Basic Concepts in Modal Logic
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Lecture – 13
Language of Modal Logic, Modal Sentences-1

Welcome back. In a last few lectures, we discussed about the origin of modal logic and we gave emphasis to the state implication and it is with respect to the state implication is more while studying state implication and get a detail, Louis has come up with the 5 axiomatic systems. Out of that first 2 he is consider non normal modal logic and the first 3 he is consider non normal logic, and the last 2 is 4 and is 5 he calls it modal normal modal logic. So, in this lecture we will be talking about in the language of modal logic. Particularly we study the logic of possibility and necessity and how this possibility and necessity behaves. So, particularly we will be talking about the syntax of modal logic.

So, first of all, what exactly we mean by modality. Modality is concerned to be any word or phrase that can be apply to a given statement x, to create any other statement, so that is makes in assertion about the modes of the truth defend modes in which a sentence can be a true. There were we talking about different possibilities. For example, if you say x dash happy for example, if you write like this x, is happy.

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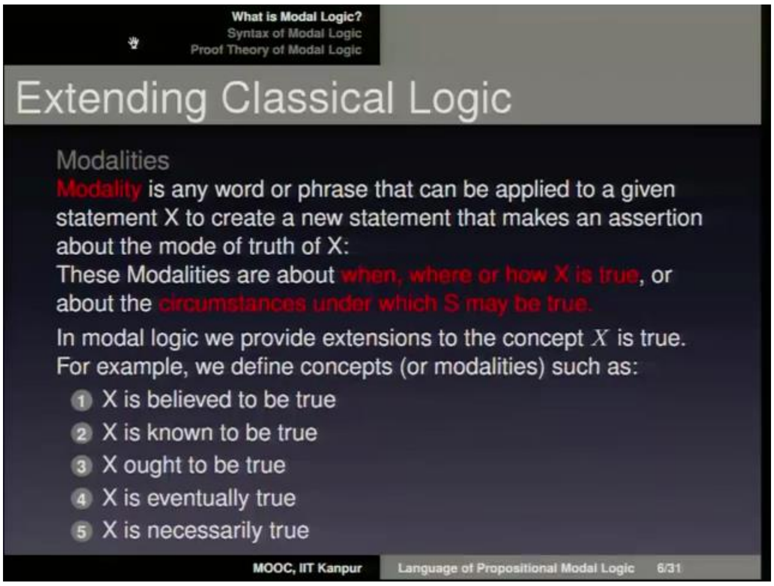

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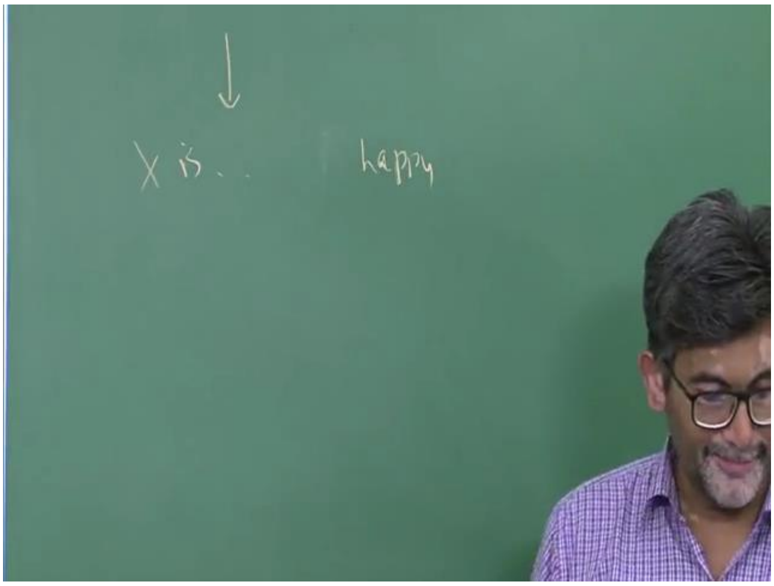

Suppose if you write like this, this is a sentence. Now depending upon what you put, what you fill the blank here, we have different modes of truth. For example, x is believed to be happy. Or x is known to be happy. Actually it should be like this. X is known to be happy. X is believe to be happy, x is actually happy or x is thinking that he is happy or x are to be happy all these things. X is eventually true. X is eventually happy after the executing some kind of actions it turned to the case that he is happy.

So, these sentences are about different modes of truth. The same sentence can between different ways.

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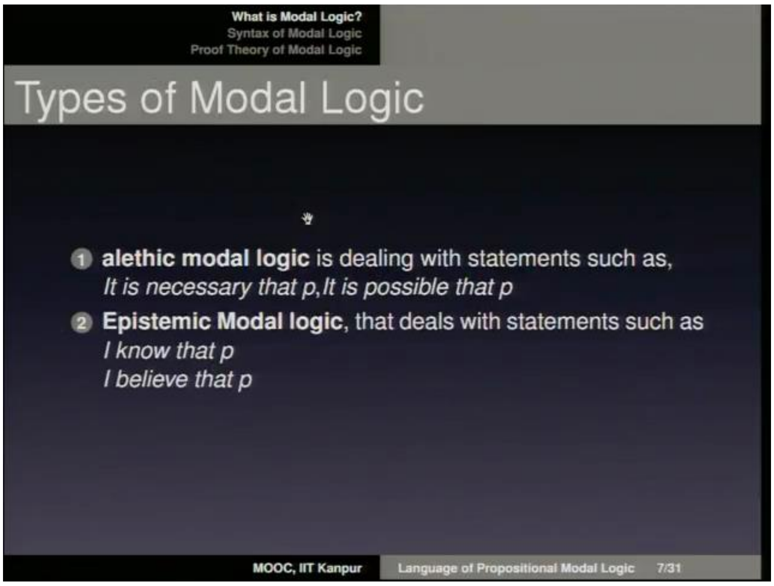

The 2 logic that we are going to cover, is the first one that is alethic modal logic. Alethic modal logic the word alethic talks about something to be with the truth. So, alethic modal logic is dealing with statement such as it is necessary that p and it is possible that p. And then there are various other kinds of operators which act like necessity of p and possibility of p. For instance, I know that p, I believe that p. So, when it at inverse knowledge, it is called as epistemic logics.

