Basic Concepts in Modal Logic

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Lecture -16
Semantics of Modal Logic: Relational Structures

Welcome back in last lecture we have seen the language of modal logic was we have seen how to translate a given English language sentence appropriate into given language of modal logic. And then when we can say that the sentence is analytic and when it is analytic how we can translate into the language of modal logic, when it is contradictory how we translate into the language of modal logic this is some of the things which we are seen earlier. And we also seen how to draw passing tree for a given modal logical formula and we have seen that no 2 formulas has same synthetically structure, if they have and then they are logically identically to each other.

So, in this lecture we will be talking about so the axiomatic system does have been developed from the year 1930 to 1960, and these are widely studies modal logical systems and this are due to some legislation or the. We start with minimal logical system K and then we will move on to other logical systems which are constructing to be extension of K. So, anything which is invalid in K, we always are automatically involved in all other logical systems because all construct to be extensions of K.

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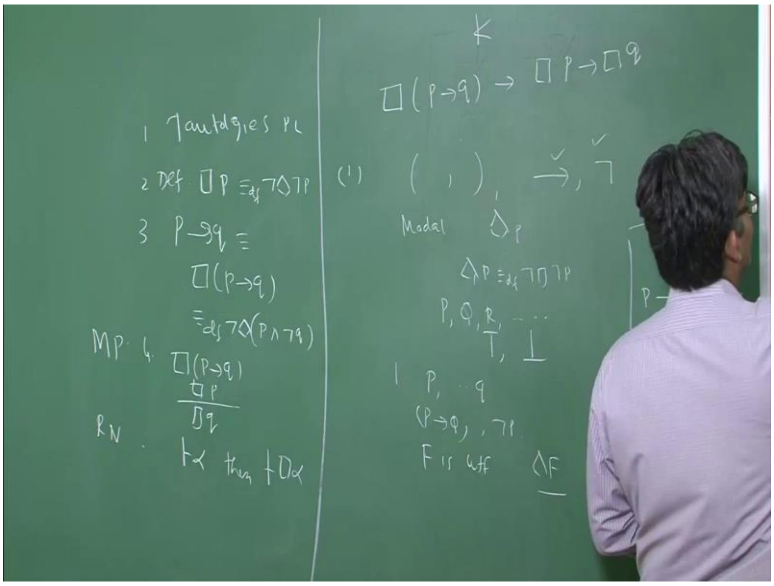

We have all the tautology of propositional logic there intact her. And second thing is that we have some definition they are like this necessity of P my definition it is same as it is not possible that not P is the case. And, strict implication defines P sense. P implies q, it is written as P implies q which is consider to be necessary and it is by definition it is same as it is impossible that P is true and q is false.

And there are some other rules which consider more response rule which states that it is necessary P implies q. It is necessary that P hence, it is necessary that q and then we have rule of necessitation, which tells us that if something theorem than obviously, it is necessary that alpha is the case and this are the some of the rules we already have.

So, now going to be there in our minimal modal logical system K? All this things are intact for us the only think is different in the case of modal logic are you have set implications and we have rules like this thing modus pones and we have rule of necessitation. So, now, K has this particular characteristic axiom which tell us that P implies q if the case than necessity of P and necessity of q. For, now any given axiomatic system what we need to have is this thing four thing is to require the first one is that we need to have some kind of a parenthesis; left parenthesis and right parenthesis this is used to read the formula in a better way.

And then we need to begin with at least 2 connectives in our case we being with only this two things, the choice of our connectives is can be any other thing for example, (Refer Time: 03:59) they have taken into consideration this 2 connectives because, implication can be define in the sense in there logical system. P implies q is defined in terms of r and negation like this not P R q r P and not q so this needs to be removed.

So, you need to have at least 2 logical connectives implies negation r you can take even r are negation in our case we have taken implication and negation. And then one particular modal operator we need to have. So, now, we are taking into consideration possibility of something P. So, these are unary operator we can vary will choose necessity of P also.

