**Basic Concepts in Modal Logic**
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**Lecture – 02**
**Propositional Logic: Syntax**

Welcome back, in this lecture I will be talking about the language of the propositional logic there is consider, we focusing on attention on syntax. Any language will have it is own syntax and semantics, and then the relationship between in logic what will be doing is that we will be talking about the relationship between syntax and semantics. Syntax as something do with the proofs whereas, semantic is something do with the truth. So, let us begin with what is considered to be the language of minimal things that we need to know, before proceeding further we need to know something about the notation etcetera.

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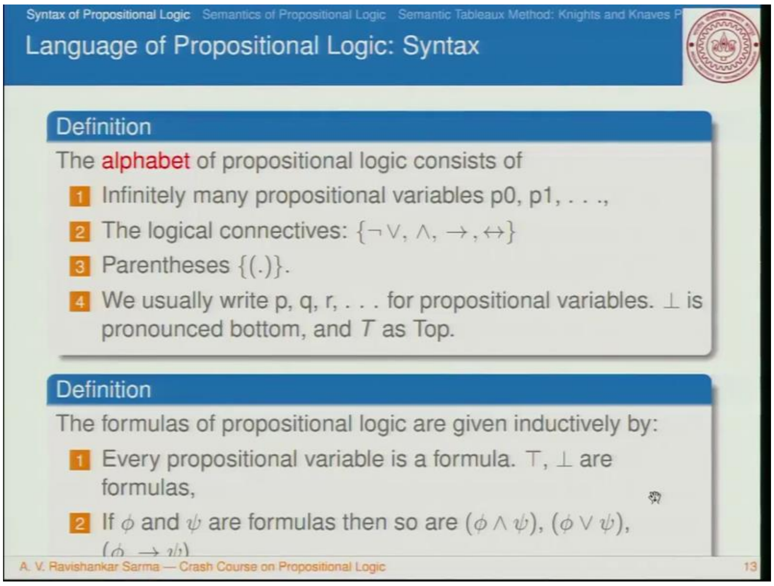


So, any language will have it is own alphabet, an alphabet of propositional logic consists of these things there are infinitely many propositional variables such as P0, P1 P2 etcetera Pn. So, many things are available to us. So, that we can represent sentences the basic unit's of propositional logic are consider with propositional or sentences. Proposition sentences all this things are used in the same way as long as propositional logics are concerned.

So, of course, there are some debates as to what exactly mean by proposition? Especially when you move to moral logics will we will be talking about possible worlds etcetera, but till then as If now, proposition sentences statement etcetera are all same a proposition is considered to be a sentence which is either true or false. So, we cleverly choose the sentences in a such a way that they are either true or false; that means, you can clearly draw a line between for example, let us say mortal and non-mortal that is, why if you observe classical logic the examples are quite clear where you can clearly draw a line between what is the case and what is not the case something is true and something is false those things which can be spoken as either true or false are under over purview.

So, there many things which you cannot spoke about those things to be true or false for example, if I ask someone what is your name or shut the door are you expressing some kind of emotions etcetera, there is no when which we can talk about whether the sentence is true or false there combines some kind of questions etcetera. So, logic can be used in the three different ways it communicates something information etcetera. There one which we are using it and the second one is evokes some kind of emotions. The third one is it talks about some kind of commands etcetera. So, we are not interested in the other two things, but we are interested in declarative kind of statements such as the sun rise in the east or this chalk piece is yellow in color. So, we have infinitely many propositional variables representing all the propositions and we have logical connectives these are the things that we have negation and RN implies and if and only if and just like in a case of English language we need to have some full stops commas etcetera.

So, in this case this is only use for convenience otherwise will have some confusion in reading the formulas? So, we have left parentheses left bracket and then full stop for brackets and right parentheses and we usually right p, q, r etcetera. Propositional variables and there are two functional and there are two symbols that we will be using it bought [FL] T that is stands for those sentence are which are always false or contradictions and T there those sentences which are always considered to be true are represented as T. So, there are tautologies. If we can classify the our sentences it can be like this that there are some tautologies and there are contradictions there always, false tautologies are always true and there are some sentences which can which are sometimes true sometimes false they are called contingent statements.