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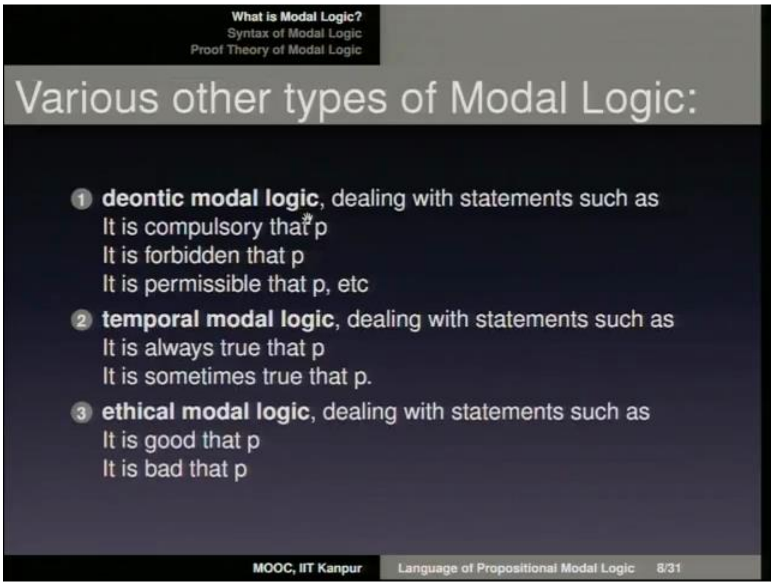

And if involves beliefs it is call (Refer Time: 03:13) logic. And there other kinds of modal logics and each modal operator come in some kind of pace. Like possibility necessity, this like in the case of deontic modal logic, it comes in 2 phase it is forbidden that p and it is permissible that p and it are to be in the case that p is a case. There are in the case of temporal modal logic, we can say that it is always true that p, it is some time true that p are not means x, in the x true in the past, are may be x true in the future, or something which is always considered to be true, which acts like a necessity operator. This is the reason by we need to, we are just we are just made a beginning and using this beginning, we can understand various other kinds of modal logic little bit later.

So, understanding alethic modal logic is considered to be most important to study all these logics deontic, temporal, ethical, modal logics etcetera.

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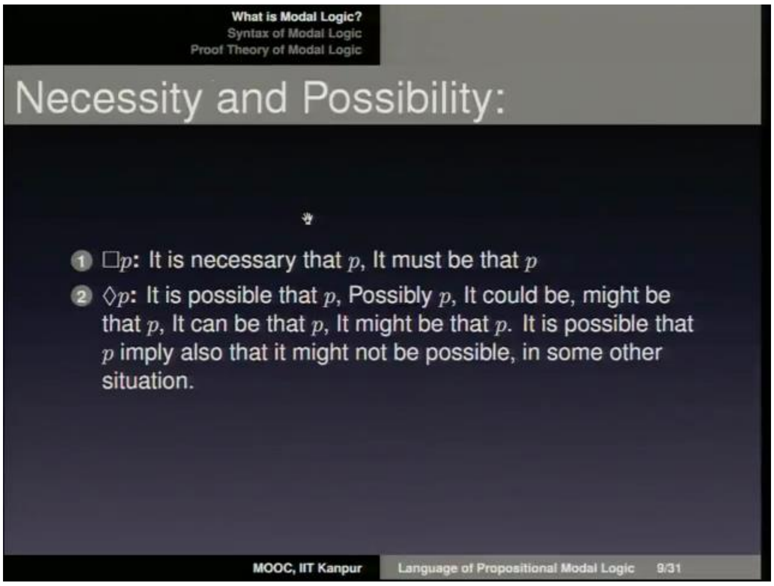

What is necessity and possibility? So, when we deal with semantic of necessity and possibility, we talk about that, we postponed it for a while. But the time being box p is represented as it is necessary that p, or it may be or it can also be read like this it must be the case it is necessary is that p etcetera, and possibility of p, is it possible that p and possible that p etcetera, it could be, might be, can be, all these things can be translated up to appropriately into possibility of p?

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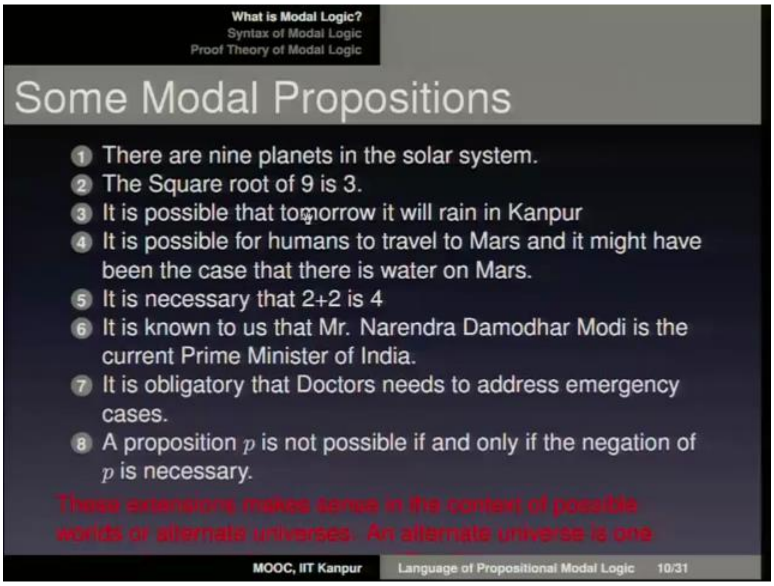

