Basic Concepts in Modal Logic

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Lecture - 14
Language of Modal Logic 2: Syntax

Welcome back, in continuation to the last lecture, where we have seen how to represent various mean how to represent modal sentence and we spoke about, what are these modal, sentences where it occurs, for example, counterfactual sentences and other kinds of sentences. So, where will be mostly focus on our attention on alethic modalities, which refers to necessity and possibility of any given statement.

So, in the last class we just started with the syntax of modal logic, that is language of modal logic it has all the logical of propositional logic and then together with that we have two different kinds of unary operatives possible that p and unnecessary that p is the case; now, just as in the case of classical logic.

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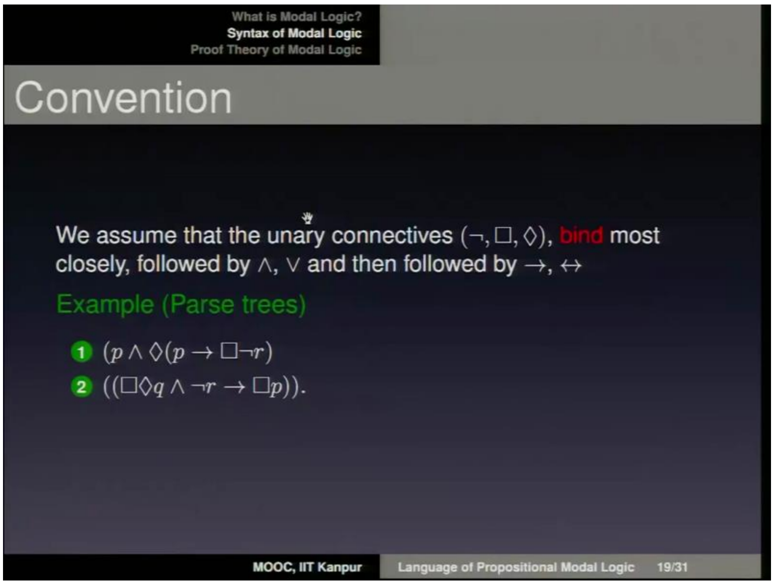

We follow same kind of convention when, whenever you come across modal logical formulas like this for example, if you have a formula like this one for example.

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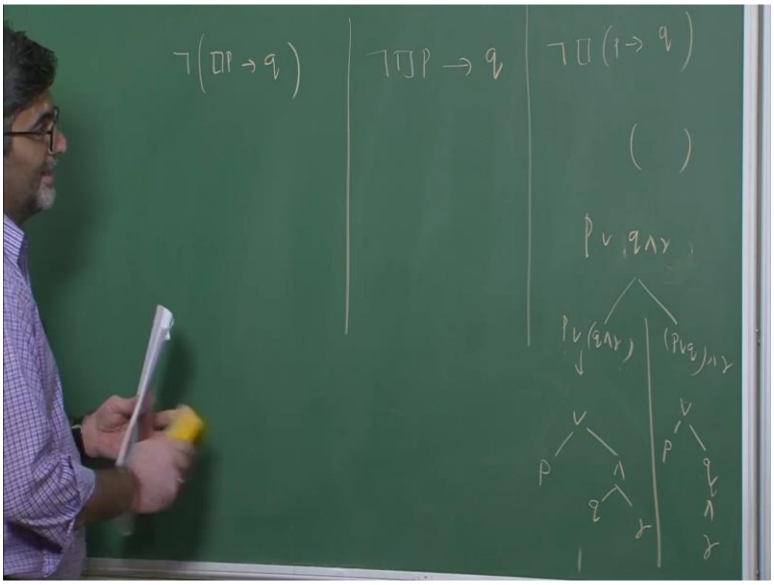

Let us take a simple example where we have a formula, it is not necessary that p implies q. Now, this formula suppose if there is no parenthesis which is given to us for example, you can have another formula like this it is not necessary p implies q, this could be another formula and then, you can put negation here and then you talk about necessity of p and q for the same kind of formula it is necessity that p implies q, you can have three different radiance.

So, now all these formulas how do we know that these three formulas are consider to be different? So, now, the passing tree comes into picture. So, in the case of propositional logic, we have said that suppose if you have a formula like this p or q and or suppose, if we did not any parenthesis.

