

Basic Concepts in Modal Logic
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Lecture – 12
Language of Normal Modal Logic

Welcome back, in the last lecture we discussed something about strict implication and it is the strict implication which led to the development of various formal Modal Logical Systems S 1 to S 5 and there was lot of attention on the strict implication till some years, but the later ever since semantics has come in to existence, the focus has been shifted to proving the validity etcetera validity of a given Modal Logical formula. But again it got renovated in particular when we are discussing about the counterfactual.

So, they are considering being variably strict conditional again the ideas it is there in the strict implication again there is coming to picture. So in this lecture I will be talking about the language of Propositional Modal Logic we are restricting ourselves (Refer Time: 01:11) our self to Propositional Modal Logic. I will be talking about the language of Propositional Modal Logic, Logic needs to be viewed as a language, and an every language has its own syntax and corresponding semantics.

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The slide is titled "Limitations of Classical Logic" and is part of a presentation on "What is Modal Logic?". The slide content is as follows:

- 1 Classical logic studies formulas that are true (especially those that are true in all interpretations, i.e., valid) and how truth is preserved in reasoning such that true premises only have true consequences.
- 2 We write $I \models A$ if formula A holds in interpretation I . We just write $\models A$ if formula A holds in all interpretations and say that A is valid.
- 3 Formulas of the form $A \vee \neg A$ are always trivially valid in classical logic, because each interpretation I satisfies either $I \models A$ or $I \models \neg A$. Consequently, $\models A \vee \neg A$.
- 4 Classical Logic is extensional whereas Modal Logics are intensional.

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So, some of the limitations that we have seen in some of the lectures, is this that the very main thing which you need to notice is that, Classical Logic fails to differentiate between

'something which is actually the case', and 'something which is possibly the case' and something which is actually the case. That means possibility of the P necessity of P and something which is considered to be P.

So, it is in that sense, when you are trying to talk about, extensional of Classical Logic with this 2 modalities 'possibility' and 'necessity' of P we are talking about Propositional Modal Logic's. So, Classical Logic, we need to know that it is considered to be extensional whereas Modal Logic is considered to be intentional.

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What is Modal Logic?
Syntax of Modal Logic
Proof Theory of Modal Logic

Example

Example (Example1)
If a new course is to be offered next year, then submission **must** be made to the faculty board before April. If submissions are to be made to the faculty member before April, then Departmental meeting **must** be called. A week's notice **must** be given if a Departmental meeting is to be called. Since it is not **possible** to give such notice, it follows that it is **not possible** to offer a new course next year.

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So, what is the difference between extensional and intentional? But before that let us consider this simple example again; I will be looking in to this example in greater detail. When the context comes so the example goes like this if a new course is to be offered next year, the submission must be made to the faculty board before April. Now you see this particular kind of sentence, your viewing whatever is there in red color qualifies this sentence to be a 'Modal sentence'.

If the submissions are to be made in the faculty member before April, then the Departmental meeting must be called. (Refer Time: 02:54) this is the kind of necessary kind of connection it's like necessity of P implies Q; where P is represented by this following sentences, this sentence that submission are to be made and faculty members. Submission before the faculty member and Departmental meeting is called as is represented as Q. A week's notice must be given to a Departmental meeting if a

Departmental meeting needs to be called. Since it is not possible to give such a notice, it follows that it is not possible to offer a new course next year.

This is simple argument that you come across in our day to day discuss, if you represent it in the Classical Logic. We might get some different kind of conclusion and if you represent it in Modal Logic, you might get a different kind of conclusion.

So, just as in the case in the last class we have seen that in, if Ravi is guilty, then he had an accomplished, then the prosecutor the defense lawyer is disagreeing with that and then judge is agreeing to what a (Refer Time: 03:59) defense lawyer is disagree so ultimately we came up with two different kind of conclusions if you follow Classical Logic, we are definitely establishing that, Ravi is consider to be guilty there and then if you follow Modal Logic's and we have come up with a conclusion that it is possible that G is the case; that means, possible that Ravi is guilty. If you say that Ravi, 'it is possible that Ravi is guilty' then 'it is also possible that Ravi is not guilty also' so if evidence is shifting more towards it is possible that not be, there is a way to escape some kind of punishment.