Now we have these alphabets and then we know how these alphabets combine together and founds some kind of strings meaningful some kind of strings which are considered to be well formed formulas just like say in the case of English language. We have known whatever alphabet we have it will not combine in a form a meaningful word. So, c a t makes sense to us t a c. For example, make any sense to us in the same way in the case of formal logic in the case of propositional logic this strings will combine certain way and form meaningful kind of things. So, these are the definition that we have how can we say that a given formula is considered to be a well formed formula this well formed formulas are generated by p, q, r, n then the logical connectives and implies r etcetera and then we have parentheses left bracket and right bracket.

So, every propositional variable is considered to be a well formed formula suppose, if you write p and q etcetera there is also these are considered to be well formed formulas and this Logical Constance. Like top and bought are; obviously, well formed formulas and there are two well formed formulas phi and psi then anything which you combine in this way only in this way will become well formed formula otherwise it is not considered to be a well formed formula. For example, phi and psi are well formed formulas; obviously, conjunction and disjunction are also considered to be well formed.

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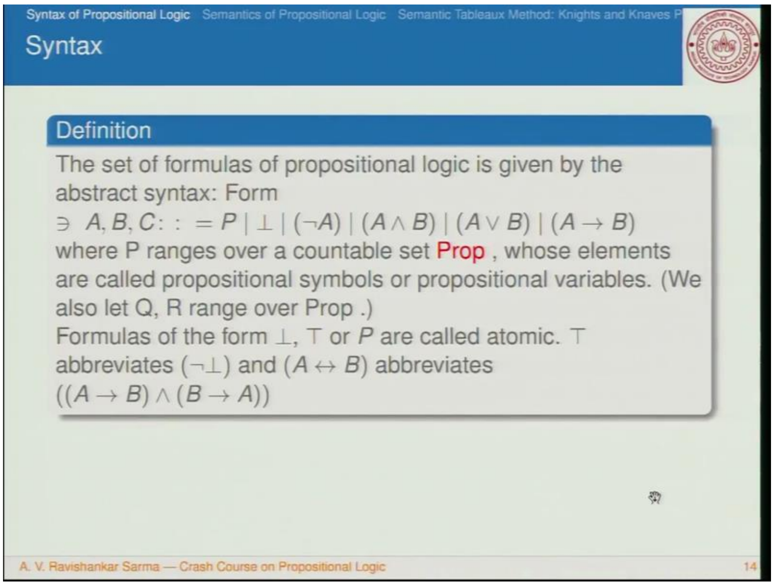


This is a routine kind of things that will be noticing this case. So, the language of propositional logic is like this it consists of A, B, C some formulas which consist of propositional variables and then something which is consider we contradiction negation of a and a and B are combined the help of conjunction a and B a or B a implies B and A. If and only if B where p ranges over counter effect of propositional variables and whose elements are also called as propositional symbols or propositional variables we also let q and r to range over propositional variables.

So, formulas of the forms single formulas like bought top or p are consider to be atomic propositions. So, these are the propositions these propositions combine with the help of a logical connectivities and form compound propositions. For example, a is atomic proposition B is another atomic proposition these two make a combine various phase and form atomic sentences like a if an only B which is interpreted as a implies B and B implies A.

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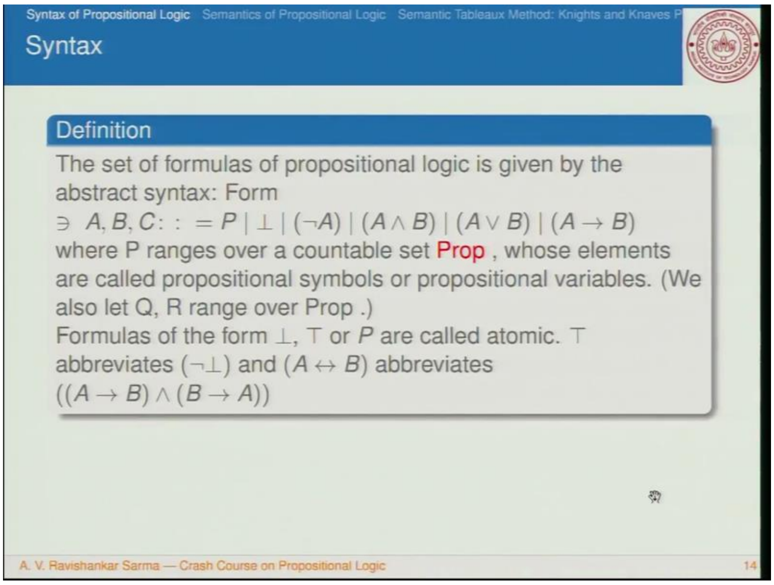


There are some kinds of conventional that we follow in the case of propositional logic and which are like this, we do not use excessively this outer parentheses etcetera you can just simply remove it for example, you have a formula like this.