But, just enough to take 1 operator because possibility of P can anybody can be define as this thing this is not necessary that not P that the case. Now, in addition to that we have some proposition variables P Q R S etcetera they represent some time of sentence that it is raining or sun raises in the east etcetera all stands for some kind for atomic sentences represented by proposition variables.

So, now in addition to that this appears to any axiomatic system, if you are developing any axiomatic system we need to ensure that we need to have these things. So, now, we need to know now this proposition variable combine with the help of these logical connectives and then we will be generating various formulas. So, this formula these proposition variables cannot combine in whatever way we want, but it has to follow some kind of rules and these rules are like this if we simply write P q etcetera, than it is construct to be a well formed formula.

All the proposition variables are considered well formed formulas. One thing we need to mention it here, there are 2 symbols that will be using it T stand for the statement which is always true and the bot ulta T stands for the sentence which are always constraint to be false say we use this things especially the proposition logic like this starting from a you end up with the contradiction that means, it has to be not a rather than a.

So, we have these proposition variables if you just write like this it is contract to be well form formula. So, this are the logical connective that we have, P implies Q or not P is also consider to be a well formed formula. And, then it is a well formed formula of course, instead of P I will take f into consideration and it does not make any big difference.

So, we can represent this atomic sentence are there (Refer Time: 07:01) it is all over's convenience. If F is a well form formula than so as this files, possibility of F is also going to be a well formed formula; why we require this things we want to tell the computer that this are the things that we exact thing that we are explicitly stating ever thing on the board.

So, now this all the things that we have and then F implies G, P implies q is also well form formula. So, now, in this case I will change it to F and Gs.

So, now this steps these are all formation rules is also called as well form formulas the rules to generate well form formulas. Just like in the case of natural language cat is on the mat make sense to us if he is mat is on the cat something like that it make little bit of sense, but if he say mat cat on the etcetera it does not make any sense to us it has to combine in certain way and then generate some kind of numerical formulas here in this contest we call it well form formulas.

So, in addition to T we need to have some kind of axioms and these are all axioms in propositional logic.

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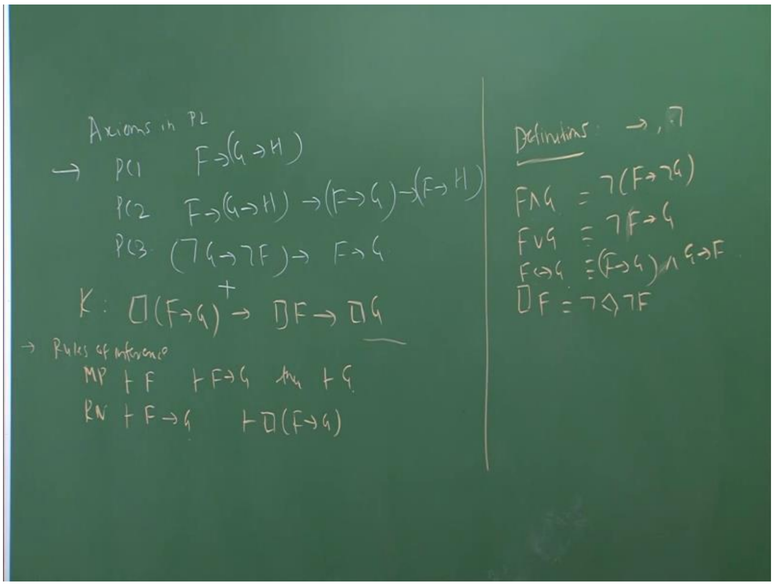

Now, we take a particular kind of 3 axioms we take into consideration DC1. So, this is like this F implies G implies H. And PC2 if you deny this formulas and contract tableaux three all the branches closes this is a tautology. F implies G implies H implies F implies G implies F implies H so these are second one.