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What is Modal Logic?
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Intension vs Extension

- 1 The Morning star is the evening star.
- 2 The morning star is the Morning star.

1 The two phrases, "morning star" and "evening star" may designate the same object, but they do not have the same meaning. Meanings, in this sense, are often called intensions, and things designated, extensions.

2 Contexts in which extension is all that matters are, naturally, called **extensional**, while contexts in which extension is not enough are *intensional*.

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So, definitely there is an advantage in interpreting the things by using this Modal Logic's. But before that, let us again comeback to the difference between "intention" and "extension". This is age hold distinction which you will find it in the works of a (Refer

Time: 04:54) made this particular kind of distinction that is the distinction between the "sense" and "reference". Suppose if I say the "morning star" is an "evening star".

So, these 2 different sentences, "morning star" and the "evening star", so they are referring to the same object, let say Venus something like that. So, these 2 are having different senses "morning star" and the "evening star" but the meanings are also different, but they are referring to a same object that is Venus so whenever you are referring to something that is kept to be different and these used in 2 different senses, "morning star" and the "evening star". So, now, if you replace if you morning star is equivalent to evening star, then it be the statement will become the morning star is equivalent to the morning star is the morning star.

So, now the 2 phrases, "morning star" and the "evening star" may designate the same object that is Venus, but they do not have the same kind of meaning, morning star means 'whatever occurs in the morning', evening star is means 'whatever you seeing it with the evening', these 2 are different things, the main 2 different things, meanings in this sense, are also called intentions, and things with things designated, that means here "morning star" and "evening star" are designating, Venus that is one particular kind of object, that referring to some object.

So, they are considered to be extensions. Classical Logic is extension which is that truth value of any compound proposition is only determined by the truth value of its individual constituents. So, the Context in which the extension is all that matters are consider to be extensional, it is in that sense Classical Logic are Logic's are consider to be extension in nature, suppose if you say that it is a same example which I am trying to give, 'I became sick and I went to see the doctor'. That is A and B the A and B is same as B and A. So, now, it is say if that is same (Refer Time: 06:57) you are using it as extensional sense, so I became sick and went to see the doctor, and I went to see the doctor and I become sick, these 2 in our 'day to day' discuss these 2 are considered to be totally different things nobody goes to a doctor to become sick.

So, we need to understand the second argument this is some kind of order which is followed in this particular kind of sentence, so temporal things is needs to be taken into consideration to understand the sentence in a better way, the first sentence: is ok for us there is a second sentence: 'I went to see the doctor and I became sick' is not acceptable

to us if you follow Classical Logic there is no such kind of distinction A and B is same as B and A, because the connective end is consider to be extensional there. It is referring to a something same object, context, context in which extension is not enough particularly in that case 'I went to see the doctor and become sick' need to know more about the truth value of not simply the truth value of, 'A' and truth value of 'B' we need to know something more.

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Remarks

- 1 Mathematics is typically extensional throughout. we usually write $3 + 2 = 2 + 3$
- 2 In classical first order logic intension plays no role. It is extensional by design since primarily it evolved to model the reasoning needed in mathematics.

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So, whenever you use "intention" and "intentional" operators of course it would become like possibility necessity etcetera and becomes intentional Logic. So, Classical Logic I said in the begging of this course, that it captures Mathematical reasoning in a much more better way much better way captures Mathematical reasoning. Mathematical reasoning in particular in Mathematics, which is consider to be extensional throughout, it does not matter when even if you write 3 plus 2 and 2 plus 3, 3 plus 2, 3 followed by that you added 2 here, And the second case: 2 followed by that you added 3, these 2 are 2 different using 2 different senses, but Mathematical does not make any sense it is a commutative to property, plus operations follows commutative properly, A plus B is same as B plus C does not make any difference is followed (Refer Time: 08:55) order.