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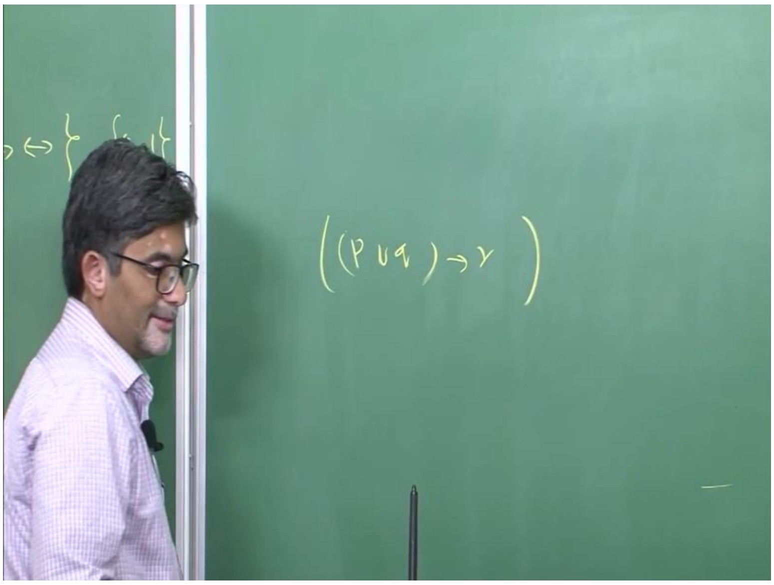


P or q implies r you do not have to use this outer brackets and you can simply talk about it has this thing p or q are r. So, outer parentheses can be simply removed from this one and there is another way in which is important here that is this.

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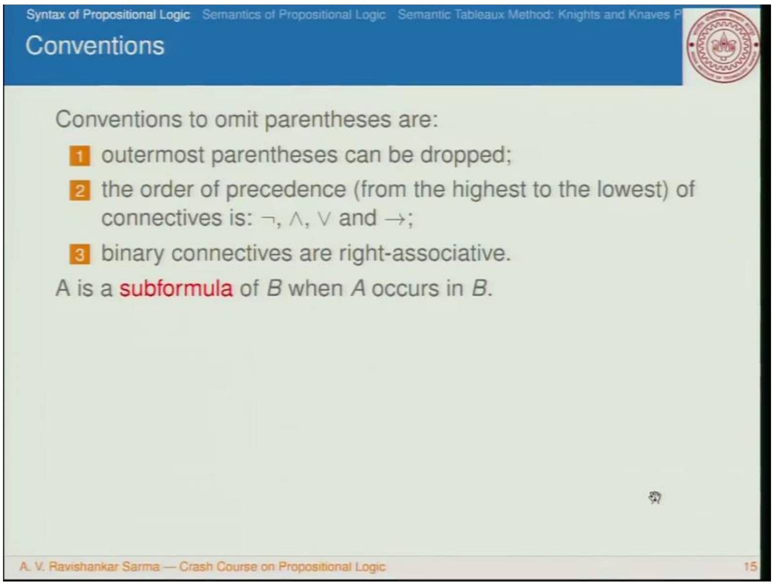


There is some kind of order of precedence that, we follow that is like this first you start with the negation and conjunction disjunction implication and if and only if.

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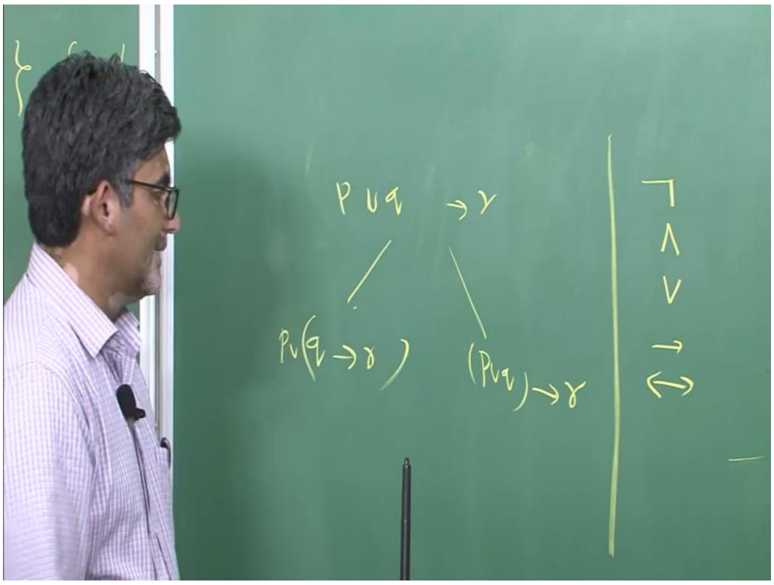


