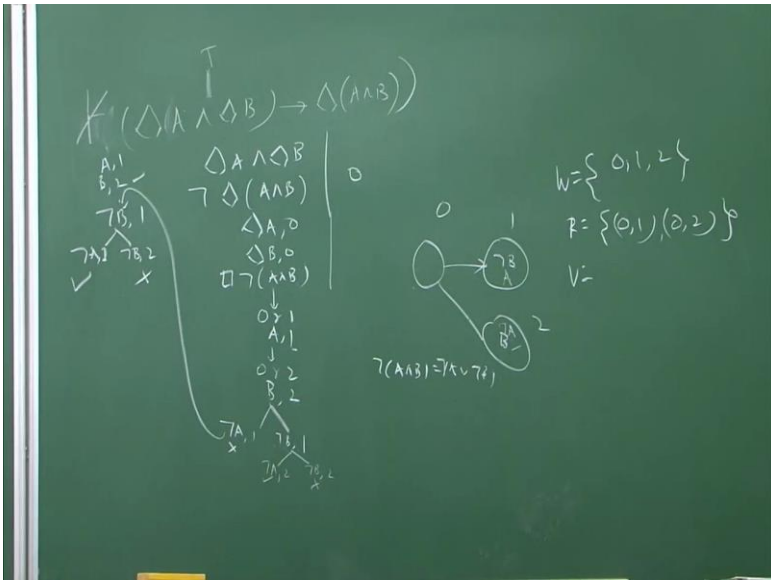


So, for the counter example you need to draw like this A 0 and A and B are true here and since there is no such kind of relation, I mean suppose if you have a formula necessity of not a there is no only move to another world.

So, I said in the last class, that whenever you have a formula necessity of A it does not imply that you should always go the another world, but whenever you come across a formula like this, it prepuces that you need to go to a different world and in that world a is true, but this does not give guaranty to you should go to another world where a is true. So, this is the thing which you need to note. The same type of situation is arriving here. So, you have necessity of not A 0. Necessity of not B 0, that it does not imply A is 0 there unless you, until you invoke t rule. T rule involves the reflexivity. So, unless until 0 is accessible to 0 and 0, is accessible to 0. There is no way which you can close the branch. So, the counter example can be constructed from the open branch. Suppose if we are talking about k, you are not supposed to use this t rule. So, this branch ends here itself. So, there is no way in which you can move further. So, this itself a relational structure this itself will survive you a purpose.

So, there is a world in that world both A and B are true, but you need not have to be the case that possibility of A is true possibility of B is true for the possibility of A to be true, there should be some kind of world which is accessible to this one, but there is no world which is accessible to this one. So, that is why this formula is going to be false and even possibility of B is going to be false that makes the sentence t and this as false so; that means, they can construct a counter example especially when you are talking about system k.

(Refer Slide Time: 09:35)



So, now what will happen if you reverse the order? So, that is this one it is possible that it is possible that A and it is possible that B from this let us see whether in what conditions this formula holds A and B. So, it is like it is humid and it is raining. It is possible that there is a combined sentence. It is possible that it is raining and it is humid, does it imply that it is possible that it is raining and it is possible that it is humid. So, there is a question which we are trying to see. If it holds under what condition it holds. So, these are the things which are we trying to see.

Again this is x and not x is you negate the formula. Then this is going to be like this. So, this is you put the negation inside this is constructed to be the denial of the conclusion. So, now, x implies y not of x implies y possibility of B. Now it is not possible that and B all this thing is true in a world 0 for example, we will begin with you construct it at as actual world. So, now, possibility of A and possibility of B can be written like this. Possibility of B true in a world 0, if you simplify this one; you will get this one necessity of not of A and B. So, know when we have to write immediately. Here if we draw a line like this and say 0 and all the formulas that are there. And true in a world 0 possibility of A is true in a world 0; that means, you have to go to a world 1 where in that world a has to be true.

Now, another formula is there which is starting with possibility operator. Say when you remove this possibility after for next time which is occurring in the branch. Then you need to another index later. So, one is the one exhausted already, you have go to another world other than the world 0. So, you have started from here. First time he went to this one, 0 one and now you need to go to another world 2. So, now, if you remove this thing, it is again you are moving from 0 to 2. Now rather than, one we already gone, in order to go to you need not have to go there again. So, in that thing your B has to be true in a world 2.