Some examples of modal propositions, there not actually true, that different modes of truth. For example, the first 2 sentences there are 9 planets in the solar system is actually true kind of a true statement. The square root of 9 is 3. There are propositions, but if it take third one into consideration is possible, that tomorrow it will rain in Kanpur we are only talking about modes of truth. When I say it is possible, that it will rain tomorrow in Kanpur. It is also possible it may not rain tomorrow. So, this same if to 2 plus 2 is equal to 4, then it you can say that it is necessary that 2 plus 2 is equal to 4; that means, it cannot be false. 2 plus 2 has to having 4, but it cannot be 5 and it is known that mister Narendra Modi is consider with the current Prime Minister India, it is obligated that doctors need to address emergency cases. He talked to be the case that p is the case.

So, we need to note that an axiom, which is an axiom of one axiomatic system, cannot mean that have to be true in other kind of p.

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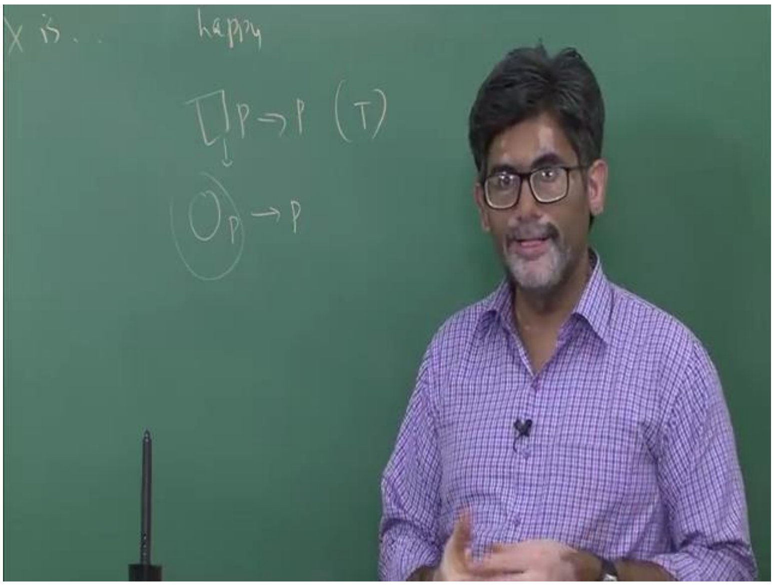

For example, if you say necessity of p implies p. In alethic modal logics particularly an operating a modal logical system T this holds. It is saying that all necessary trues are actually true, but the same thing necessity is translated into deontic logic for example, logic of obligations. I talked to be the case that p implies p. In our day to day example we can say that you are to fallow the traffic rules, means you have to actually follow the traffic rules, but you can always come off with the counter example for the same. We talked to be the case that we need follow some strict rule, but it need not have to be actually truth.

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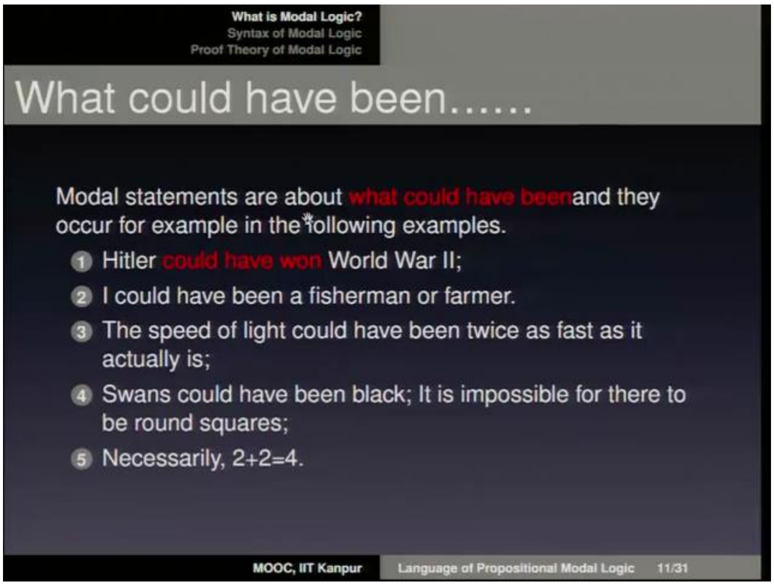

So, in the case of emergency etcetera and ambulance can the work person who riding the ambulance can violate this particular kind of thing. Although, we have to follow the rules, things of what could have been, what might have been etcetera, all are in the purview of modal logic. Why we have doing this particular kind of thing? We are trying to distinguish necessity of possibility of p, and something which is actually the case are p is the case. Classical logic deals with only this one whereas it leaves out the important things that we are going to talk about in this course that is necessity of p and possibility of p. Some examples have it is possible that p are like this, this is always been interesting to history would be interesting particularly when we invoke various kinds of possibilities, are we talk about various counter factual. Counter factual are the conditional variants. So, in which the antecedent is always false.