So, why you required these things in the case of propositional logic; for example, if you have a sentence like this, where we do not know where we need to put our brackets in any good text book can logic it will have all the parentheses this is given suppose if you do not somebody writes a formula like this then there are three two ways in which you can read this formula the first way which is that p or q implies r; that means, p it is combined with another statement with r p or q plus r or you can read this statement in a different way that is p or q implies r. So, these 2 formulas are totally different. So, how do how can we say that, when you have a formula like this, all of us read it in the same way such as we are following some kind of conventions. So, these conventions and the convention that we follow are like this.

So, the idea here is that every formula in propositional logic has it is own tree structure suppose if you draw a tree diagram for a given formula. It looks different for different formulas if you looks same then these two formulas are considered to be logically equivalent to each other. So, here two formulas that we have now we do not know, how to figure out.

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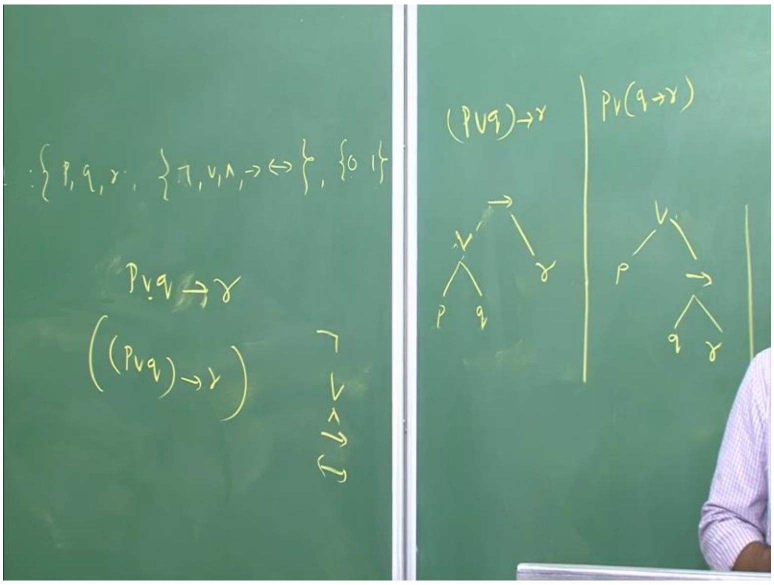


How to draw parentheses here p or q implies r. So, now, if you say it p or q implies r and it has some kind of tree structure and if you say p or q implies r, this will have it is own structure tree structure. So, every formula comes of it is own corresponding tree structure you know two formulas will have same kind of tree structure. So, this formula needs to be represented like this now we give some kind of preference to these connectives.

For example if you have a formula like this now how do we right in a proper way. Now, the first preference should be given to negation which is not there in this formula and the second preference should be given to or so now, wherever this or occurs you just put a bracket like this now the next preference should be given to and is not there here implication if and only if. Now, we have implication here and then you put bracket here. So, that is the way in which you can read this particular kind of formula. So, these two are the two formulas p or q implies r and p or q implies r. So, these two have different kind of structure. So, this is this needs to be write like this p or q and then. So, this is r. So, this is considered to be the tree structure for this one it has to be read like this p or q implies r.

Now for this it is like this, or p and implies and q and r. So, this is where you can draw a diagram for these two things. So, this stands for this p or q implies r that is the way you read the formulas so every tree ends with atomic formulas. So, these two diagrams are different; that means, every formula will have it is own tree structure no tree no two formulas will have the same kind of tree structure. This is the convention that we follow, but in the most of the logic text books all this parentheses are given if it not given to us then, how do we read the formula in the same way we follow this precedence first you give importance to the negation and then followed by that and or implication and if and only if another important thing we need to notice is, that in our order of preference we started with this thing and or implies and if and only if.