Parentheses are left bracket and the right bracket. So, now, this formula if, you do not give any parenthesis here then this is viewed in a difference sense that is like this, which it can be read in two different ways p or q and r or you can just write p or q and r. So, this two are different formulas. So, in that context how do we know that these two are different formula, then we are drawn some syntactic trees for this formulas and then we have seen that they have different syntactic structures.

So, now the first formula can be written like this. So, usually when you draw a syntactic tree, you will end up with atomic propositions. So, this is going to be like this and q one second. So, this is going to be for syntactic tree for this one, is going to be like this. So, this going to be q and r and followed by that. So, we have sorry it should be drawn like this one second. So, now, we just want to say that these two formulas are different p or and then n q and r. So, now, in a syntactic tree this is the way of representing this particular kind of formula, it should be read in this way p or q and r. So, now, this is the syntactic structure for this formula. So, now, corresponding formula of syntactic tree for this formula is going to be like this. So, we have p q and then p or q then come n r, q n p r, q n r. So, of these two are having different syntactic structures.

So, for that reason you need to note that, that is why in the case of propositional logic two different formulas will have different syntactic trees. There is the way of drawing this syntactic trees you need to ensure that in the end of this notes. I mean in the branch, we always have atomic propositions. So, now, these are also called as parsing trees this gives us indication of how to read a given formula.

So, this reads p first and followed by that r and then q and then n and r in that way the computer reads this particular kind of formula. So, now, this formula first computer reads p and then followed by that r and then q and then followed by that n and r. So, just change in the case of this one, in the case of proposition logic we have different syntactic trees note 2 formulas we will have same kind of syntactic structure that makes this 2 formulas different.

Now, these three formulas are considered to be totally different. So, now, let us drawn syntactic trees from these formulas and see how we can say that these three are different set of formulas. If the syntactic trees matches are the syntactic structure matches, then we are talking about the same kind of formula.

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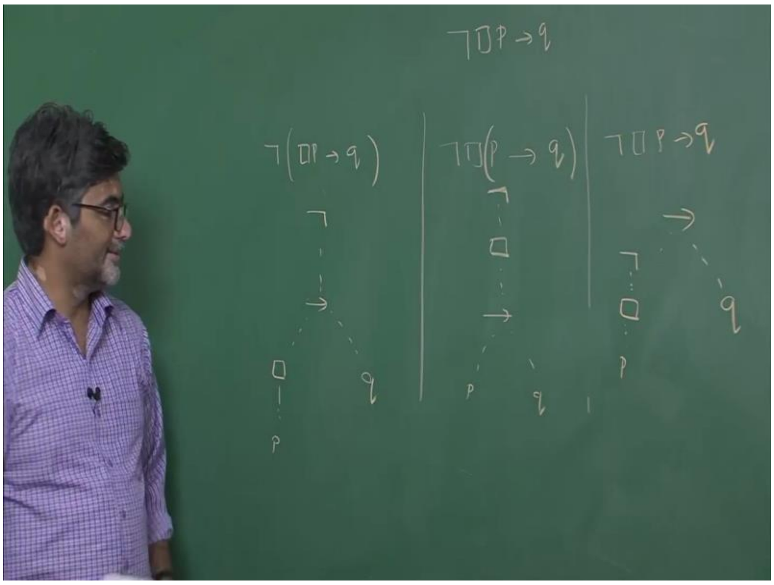

So, now the first one I can be written like this. So, now, the major connective is here it is implies now followed by that we have necessity and we have p. So, this is necessity of p implies, on the other hand we have q then, the whole thing is viewed this way. So, negation of necessity of p implies q. So, now, this is the syntactic tree for this formula now for this one again necessity is considered to be the major kind of connective. Now this connects p and q, in the syntactic tree you always end up with atomic propositions p q. So, this is taking care of that one and then followed by that you have necessity and then negation of necessity of p implies q is this one.

Now, when you when you come back when comes to this one implication, then negation then necessity and then p So, this is can be read as negation of necessity p, now implies just one second this is for this one just one second necessity of t implies q. So, this is going to be like this.

So, now, the main connective is necessity and then you have to write this formula, ultimately you have p here this way it is here and then necessity and then negation. So, that is one thing and then followed by you have q here. So, now this one p implies q is the one which you have written here and then for the whole p implies q necessity operator operates and for whole necessity p implies q is negation operates. So, negation of necessity operates on the whole formula p implies q, that is what is this formula.