So, in Classical first order Logic intention plays absolutely no rule at all, there is no distinction between possibility of P and something is actually, the case that P, it is raining out, it is possibility it is raining out outside and it is actually raining, there is no

difference between these 2 things, because they are same not possibility of P necessity of P is equal to P. In Classical first order Logic intentional plays absolutely no rule, and it is extensional by design, since primarily it evolved to modal the reasoning needed in Mathematical this is consider as to be the starting point, for understanding various other kinds of reasoning.

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The slide is titled "Extending Classical Logic" and is part of a presentation on "What is Modal Logic?". It defines modalities as words or phrases that modify a statement to assert a specific mode of truth. It lists five examples: X is believed to be true, X is known to be true, X ought to be true, X is eventually true, and X is necessarily true.

What is Modal Logic?
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Extending Classical Logic

Modalities
Modality is any word or phrase that can be applied to a given statement X to create a new statement that makes an assertion about the mode of truth of X :
These Modalities are about **when, where or how X is true**, or about the **circumstances under which S may be true**.
In modal logic we provide extensions to the concept X is true.
For example, we define concepts (or modalities) such as:

- 1 X is believed to be true
- 2 X is known to be true
- 3 X ought to be true
- 4 X is eventually true
- 5 X is necessarily true

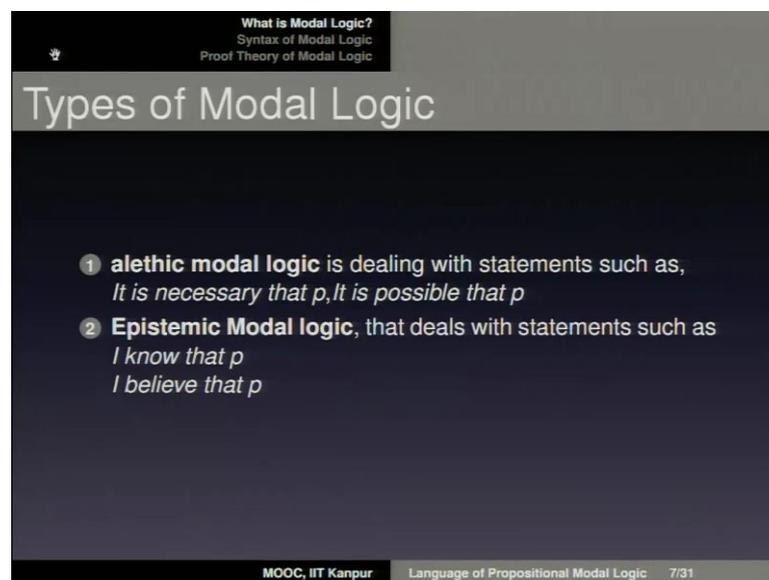
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So, it is in this way we are Extending the Classical Logic's with Modalities. Modalities are considered to be different modes of true. What do you mean by Modality? Modality is considered to be any word or phrase that can be apply to a given statement. X is to create a new statement, and that new statement makes an association about the modes of truth of X , it is the same sentence, but it will be we are talking about different modes of truth. So, these Modalities are about when, where or how X is true, or about the circumstances under which S may be true. Or, in the latest stage will be using possible worlds, and which possible worlds it is consider to be true.

See, in Modal Logic we provide an extension of a concept that is in Classical Logic we view simply as X is true, now we are talking about X may be true, X might be true, X (Refer Time: 10:38) believe to be true X . So, X dash something is true, that can be filled up with these things. X is believe to be true, X is known to be true, there is lot of different lot of difference between 'something is believe to be the case' and 'something is known to be the case'. I believe at (Refer Time: 10:59) might exist is true for me, but

does not imply that I know I cannot say that I know that (Refer Time: 11:04) exists is actually true. So, and in other instance is that X are to be the case, you have to follow the traffic rules, but does not imply that you actually follow the traffic rules. A person is traveling in of course; in ambulance we know how to obey the laws of traffic rules. Ok, X is eventually true, x is necessary true, the same you are taking about the same sentence X but we are viewing this X in different ways, X is believe to be true, X is known to be true, X is are to be true, X is always true, X is eventually true, all this things all this thing are set to be different Modalities.