So, Hitler could have won the world war provided so and so, or I could have been a fisherman or a farmer. Or speed of light could have been twice as fast as it actually is. Always since are possibilities, but we known that our current laws etcetera restricts us to have a speed of light to be some within the limits. We know that is all swans are white in color, but it could be the case that some swans could be black in color or it is impossible to have round squares. It is upset to talk about square circles, round squares etcetera. The same way 2 plus 2 is equal to 4 if you want emphasize that particular kind of thing you can say that necessarily 2 plus 2 is equal to 4 because it cannot be false. That 2 plus 2 is equal to 4 is 2 plus 2 cannot to 5. The other interesting things which for another under the purview, and which will be going to talk about it, while we deal with conditional statement are the special kind of conditional statement which is called counterfactuals.

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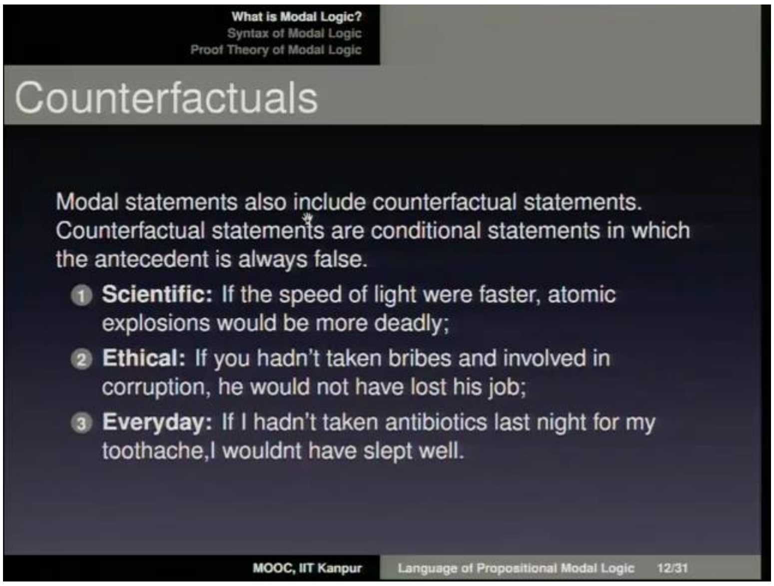

So, these are also considered to be modal statements. Anyway David Louis use counterfactuals as variables strict conditionals. Then modal statements, includes counterfactuals statements. Counter factual statements are conditional statements, in which the anti-semantic is always false. Example if we say if I drop this chock piece it would fall on the ground. So, I did not drop this chock piece, it is still in my hand; that means, the antecedent part of it is still considered to be false. Provided I drop if I drop this chock piece it would be fallen on the ground. So, it makes sense to talk about those counterfactuals whereas, suppose if I say that if I drop this chock piece, it would fly or it would turn into some cat or donkey or something like that, it is not acceptable to us. But if we take material implication into consideration, material implication uses all counterfactuals.

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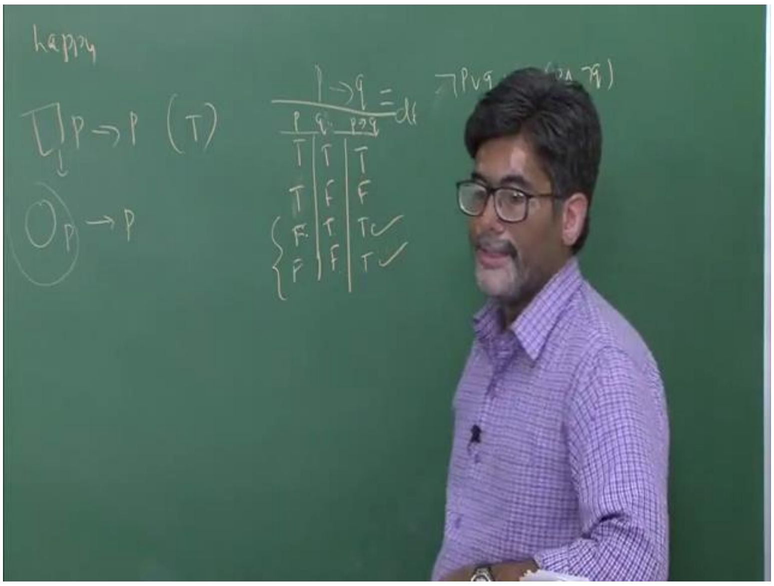

In the same kind of definition, so that is p implies q. By definition it is same as not p or q and it is same as it is not the case that p is true and q is false. If you take this definition into consideration, in the case of counterfactuals, this is the truth table of p implies q. T F and T F here, we have p, you have p, and we have q here, and then p implies q T F T F Alternative T, and alternative F. You need to write T F T F and then. So, this conditional is going to be false only in this one when we have an antecedent T and the consequent false.

The condition is going to be false in all other cases it is going to be T. In this context suppose if you define p implies q as this one - counterfactuals are falling under this category. Sorry this is this should be f. So, when the antecedent is false, irrespective of this consequent whether it is true or false, your conditional is always going to be truth; that means, if I drop this chock piece if it fallen on the ground is true, if I drop this chock piece it would fly will also going to be true.

Now, we need to distinguish between these 2 statements. When the counterfactuals are true and when the counterfactuals are false, we need to have appropriate interpretation are the meaning of this conditional. Special kind of conditional statements in that context we will talk about conditional logical little bit later which involves again the modal operators. These kinds of counterfactuals are already discussed. We use it in the scientific discourse.

Ah for example, in this case if the speed of light was little bit faster. In atomic explosions are going to be deadly would be more deadly; that means, e is equal to n c square and c value increase; obviously, energy that you produce it will; obviously, we considered to be very deadly. More deadly or in the case of ethical kind of domain you can talk about counterfactuals light, if I had had not taken brides and involved in corruption then he would not I would not have lost my job. If you taken loss his job are in the everyday discuss, we can have example like if had not taken antibiotics last night for my toothache, I would not slept well. So, these are all useful kind of counterfactuals and you require careful analysis somehow.