Now, we have this formula. Necessity of not of a in this true in 0; that means, not of A and B has to be true in all worlds. So, know first you applying this one. Not A has true in a world one not B is true in a world 1. Because not of A and B is not A or not B. So, now, we need to see whether a lateral need a negation in the branch this closes here. And now a you have B is to be in a world 2. So, so B is true in a world 1 sorry B is true in a world 2, but not B is true in a world 1 it is like here B is true and not B is true here there is no way in which you can close the branch here.

So, now again once again you apply the same thing, because it is not of A in B is necessary that is true in all possible worlds. So, once again you apply I will write it here, not of B this branch closes; now what is left is this one. Under this not B implies 1 again you need to write this one. Not A is true in a world 2 and not B is true in a world 2. So, now, above that we have all this things B is true in a world 2 A is true in a world 1. So, now, not B is true in a world 1, and B is true in a world 2. So, a 1 and this is A 2 A 2 branch remains, open now not B 2 and B 2 this remains closed.

So, now we have this open branch so; that means, if you take individually the possibility of A and possibility of B. It does not need at to need to B. So, now, if you are ask to construct a counter example. So, let us check it again here, we have exhausted rule for.

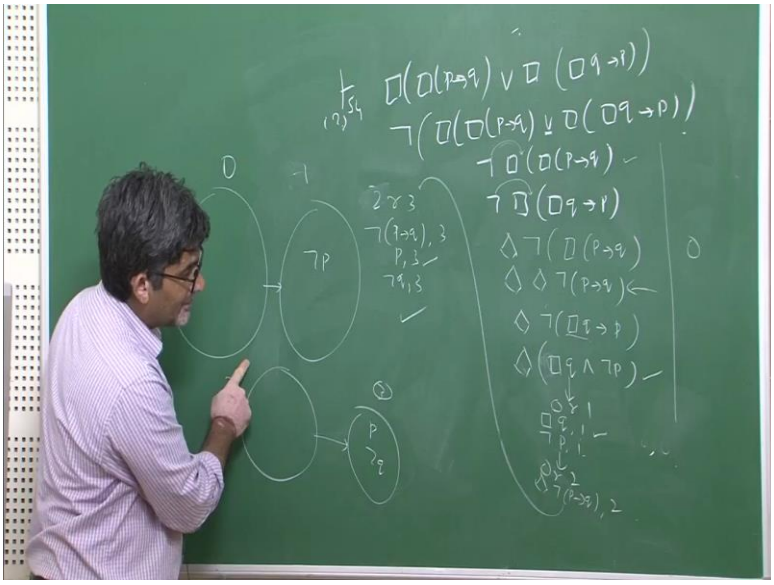
So, B is true in a world 2 not B is true in world 2 now, here if you write like this. So, now, you check it here that you know, suppose if you apply the second rule here. It will become not A is true in a world 2, not B is true in a world 2. So, B 2 and not B 2 this closes and then this remains open, not A 1 not B 1, there is no way which you can close the branch.

So, now what could be the counter example for this one? So, now, we need to see from the open branch open branch is like this. So, what is true in a world 2 not a, and not B is true in a world one, and then B is true in a world 2. That is what we have written and then A is true in a world 1. And it goes for like that. So, now, observe this formula now this is the relational structure of course, you need it written in a systematic manner. So, that is like this first we have set of world 0 1 and 2 where possible worlds that you have and accessible relation is like this 0 is accessible to one, and then 0 is accessible to 2. This is thing which we have and v valuation function is that even atomic sentence is A B. A is true here and B is true. In the world 2 all these things are listed here. So, that is way we have a complete fifty structure.

So, now this could serve as your counter example, now apply this one here, now, if you go the original formula. This is going to be like this. Now possibility of A and possibility of B, possibility of A is true and possibility of B is also true. So, this makes this formula 2 now whatever possibility of A and B. So, now, if I take the possibility of A and B, suppose if we go to this world where A is true, but B is false so; that means, this is going to be false, in the same way although B is true here A is false, here that is makes A and B false. So, now, for that we using, this is not going to be theorem and that is going to be the counter example.

So, let us consider some more examples. So, that you know we will understand this thing in some more better. So, assuming that let us consider some world little bit of complex kind of examples and they are here like this. So, with this we will end this lecture. So, imagine a situation that we have this thing.

(Refer Slide Time: 17:27)



Now, we are try to construct S4 counter examples. So, this is little bit of difficult formula and with this we will end this thing. Necessity of necessity of p implies q necessity of necessity of p implies q, or necessity of necessity of q implies p, we need to be careful about the brackets. So, this tells us that it is a design q kind of formula necessity of necessity of p implies q, where as to be the case r necessity of necessity of q implies p.