And third formula is this thing not G implies not F implies F implies G. So, this are the things we have plus. So, this is the one additional thing, but we have and this is contracting to be axiom K. Which tells us that F implies G implies it is necessary that F implies it is necessary G. Now, in addition to that thing we have some kind of rules of inferences which we have stated it already they are like this: suppose if F is consider to be a theorem F implies G is also theorem when if we have G in the case this is moro sponus rule it is applies to human normal modal logical system also.

In same way F is considered to be this is moro sponus and this is the rules of (Refer Time: 10:05) if F implies G is already a theorem then the city of F implies G is also consider to be a theorem. So, the list is big like this first everything you stated it explicitly now we have some definition.

So, what we are going to do with all this symbols etcetera. So, what we are trying to do is that using the axiomatic system K and following the rules and the axiom we will be generating some kind of theorems. Now, I will not be exhaustive proving the all the theorem, but I will be considering some examples. So, that rest of the examples you can do it on your own.

So, these are some of the definitions F and G. Since, you have only implication and negation are there in our logical system we began with the implication and negation. Now, we need to define convention (Refer Time: 11:05) and by implication so this are the things we need to define.

So, F and G we need to decide in terms of implications. So, that is negation F implies not G and F R G you have to define in terms of implications. So, that is negation of F implies G. F implies G is simply F implies G and G implies F. So, this are some of the definitions and other definition is this it is not possible that F.

So, these are the things that we have everything we have stated on the board. So, now, using only these things now we need to show whether particular things consider to be a theorem or not. So, let us prove some of the simple theorems within the modal logical system K then we will see some extensions that how other theorem can be code in other systems. So, now we are trying to prove this particular theorem now this I am stating it here. So, this is simple theorems which tell us this. Suppose, if you want to prove this things we have something like if F implies G is a theorem.

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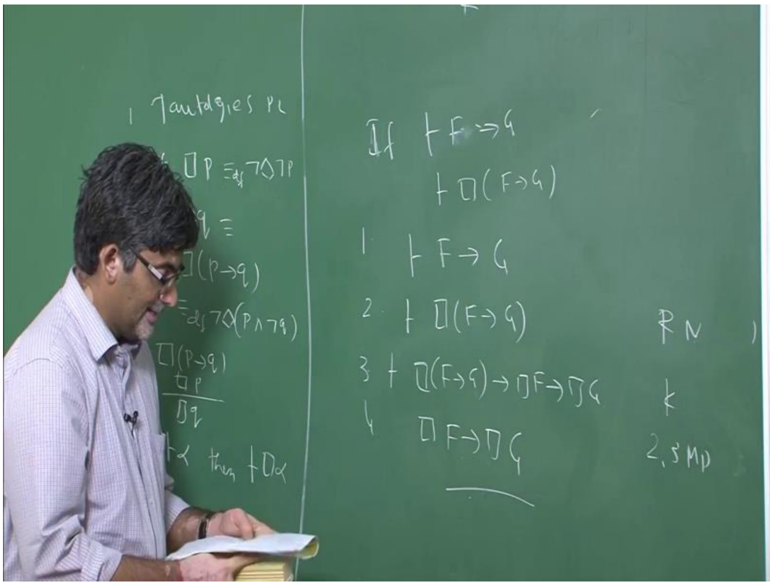

Now, we need to prove necessity of F implies G is the case. So, how do we how do we prove this particular kind of thing. Suppose F implies G like thing and you have to prove necessity of F implies G. So, now, we need to begin with something like that some rules etcetera.

So, what is given to us is this one as a first step this step has to be very clear and justification needs to be written on the right hand side. So, this is what is given to us this is already a theorem and is given to us F implies G. Step 2 we use rule of necessitation then using rule of necessitation we can say that F implies G why because alpha is the theorem than by rule of necessitation it is necessity of alpha. So, now, 3 we need to use only those things which are listed on the board we should not borrow any other axiom here.

So, know we use axiom K. K tells us that that is F and G F implies G is necessity implies F implies that is G, now, this K axiom. So, we need to justification written on the right hand side if we do not write justification than things will be difficult your reader did not be able to understand it.