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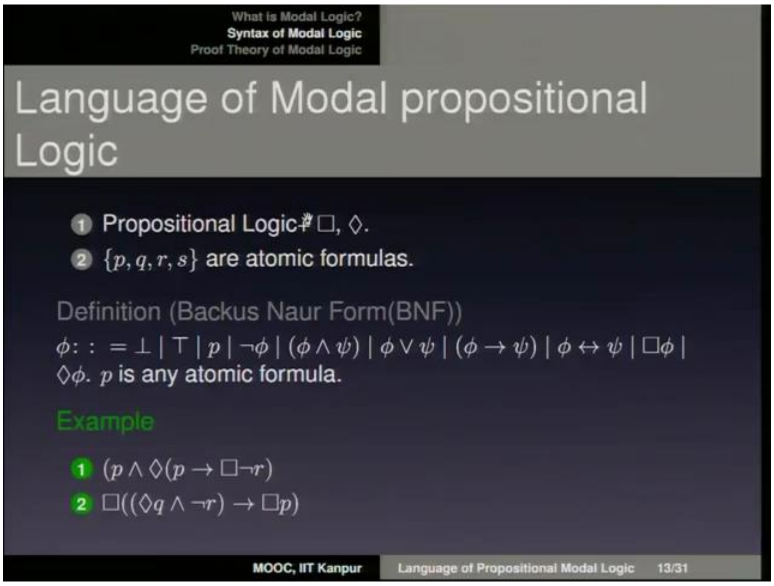

Let us come back to our topic of this lecture that is language of a proper modal propositional logic. Modal propositional logic means it is an extension of classical propositional logic with 2 operators. So, these 2 operators are it is necessary that p and it is possible that p is the case. So, this is considered to be an extension of proposition logic by adding these 2 operators. Is necessary that p and it is possible that p all other things are being a same as in the case of classical logic. So, this one definition, these are special kind of form with which you can express the whole language. So, that is like this, is also called Backus Naur form BNF. These are called BNF.

So, this is a, you can be written like this any formula 5 it needs to be either a contradiction like bot, are it should be simply a formula T, or it should be any atomic proposition p, or if 5 is considered to be a formula then not 5 also considered to be a formula. And phi and psi combined together with a conjunction it become another formula. Like this with all the other logical connectives we have various formulas. Together with that we have 2 more formulas. So, they are like this. It is necessary that a phi is the case and it is possible that phi and p is considered to be any atomic propositions. So, any other kind of way you write it, it is not going to be a formula. In this example p and it is possible that p implies a necessary or not they are all considered to be well formed formulas. And it is necessary that possibility of q and not implies necessity of p is also considered to be a formula.

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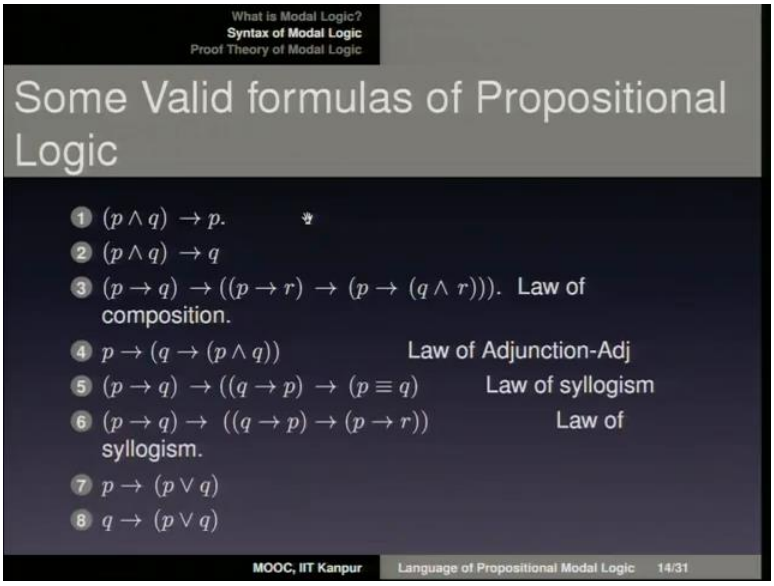

So, gesture in the case of classical logic, we have some kind of, some way of telling us how to read a given formula. No 2 formulas have same kind of synthetic tree structure. So, you translate the formula and draw a corresponding synthetic, it tree it appears to be if it is same then these 2 are considered logically equivalent to each other. Otherwise any 2 given formulas will have different synthetic tricks. So, to begin with our logic are the language of classical proposition modal logic has all the tautology is that are already there in the case of classical logic. For instance, p and q plus p, p and q, you will arrive at q in all these this several thing laws of adjunction law of syllogism etcetera there all considered to be tautologies in proposition logic and hence are also in any given modal logic. Why because any given modal logic is considered to be an extension of classical propositional logic. Only extension is that we are adding into more operators that is possibilities of p, and necessity of p, and modal logic is all about the study of necessity and possibility, how it behaves.