So, now how to go whether it is it holds in S4 or not. We will question mark that we have to start with. So, now, for this we need to negate the formula. And this is the one with which we do, and then it is going to be like this p implies q or not of q implies p. So, this is a there is a dis junction here, negation of dis junction this conjunction. So, it is going to be like this negation of necessity, necessity of p implies q now negation of necessity and necessity of q implies p necessity of necessity this thing. So, we need to little bit careful about the brackets.

Now, negation of necessity is this thing. It is possible that not and you need to push this negation here. So, now, this will become formula S4. Now further if you simplify it, it will become this thing negation of necessity, will become possibility of not of p implies q. The same thing we have simplified it, by using modal negation rules. Now this is done now coming back to second. One negation of necessity is possibility and push this negation inside and it will become the first step it will become like this. So, we ensure that we should not write p for q, q for p etcetera sometimes. We will confuse our self and we will write it that way we need to check it carefully.

So, now negation, you put this negation inside and this will become negation. So, we have this formula, x implies y. This is x and this is y negation of x implies y is x and not y. So, that is. So, x is q and not y is this thing not p here. So, this is one which we have. Now all those things are true in a world 0. What is that we are trying to check? We are trying to check whether is given formula holds in S4 or not. S4 already includes t, t rule already. So, we do not need to use this t rule here let see whether you can use it. So, now, possibility of possibility of not of p implies q and here you have possibility necessity of q etcetera.

Let us remove this possibility operator, then you have already have 0 we are moving to a world 1. And in that world this whole sentence as to be true. It as to be true in a world and it as to be a true in a world 1, because not necessity of q and not p is written in this one after another; now coming back to this one. So, the strategy here is that always you work on possibility operators. And then you move on to necessity operators if you understand one particular kind of problem you will you will able to solve all other kinds of problems easily. And you need to note that whenever you have your branch open remains open from that you need to construct a counter example.

Counter example in the sense that your premises are true and the conclusion is false. Coming back to this one, this is the one which we need to see possibility of possibility of this. So, now, once you remove first possibility, we are not supposed to the go the same world 1, but you have to go to a different world 2. And in that world possibility of not of p implies q, as to be true in a world 2. Now once again you apply this one. If you remove the possibility operator, then it is going to be like this I am writing it here. So, if you remove the possibility operator here and this will become. So, right now you are in a world 2 and you need to go to another world 3. And in that world this p implies q has to be true; this one which we need to use.

Now, we have a formula, which says that necessity of q is true in a world 1, but we do not have we do not have anywhere to go from 3 to 1. So, now, this can be further simply into p 3 and not q 3. So, S4 requires that you are accessibility relation has to be transitive or it can be reflexive also. Reflexive transitive holds then that system is called S4 now. So, in this case possibility of not of p implies q which holds into and now you have this thing. So, the problem is that p is true in a world 3, but you have formula not p is not true in a world 1 you cannot close this one, because a formula is to be in one world, another formula is true in another world and unfortunately there is no connection between 1 and 3, because we do not have such thing called 1 is accessible to t and then 2 is accessible to 3 of course, 2 is accessible to 3, but there is no way in which we have this thing 0 is accessible to 1 and 0 is accessible to 2, but we do not have such kind of property that is one is also accessible to 2 in that sense we do not have this particular kind of this. So, that we can say one is accessible to 3.

So, now this will serve as counter example. So, you need to invoke some other kinds of properties like in this case for example, if one as to be accessible to this. One when we have some kind we have seen Euclidean property. That is 0 is accessible to 1 0 is also accessible to 2. Then 1 is accessible to 2. So, that is not available here. So, we are not invoking Euclidean property, equivalence relation involves reflexive transitive and symmetric.

Now, the branch remains open like this. From the open branch we need to construct a counter example. Since we have taken 3 worlds in to consideration and let us say this is 1 2 sorry 0 1 2, and even you have world 3, also what is accessible to what you need to write. So, 2 is accessible to this is 3, 2 is accessible to 3 in that world your p is true and not q is true. So, now, here in a world 1 not p not p is true in a world 1. So, this could this this will serve as your counter example. The only thing you need to do you need to draw accessibility relation like this, but we do not have accessibility relation from now we have accessibility relation from 0 to 2 also 0 r 2. So, in this way this branch remains open and this will serve as the counter example.

So, we will stop here. In the next we will be talking about possible worlds, and what is the there is an important position in with respect to the possible worlds, and it is popular in this days and it is highly debate able and that position is called modal relation.

Thank you.