So, now, observe these three formulas. So, now, we can say that you know this has the different structure then this one and this is totally different from the other one. So, now, the same formula negation of necessity of p implies q. If you change the brackets, it make me a different formula, how of course, we talk about semantics of this particular kind of formula we will see what are the truth conditions of these three formulas these three formulas are considered to be different formulas, where since we are talking about syntax. So, structurally these 3 sentences are considered to be different, because they have different syntactic structures.

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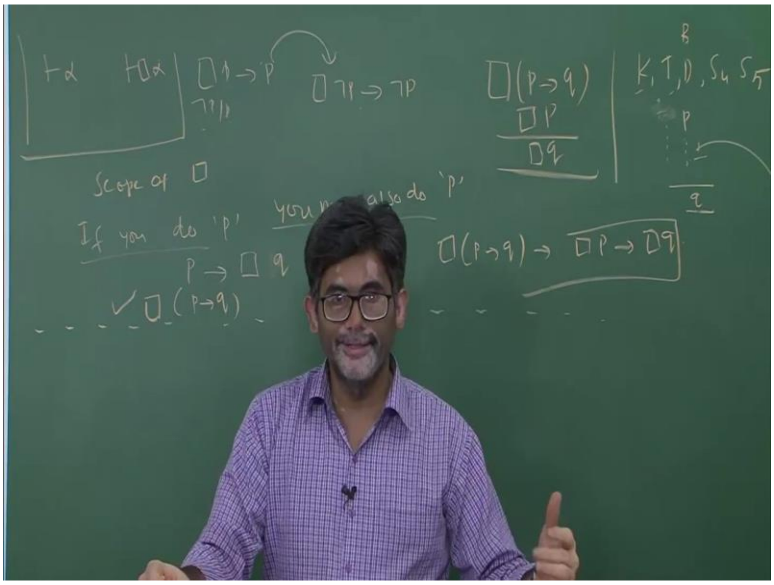

And there is one more important thing which you need to note that is the scope of necessity of return. So for example, if you give an example like this, if you do p if you do action p then you must also do p. So, this sentence can be if you literally translate this sentence, this is going to be like this. If you do p is represented as p and then you just translate it as you must do also p of course, you represented it as something like q necessity of q.

So, now the question arises to us is that where to appropriately inside this necessity operator does this necessity operator operates on the whole conditional first of all this is a this is considered to be a condition. So, this involves the necessity your consequent has this particular kind of necessity. If you go with literal translation this is going to be the translation of this one, but in the case of modal logic, necessity operator it is only again the convention, this is that it operates on the whole conditional rather than part of the conditional it will not operate just on the consequent of your conditional whenever you have a sentence like this if you do so. If you do p then you must also do p this is appropriately represented as this one this has narrow scope, but it has wider end of scope. So, this confusion has been there for a long time.

So, right from the antiquity till the development of modal logics this kind of confusion where to put this kind of necessity. So, when we talk about Aristotle, see battle example that little bit later then we are going to see two different translations for that particular kind of example and then see, we have going to see that under one particular translation this the argument is going to be valid in other case it is going to be invalid. So, we are going to see after a few classes.

So, this is usually this confusion was almost there is no such kind of confusion especially when there was since introduction of k axiom which is also considered to be a distribution axiom. So, this tells us that p implies q is necessity then individually p is considered to necessity and then q is also considered to be necessity. So, this is the first thing that you need to note with respect to modal sentences in modal sentences. If you come across a conditional you should ensure that your modal operator operates on the whole condition rather than part of the conditional.