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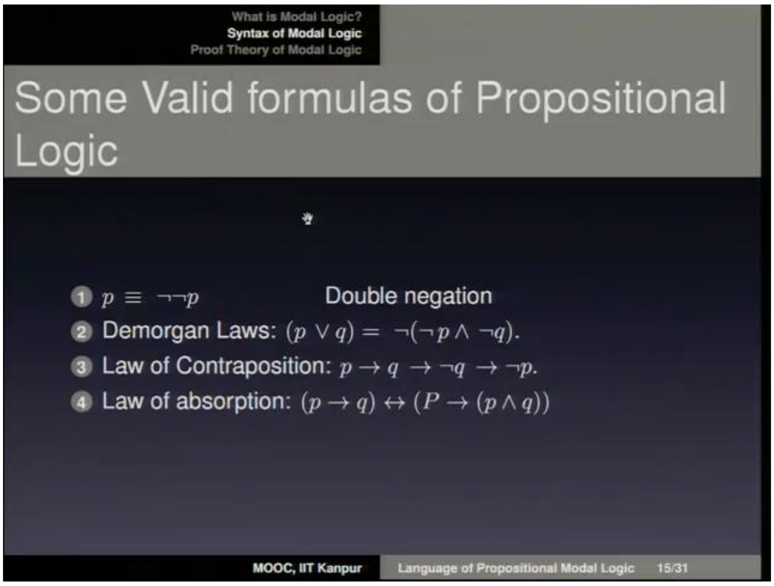

These are some of the valid formulas the list is going to be quite big, but they all considered to be validities.

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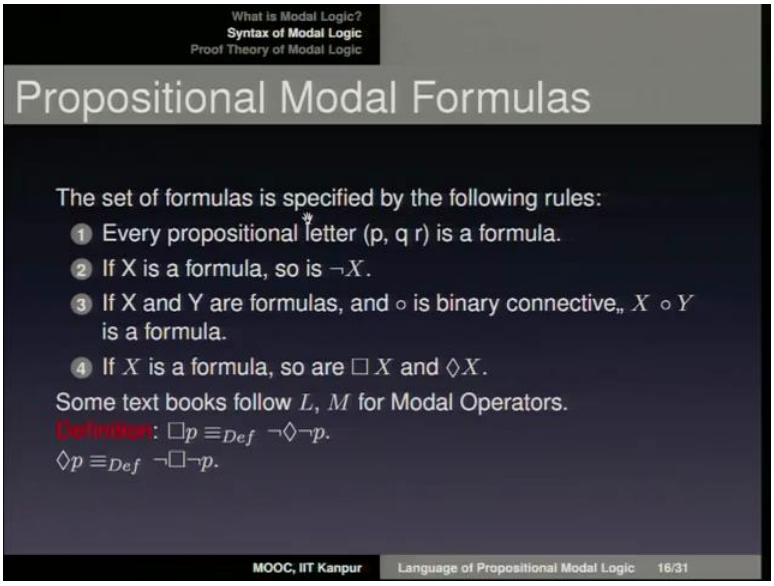

So, now how do we say that a given formula is considered to be a propositional modal or well-formed formula? A set of formula is specified by these following rules. If we just write p q r etcetera usual atomic sentences, it is also considered to be a formula. If x is a formula, it is negation is also going to be a formula. If x and y are considered to be formula and these x and y are connected by some kind of binary operator. This binary operator can be conjunction bases and implication.

And negation is considered to be a unary kind of connective, but other things conjunction disjunction implication, bi-implication they all considered to be binary connectives. Anything which is combined by this connectives is also become formula in our language and derivation to that is what is propositional logic is all about. And if you add further, if further add these 2 things, any form if x is a formula the necessity of x is also going to be formula. And possibility of x is also going to be formula. And there are some definitions like necessity and possibility becomes in a bale, necessity of p can define in terms of possibility of p. And possibility of p can also be defined in terms of necessity of p.

So, this is the one which we usually follow. We discussed it, in when we spoke about Aristotle and modal syllogisms in the in the historic origins of modal logic.

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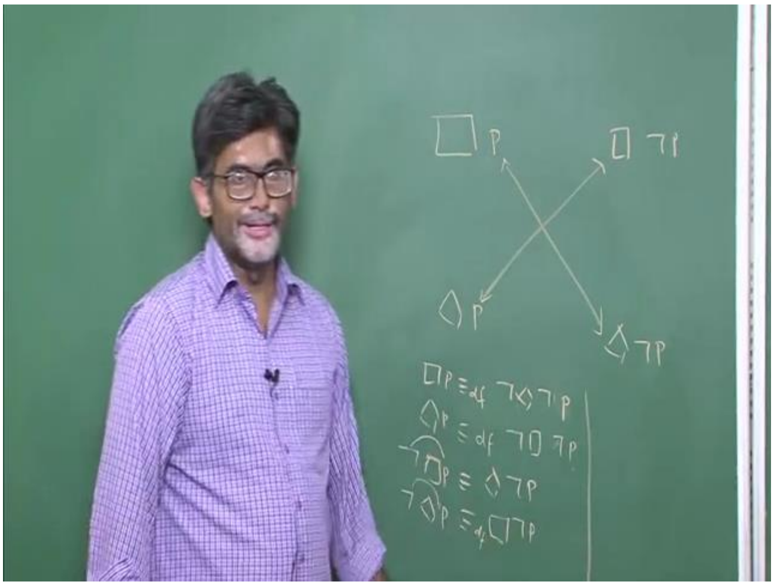

So, this is like this. So, on the one hand we have possible necessity of p and possibility of p, and exactly opposite of that one, is this one. It is possible that not p is a case and then it is necessary that not p is a case. So, these 2 are considered to be diagonal considered to be contradictory to each other. So, our definitions are like this possible necessity of p is defined as it is not possible that not p. So, this tells us that not p is impossible if you rule out not p when you render with necessity of p. So, that not p has to be impossible and the same way possibility of p is defined as, it is not necessary that not p need not have to be necessary suppose if you say that, it is possible that it is not raining outside. So, that is not considered to be necessary statement because it is not necessary that not p is the case p and then negation of possibility of p.

So, in this case you are put this negation in say and it will become not p and negation of possibility of p. So, in these cases you have to push this negation and will become not p and negation of possibility of p by definition is same as necessity of not p look at this negation goes inside negation of possibility will become necessity operate. These are the entire thing which will be of some use particularly when we deal with some of the when we deal with, semantic tabular method little bit later.