So, now, 2 and 3 Moro sponus you will have this things. Now, this is what you need to prove. So, we started with F implies G we need to prove simple thing like it is necessity that F implies it is necessity that G. Here, simple side forward things. Now, let us try to show you another you know theorem and then we will move on to another axiomatic system like this you can keep on proving various theorem. So, idea here is that first you need to list out you need to choose the logical connectives either R negation etcetera, implication negation etcetera. R which is 1 particular modal operator that is possibility or you can even choose necessity also and then you have the definition uniform substitution rules Moro sponus rule of necessitation and the characteristic axiom that is consider to be most important. So, now we are trying to prove 1 more theorem which we will try to see here and then rest of things do it on your own.

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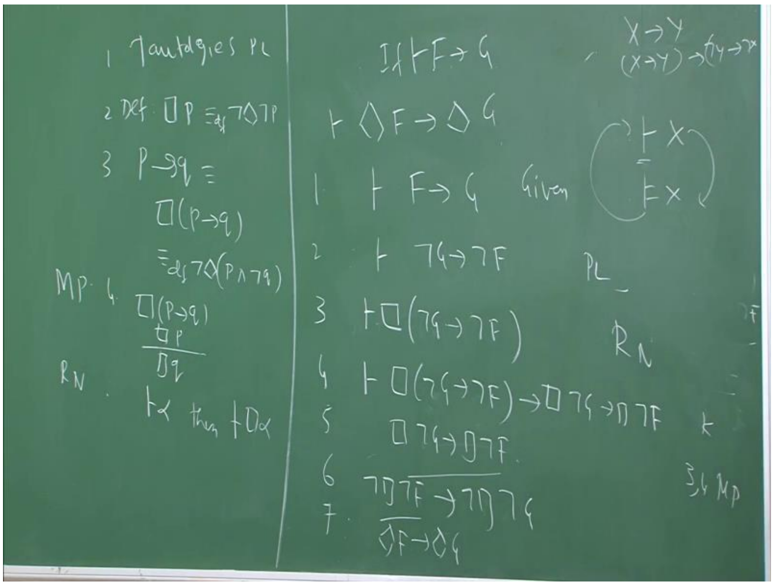

So, if F implies G is consider to be a theorem you need to note that when you write like this means it is a deduction you are reducing something. So, nothing is there on the left hand side that means, X is consider to be theorem or it is shown to be the case than not G implies not F there is some classical tautology this tautology in classical logic classical counter position F implies G in the case not G implies not F this also a case.

So, in that sense so this can be written as not G implies not F. So, actually it should be written like this F implies G implies not G implies not F and you have to apply moru sponos to one and this one and you will get this not G implies not F. I am directly writing it here. So, now, 3 if not G implies not F is a theorem than this is also consider to be a theorem. Again, rule of necessitation is the one which you have used it here. So, this is what the rule of necessitation is.

So, now, fourth one this by using K axiom this can be simplified as this thing in fact it is not G implies not F the necessity of not F implies necessity of not G implies necessity of not F. Now this is rule of necessitation we need to write it here this is PL. PL means already a theorem in PL this is what is given. So, now, this 2, 3 and 4 Moro sponus this is K axiom or it is insist of K axiom it is directly not K axiom, but it is insist of K axiom.

Now, fifth one 3 and more sponus you will get not G and not F. So, now, this is again in the form of X implies Y. So, this is like this X implies Y. So, now, from this X implies Y nothing, but not Y not X, X implies Y not Y implies not X. So, now, this in that sense can be written like this. So, not F implies not of necessity of this thing and not G. So, now this is same as possibility of F and this one it is not necessary that it is not G implies it is possible that G. So, under a 7 step we could prove this particular kind of theorem.

So, now like this you can keep on proving that does not make any sense to prove N number of theorem in this class room. But, I am just giving you an idea the idea here is that we have to use we can use all the tautological of classical logic and you can use the rule of necessitation to transform appropriate into the language of modal logic whatever is require here and then use modus pones and ultimately you will reach your destination and one which you are trying suppose to prove.