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So, there are different types of Modal Logic's, that we are going to study, but in our course is consider to be the basic starting point, it is consider with the starting point we are making our journey may be view that you know how this Modal Logic is coming to existence, and when the minimal kind of things that we will do talking about that is a normal Propositional Modal Logic, this is the one the one which we are trying to talk about is a alethic Modal Logic's, is dealing with statement such as. It is necessary that P, it is possible that P. So, they are represented by box P and diamond P.

So, we are view we are taking in to consideration some kind of logical and metaphysical necessity taking it for granted in this Logic's, alethic and Modal Logic. And the other kinds of various kinds of Modal Logic that have emerge these days including temporal logic etcetera (Refer Time: 12:48) of lots of applications, another Logic that we

commonly see is Epistemic Modal Logic, that deals with Logic of knowledge and (Refer Time: 12:57) Logic's, talks about 'Logic of belief statements'.

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The slide is titled "Various other types of Modal Logic:" and lists three types of modal logic:

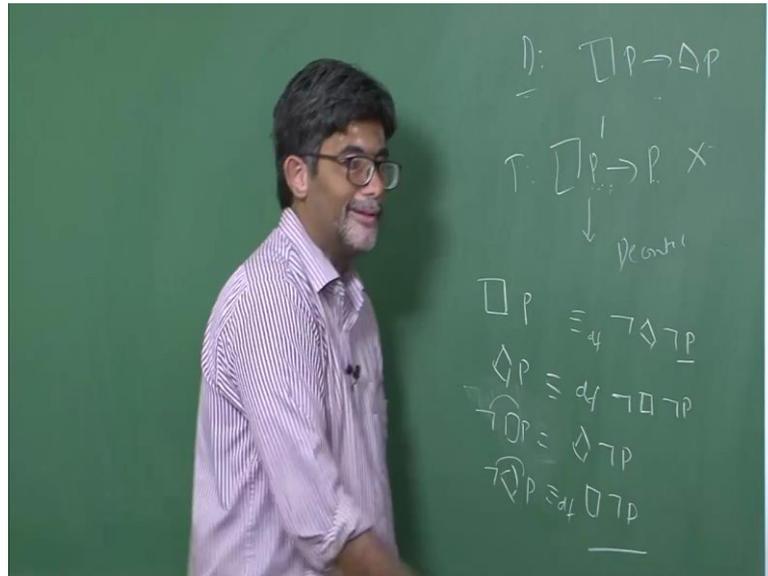
- 1 **deontic modal logic**, dealing with statements such as
It is compulsory that p
It is forbidden that p
It is permissible that p, etc
- 2 **temporal modal logic**, dealing with statements such as
It is always true that p
It is sometimes true that p.
- 3 **ethical modal logic**, dealing with statements such as
It is good that p
It is bad that p

At the top of the slide, it says "What is Modal Logic? Syntax of Modal Logic Proof Theory of Modal Logic". At the bottom, it says "MOOC, IIT Kanpur Language of Propositional Modal Logic 8/31".

And depending upon at the same kind of "necessity" and "possibility", behaves in quite different ways in all these Modal, types of Modal Logic's. So, deontic modal logic is dealing with the this particular kind of things it is forbidden that P it is permissible that P or to be, it or to be the case of P, is it is occurs in the area of ethics, you we commonly (Refer Time: 13:26) moral reason in temporal modal logic the same kind of necessity can be (Refer Time: 13:33) slightly different way so that is it is always true that P, it is sometimes true that P etcetera it was true the past etcetera. In case of ethical modal logic give you good, bad and we make value judgments so it is good that P, it is bad that P etcetera.