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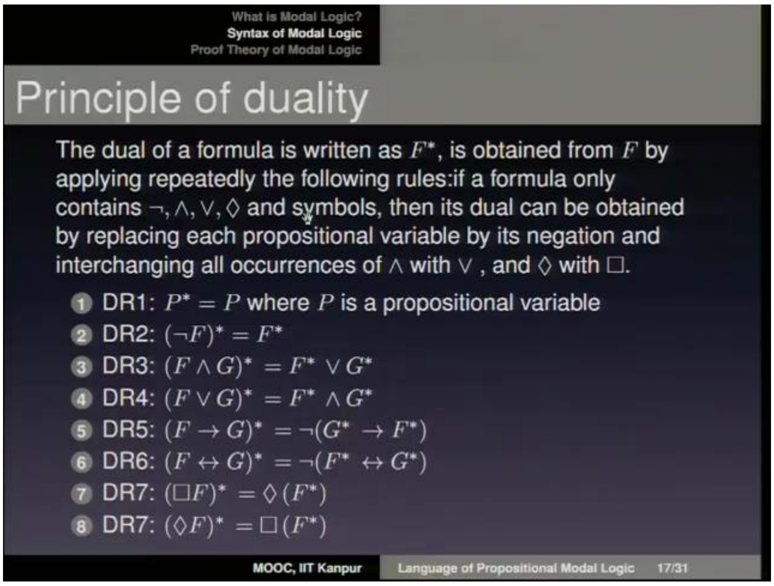

There is one principle which will be using it that is what is called as Principle of duality. So, that dual of the formula usually we write it as not of f, but here we write it as F star. So, that is up ting from F by applying repetitively the following the rules are like this.

If a formula contains only negation, conjunction, disjunction, possibility, as and some and symbols p q r etcetera, then it is dual can be obtained by replacing each propositional variable with it is negation p occurs in given formula you replace that one with not p, and interchanging all occurrences of if it is a conjunction you write it as a disjunction, if it is a possibility and you replace it with replaced it with necessity.

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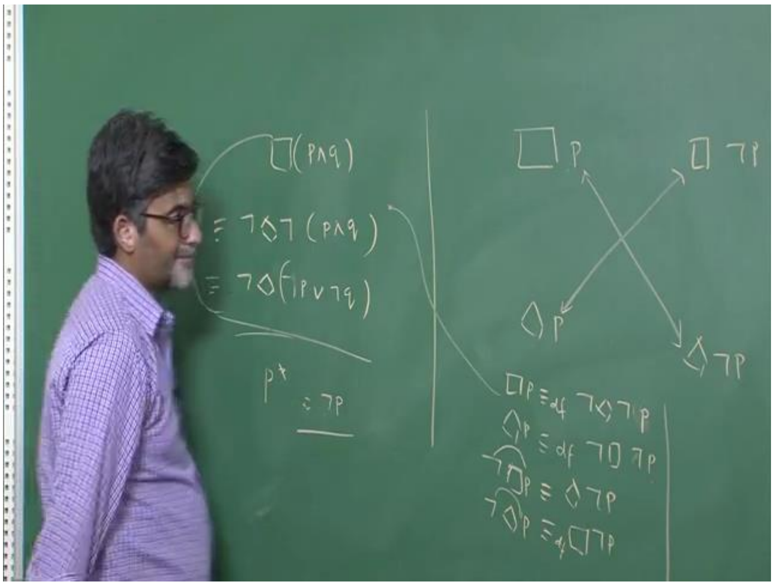

And these are setup the some set of rules; we will be a following and using that you can translate a given sentence which occurs in, for examples in the case suppose if a formula is like this p and q. Now the principle duality tells us that you replace this thing somehow you need to translate this sentence into which includes only, if necessity is should have possibility and if conjunction is we have negation, we have disjunction and you can have negation of p negation of q as also.

So, now this formula necessity of p and q, by applying these principles of principle of duality, now what we have trying to do is we are converting this formula into the formula which includes only possibility. So, now, we need to use some of this rules necessity of p is represented as not of possibility of not of p and q. So, this is the first one which comes from this one.

So, now this is same as not of not of p and q is not p are not q. So, now, you have a formula which is considered to be dual of this one. So, that is it includes possibility rather than necessity and then instead of p we have not p. So, in our notation p star means p same as not p. So, now, this is simple example, but you can transform any given formula into it is corresponding dual kind of formula, the one which includes negation conjunction and or possibility etcetera. So, they can be approximately transformed in to it is corresponding dual, by replacing this thing conjunction with disjunction and you translate the symbols p to not p etcetera.

So, these are some of the rules that we follow. So, DR1 is says that if you have p star that is in not p, p star is equal to p, where p is considered to be a proposition were it is same as p, one not of F star is equal into F star, and F and g star, where is F star or g star etcetera. So, DR7 tells us, it is necessary that F star is equal into. So, now, you see here necessity is transform to possibility and F an individual anatomy letter F appears with it is star. And possibility of F is translated into necessity of F by using DR8 formula.