So, like this till 1960s people where (Refer Time: 19:21) constantly trying to prove theorem within the given moral logical system. So, now, so far we have spoke about on the logical system K. So, now, there are different other logical system which are come into existence and this are like this. So, there are K T D 4, 5 and B.

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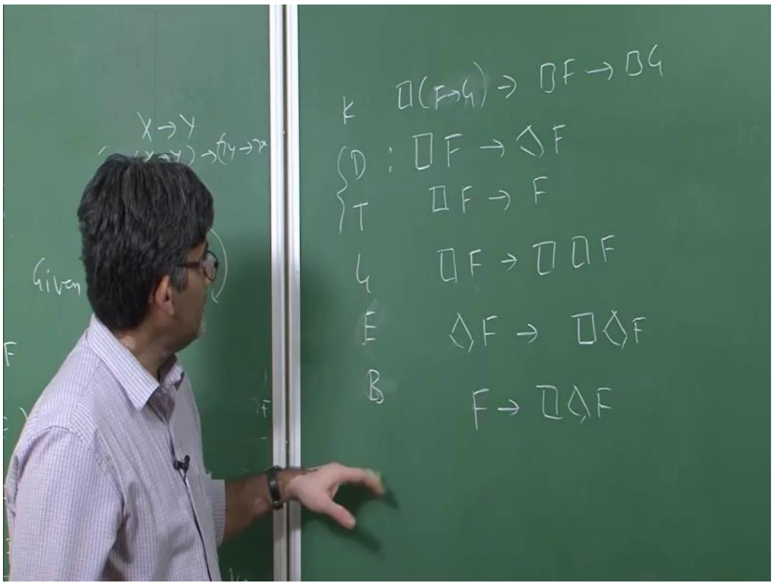

So, now K has this characteristic axiomatic that is what seen here. So, this include all the tautology etcetera together with this you have K P implies q write it as F implies possibility sorry necessity of F implies necessity of G. Now, D, D has this characteristic axiom. So, that is this thing it is necessary F implies it is possible that F.

You have to follow the traffic rule may implies you are not actually following the traffic rules it is possible that you follow the traffic rules. So, this related to deontic logic. So, in the slide D system has come and then T, T has this characteristic adjacent which talks about this things necessity of P and necessity of F has to necessity of F implies F something should be consider has to be actually true.

Now, there are other axiomatic systems that we have. So, the thing is that here D was introduced first then T has come little bit later. And these 2 define this axiomatic system D and T. There is something call as 4. 4 has this particular axiom it is necessary that F implies that it is necessary has to be necessary. So, this is what the case is.

The fifth one if something is possible if F implies that something is possible if F has to be necessary. So, you might be wondering how various logicians have came up with this various axiomatic system. So, in the process of studying various situations that you need to fit it into this logical systems for example, in this case suppose if you translate this thing as possibility as I thing that something is the case are necessity I know that P is the case.

So, now, this is represented in the epistemic logic like this something is known to you, but whatever is known to you is also know to you. There are known's. So, that is what is the case this is also called has K hypothesis. Something is known means it is known to you. So, in the context of this thing we require these additional characteristic axioms for given system this is E something called E.

Now, we have something call as B. B has this corresponding axioms F implies that it is possible that F has to be necessity. So, these are some of the characteristic axioms with respect to various axiomatic systems K D T 4 E and B etcetera. Now in the next class what we are going to do is that we will introduce Kripke semantics and then we are going to see how Kripke has come up the validity of a given modal logical formula just by means of some kind of relational structures. It is from 1960 onwards there is emphasis on the relational structure which we call it a Kripke diagrams or we call it as a Kripke frames, Kripke modals. So, in the next class we will be continuing with Kripke semantics and then where we will be taking about the meaning of a given modal logical formula.

Thank you.