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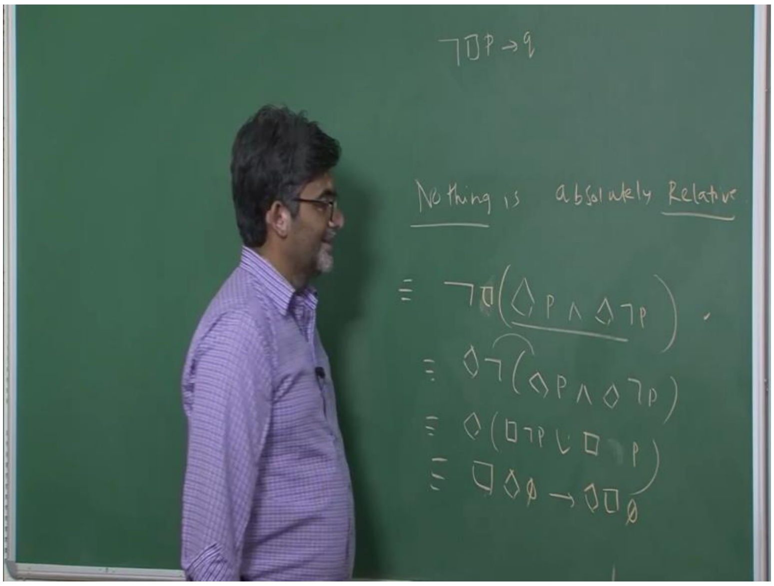

And now coming back to some translations now, how do we translate a some kind of complex sentences like the this one there are few other examples, which I will go through it, but let me write down one of the things of an have confusion nothing is absolutely relative . So, for this we need little bit translation now. So, you say that something is considered to be relative especially when something is possible and the same way something is not possible also. So, if that is the case then your p is set to be contingent or you can also be called relative.

So, now nothing is considered to be absolutely relative. So, how do we translate this particular kind of sentence this is considered to be relative absolutely relative nothing nothings takes care of this one. It is not necessary that something which is considered to be absolutely relative you have to rule out this possibility then this is considered to be the translation of this one.

So, now, this can be appropriately translated into negation of necessity is nothing, but this one necessity of p and sorry possibility of p and possibility of not p. So, now, this can be if you push the negation inside then it is going to be this negation of possibility will become this thing and negation of conjunction will become disjunction then it is going to be something like is negation of possibilities is necessity and you push the negation inside it is going to be like this negation not p is p.

So, ultimately this gets this appropriately translated into this famous axiom which is also called as Mackenzie axiom. So, this is like this it is something which is possible that phi is necessary implies. So, that it is necessary is considered to be possible. So, that we read it like this something which is possible that phi is necessary implies whatever is necessary is considered to be possible.

So, nothing is absolutely relative is nothing, but some kind of axiom. So, this is listed in one of the important books that we read it in this course that is the title of the book is interesting modal logic for the open minds; in that book this is listed. So, there some of the interesting translations and there is one more thing which we will be a coming across in the process of translation, that is we are not going to cover it in our course, but this distinction is considered to be important.

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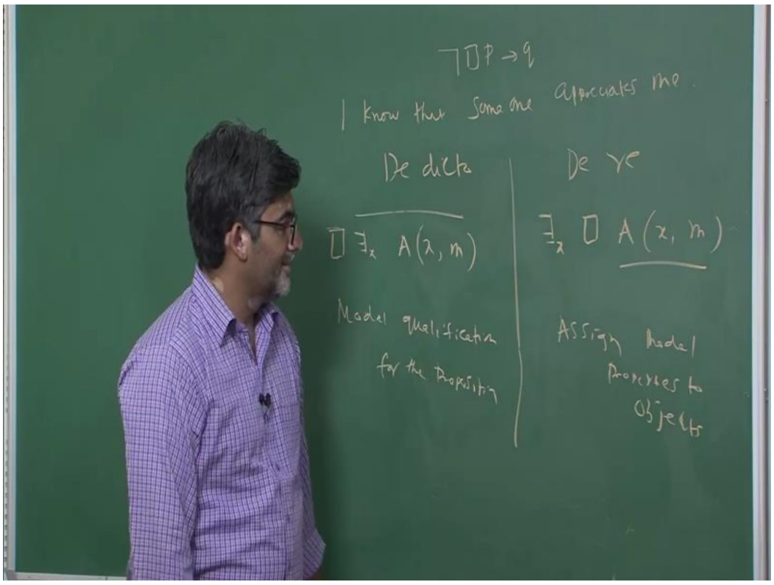

So, that is these particulars distinction De Dicto and De Re distinction. For example, if you say that I know that someone appreciates me for example, you say this there are two different readings for this, particular kinds of sentence. So, one particular reading is De Dicto reading in that you write it like this it is necessary that that exist some x a is considered to be a property that property here is being appreciated by someone. So, when you talk about predicate logic we do deal with. For example, if you say that all humans are mortal all humans are considered to be subject and then being mortal is considered to be the predicate.