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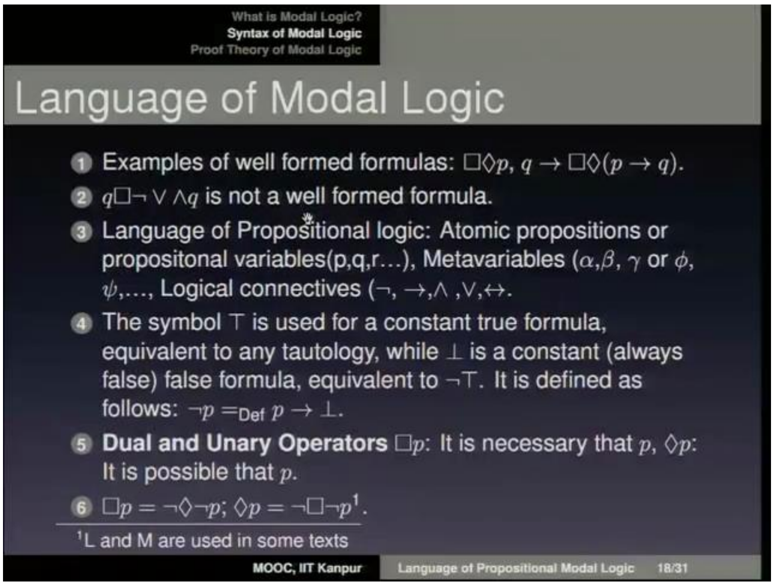

So, these are some of the examples of this thing. Some of the examples that you can take into consideration with aspect to the well-formed formulas; it is necessary that possibility of p is a well formed formula. Or if you say that q implies it is necessary that possible that p implies q here, we have individual atomic prepositions q, as well as both modal operators are there in the second sentence. Whereas you observe the second formula here, q necessity not r and q is not considered to be a well formed formula because it does not follow any one of the rules of our well-formed formula. So, language of prepositional logic includes atomic prepositions, prepositional variables, and met variables such as the alpha beta gamma etcetera, in logical connectives now in logical connectives, we have a case of prepositional logic in 2 symbols that we used top and bottom.

And there is a particular notation that we use here. If you say if you have a negation of p; that means, p leads to some kind of contradiction. If p leads to contradiction you say that it is not p, but it is not it is not p is the not p is the case and unary and dual operators are like this necessity of p and possibility of p these are all unary operators.

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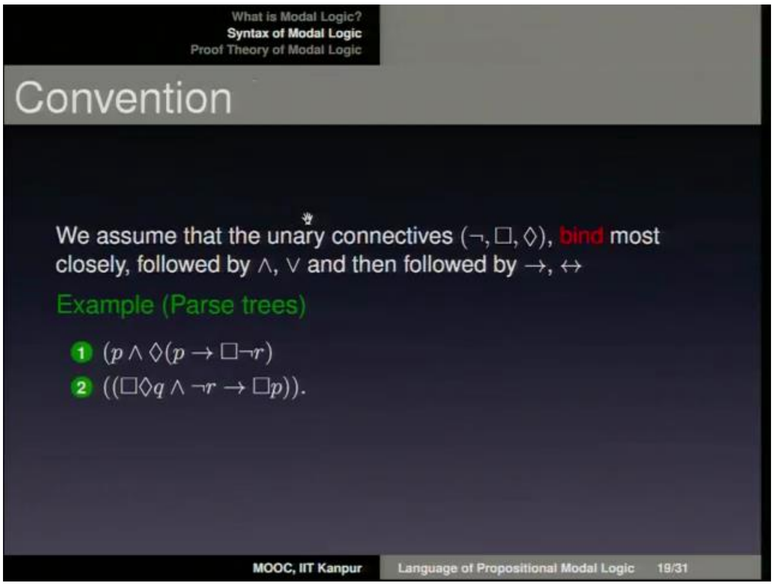

So, just like in the case of classical logic we follow little bit of convention.

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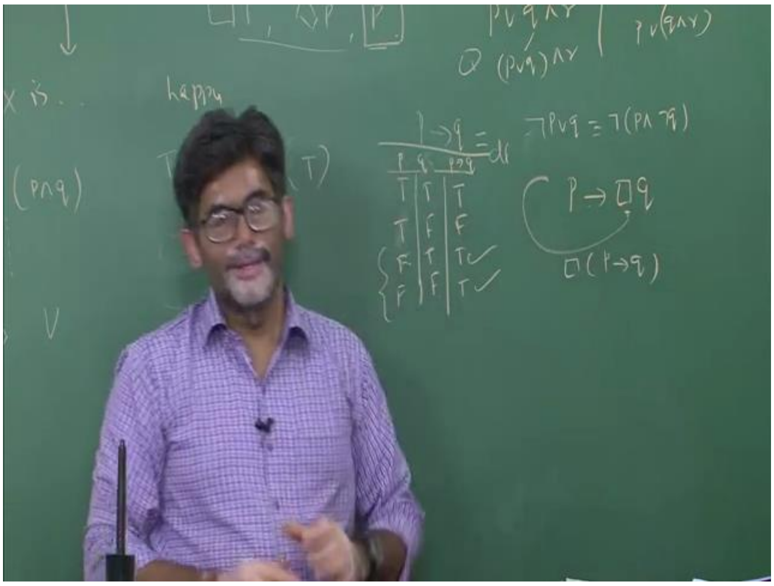

For example, if you have a formula like p or q and r. So, this formula can be read in different ways. First is it can be read simply as p or q and r, second reading is that p or q and r. Suppose if you are not given any parenthesis then we will going to this way. So, in the next class, will be talking more about syntax of propositional logic and then we will see how a given English language sentence can be appropriately translated into the language of modal logic.

For example, whenever you come across a statement which includes it might have been the case, you appropriately translate it as it is possible that p. And we are also going to see how necessity operator operates on a given condition. For instance, can we have the situation where we have this p necessity of q? Now in this case this is necessity operates, on the whole conditional like this one having big scope or in the case of this one p implies necessity of q both q as a narrow scope. So, to, we need to see how it is going to operate.

So, in the next lecture, we will be talking about the language of modal logic in more details, and then we will see how English language sentences can be appropriately translated in to the language of a modal logic. And then gradually we will move on to the semantics of modal logic.

Thank you.