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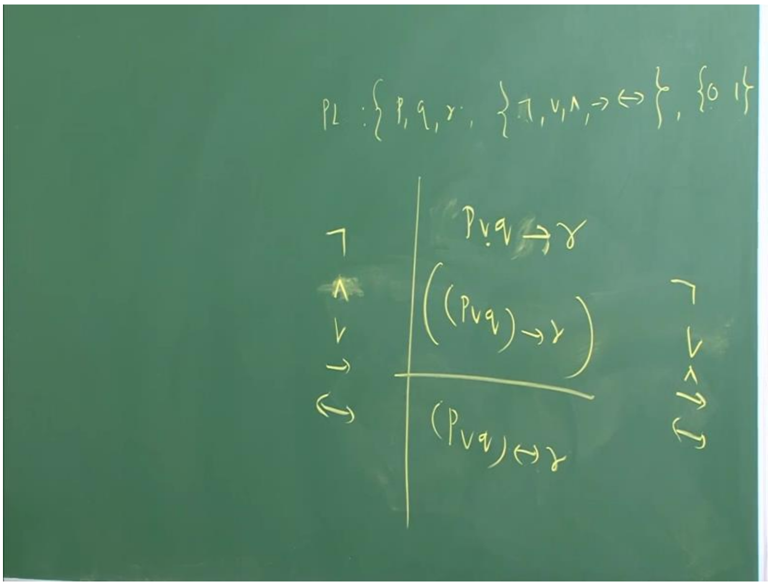


Now in any given formula let us say it is p implies q or if and only if r, suppose if you have a formula like this then the lowest precedent that we have is if and only if, now, this will become the major connective. So, why we are talking about this precedence it is just to find out what is considered to be major connective in our given formula, while constructing the truth table this is considered to be very important. So, the one which occupies the less precedence is considered to be our major connective here, this way you read out the formulas in propositional logic the same thing applies when, we talk about modal logics. We can still talk about the same kind of thing and other important thing we need to notice is that binary connectives like or implies and etcetera are considered to be right associative they always operate on a right end of thing. So, we have something called as sub formula particularly when any given formula occurs in any other formula for example, if you have A implies B and C, B and C is one formula and then total A implies B and C is another formula.

So, B and C is considered to be sub formula of that thing for example, if you have this thing not p and then all this brackets etcetera

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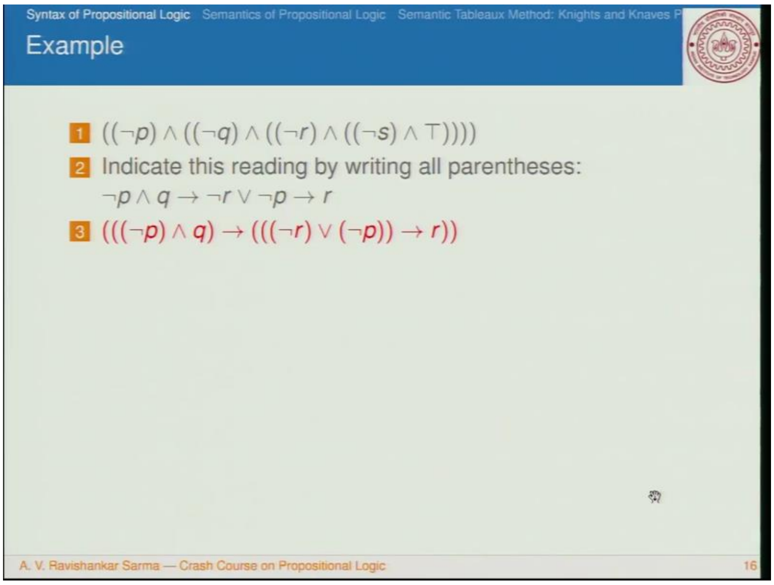


We indicate this reading we are writing all the parentheses that is like this not p and q implies not or p etcetera. So, this will look like this thing. Suppose if you have a statement like whatever is your saying it and point number two bullet number two we give first precedence should be given to the negation and then followed by that and then implication etcetera. Then ultimately this formula becomes like this as in as your seeing it in 3. So, this way we construct in this way we can say that you know any given proposition logic proposition logic will have it is own syntax, this syntax is generated by this 5 logical connectives implication negation and or etcetera.

Thank you.