So, this say sentence can be appropriately represented as this thing x if x is human then x has to be mortal. So, this happens for all x. So, now, in this case I know that someone appreciates me is translated into this thing there exist some x says that appreciates x appreciates me here means m.

So, this stands for there exist some x that is someone x x appreciates me. So, this is written in this way. So, this is the first. So, here you need you it is to be noted that necessity operator comes first and then followed by that you have codify and followed by the predicate later.

Now, in the case of de re it is like this. So, here we have modal qualification for the proposition suppose thing is considered to be some proposition. So, it is qualified by this modal proposition it tells as that it is necessary that there exist someone some x. So, that x will appreciate me. So, now, in the case of de re distinction, now we have in the other way round it is like this. So, it is there exist some x, that it is necessary that it is that x push its me x appreciating me is necessary and that kind of thing exist there is exist some x that is one x needs to exist.

So, in the case of de re kind of modalities we assigned modal properties to modal properties to objects in the form of the objects that we have. So, here modal proposition qualifies this particular kind of proposition, but this is applied to specific kind of objects like objects are here someone and me is another object individual object.

In the since modal properties to individual objects. So, this is another interesting thing we need to note here coming back to our slides. So, each and every formula in this sense comes up with its own corresponding parsing tree.

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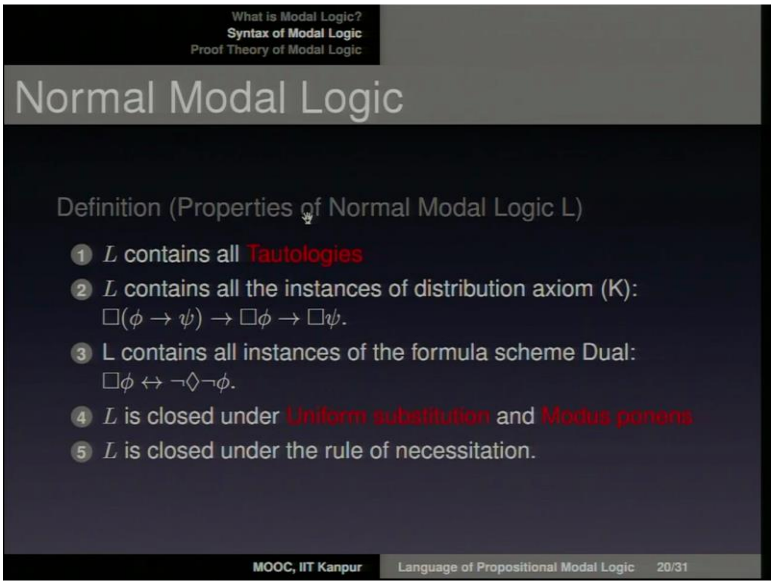

And they are also called syntactic trees. Now, normal modal logic is considered to be a modal logic it is logic of necessity in possibility which contains all the tautologies of propositional, logic which includes an axiom distribution axiom which tells us that if phi implies psi is necessary. Then individually speaking phi is necessary and then psi is also considering necessary and l contains all instances of a formula scheme its corresponding duals; that means, any given formula can be appropriately translated into its corresponding dual for example, if formula starts with necessity disjunction etcetera, then the corresponding dual kind of formula will have possibility with at disjunction etcetera just like in the case of c n f d n f. We have in our propositional logic any given conjunction normal formula conjunction of disjunction can be translated into disjunctions of conjunction that is d n f. So, like this we can have any given formula can be translated into appropriately into it is a dual kind of formula.

So, well cons contains all the formulas of the scheme that is duals; that means, if you have necessity of me you can translated into possibility of p with this definition because possibility and necessity comes in duals and l is also closed under uniform substitutions and modals ponens. So, when I say that one is closed under uniform substitution so; that means, if you substitute for any formula any given formula anything which is in uniform then the corresponding formula is going to retain is tautologies just like in the case of classical logic in the case of classical for example, if you have the formula p implies necessity of p implies p.

So, now, you substitute not p per p here uniformly, you substitute wherever p is there you substitute it with not p then this formula is going to become like this is translated into this one now this is if this is the tautologies this also needs has to be oh tautologies.

Here what we have done is simply we have substitute and not p wherever the p occurs that is what is uniform substitution and more despondence is like this. Suppose if we have something like that p implies q it is a case and then, it is also necessary that p is the case then we read to have p implies necessity that q is the case and other important thing which you need to note it and then, it needs to be used carefully that is the rule of necessitation the rule of necessitation tells us these thing if something is a theorem are proven kind of thing and nothing no how stating that this is also necessarily true. So, you came across the theorem p and then; obviously, that our theorem also has to be necessarily true. So, if these 5 things are there we call it as normal modal logic.

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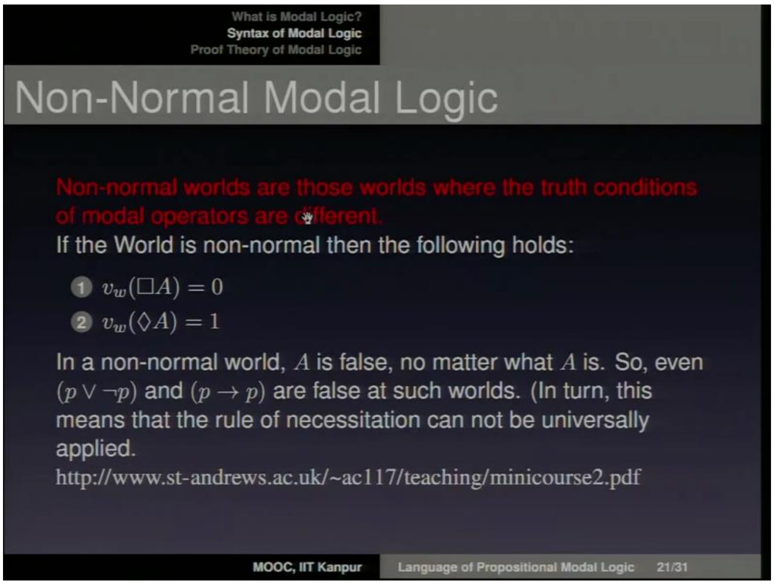

Now, how to distinguish between normal and non normal modal logic, for non normal modal logics we need to study little bit about possible worlds since. I did not introduce the concept of possible worlds between me after dealing this concept of possible worlds and we will this idea will be clear usually in a the normal worlds are those worlds, where the truth conditions of modals operators are considered to be different if it is uniform it is considered to be normal modal logics if, the world is non normal then the following holds. So, this formula needs to be write like this necessity of A is true in a world w the value of necessity of A is going to be 0.

It is false where as the possibility of A is going to be true; that means, in a non normal world a sentence A is false no matter what a is considered to be. So, even p are not p, p implies p are also considered to be false in such kind of at such kind of worlds this means the rule of necessitation cannot be universally applied if it is universally applied it is true for all kinds of formulas, but this cannot be applied. So, we are not talking about non normal modal logical system all though is hinting at, us that it is a non normal modal logics S1, S2 to S3, S1 to S3 captures implication in a better way.

So, if you now in the next class I am going to talk about some of the important theorems which are related to how to prove this important theorem within those modal logical systems. Anyway we will be talking about there will be many modal logical systems which are possible that we will be talking about some of the important modal logical systems such as k t d s four and s five and we have another system called b. So, all these logical systems are named after one logician and another. So, k is maybe wildly popular to be the logical system which was discovered by.

So, this is also considered to be minimal modal logic. So, we talk about axiomatic system axiomatic modal logics in the next class where you will be proving some of the important theorems within those logical axiomatic systems each axiomatic system will consist of a set of axioms, it can be one two or it can be three followed by that you have substitution rules and then more despondence etcetera and then transformation rules it one formula is transform into another one and then rule of necessitation and then more despondence is any how is there in those logical axiomatic systems.

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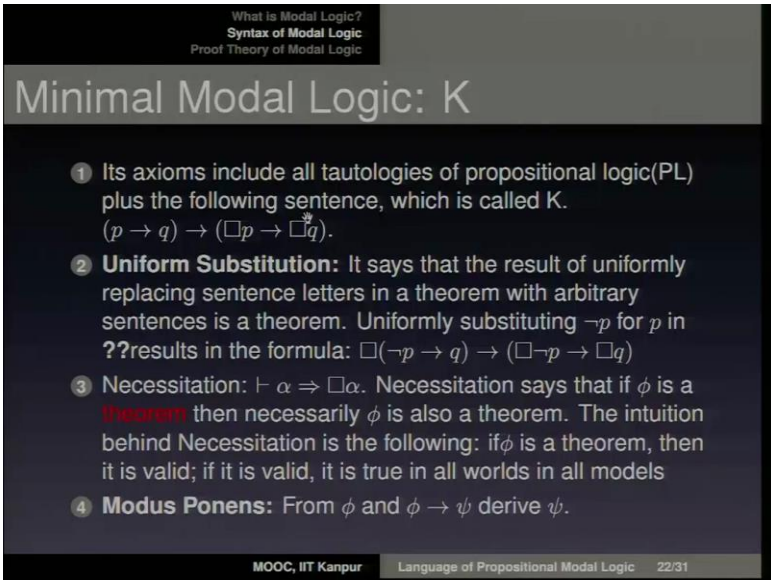

You have only these few things and then using this axiom and the transformation rules and more despondence rules etcetera we will be generating various theorems within that logical axiomatic system.

So, in the minimal modal logical system k we have only one axiom that is p implies q is necessary implies.

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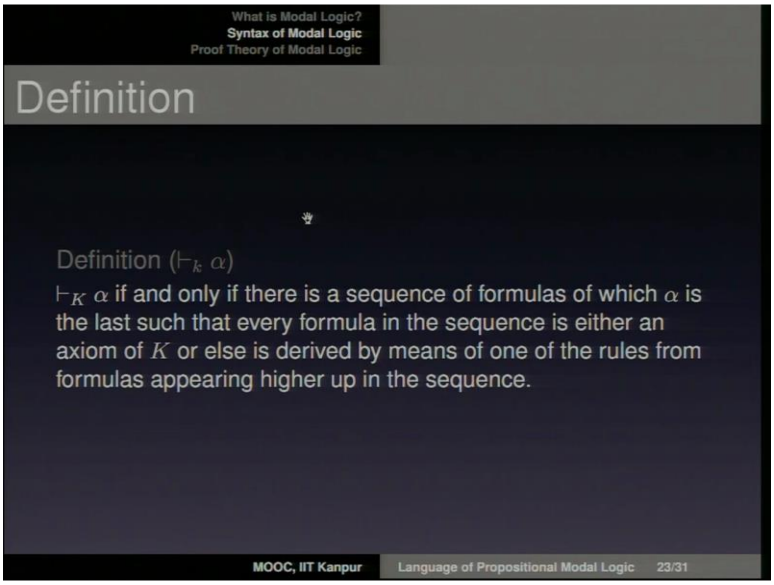

It is necessary that p implies it is necessary that q suppose if, with this I will end it if alpha is considered to be theorem in modal logic k. If and only, if there is a sequence of formula; that means, you need to start with the axioms and then solve the tautologies that we have already have in the propositional logic and we transform it are we trim this formulas in such a way that, ultimately you will you will arrive it one particular kind of proposition I mean one particular kind of thing formula at the end of your proof and that is considered to be a theorem.

Now, alpha is provided in k, if and only if there is a sequence of formulas of which alpha is considered to be the last such kind of formula, such that every formula in the sequence is either axiom of k or else it is derived by means of some other rules from the formulas appearing higher up in the sequence.

Suppose if you if you derive something like from p to all the way down to q we derived something. Now, that means, this is considered to be the final step of your proof from p all the way down to q, the final step of your proof is considered to be theorem, and then all the other step S1, 2 something their all also considered to be true statements. The only thing which you need to note is that, if it is proved in a system k, it has to follow the same rules of k I mean, sorry the axioms of k and only transformation rules substitution rules etcetera and more despondence etcetera nothing else. So, this is considered to be a regress proof, there should not be any other thing which comes from outside it can be borrowed from outside see that sense it is considered to be a regress proof.

In the next class will be talking about various theorems within these logical axiomatic system, then these are wildly studied logical systems KDS S4 and S5 and it is S5, which was wildly studied and we will see the reason why S5 is considered, S5 is studied by philosophers and everyone, why it was given much emphasis. So, in the next class we are going to continue with the axiomatic system pertaining to this system KTD S4, S5.

Thank you.