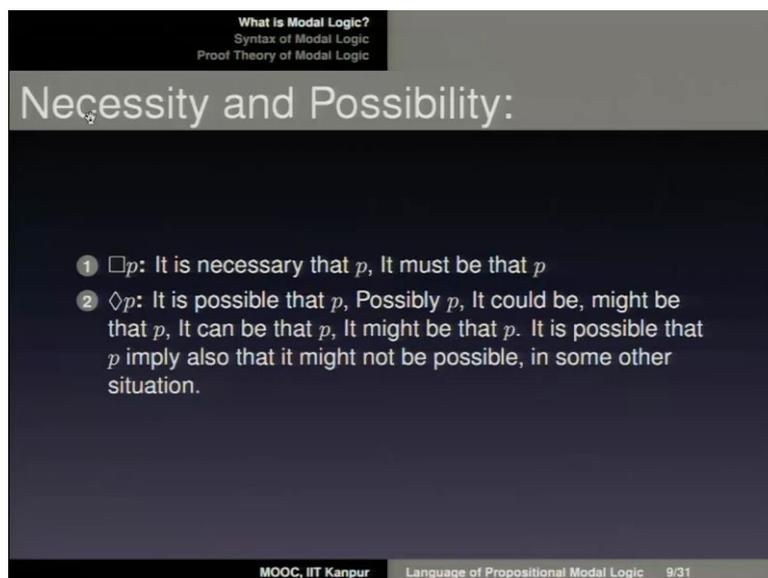
So, the idea it is that the same kind of necessity, same kind of theorems that you come across in alethic modal logic's may not be theorems in other logical systems, Any theorem in any other logical system may not be the theorem in or alethic modal logic's. For example, if you take a simple example that in a necessity of P implies, necessity of P implies P.

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So, this particular kind of theorem it follows in logical system D, but this may not apply in some other kind of systems or any other axiom that you take in to consideration, T for example, this holds in T, necessity of P implies P, and all the necessity tools are actually true. So, now, if you translate this same kind of sentence into deontic logic for example, now this is translated as, it are to be the case of you followed the traffic rules, implies that you actually follow the traffic rules, this may not be theorem in the deontic logic's; that means, same kind of theorem need not have to hold in other Modal Logical Systems.

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So, now there are 2 things which we are trying to talk about 1 is Necessity of P and other one is Possibility of P, and these are represented like this, it is necessity that P, or it must be the case that P, it is necessary that P all these things are translated as box P. Diamond P is like it might be the case, it could be the case, can be all these things comes under the category of Possibility of P, could be, might be, can be the case that etcetera this is list is be all these things can be translated in to diamond P.

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What is Modal Logic?
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Some Modal Propositions

- 1 There are nine planets in the solar system.
- 2 The Square root of 9 is 3.
- 3 It is possible that tomorrow it will rain in Kanpur
- 4 It is possible for humans to travel to Mars and it might have been the case that there is water on Mars.
- 5 It is necessary that $2+2$ is 4
- 6 It is known to us that Mr. Narendra Damodhar Modi is the current Prime Minister of India.
- 7 It is obligatory that Doctors needs to address emergency cases.
- 8 A proposition p is not possible if and only if the negation of p is necessary.

These extensions makes sense in the context of possible worlds or alternate universes. An alternate universe is one

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Let us consider Some Modal Propositions: so we are getting ourselves familiar with this, language of Modal Logic, Modal and the Modal sentences or Propositions. So, There are nine planets in the solar system. Square root of 9 is 3. Now these are considered to be the sentences which are actually considered to be true that is represented as $P \wedge Q \wedge$ etcetera represented by some proposition variables. Third one for example, if we say is possible that tomorrow it will rain in Kanpur, so this is different from simple sentence like P in Classical Logic we simply represent the sentences P M.

And it is possible that humans to travel to mars, and it might have been the case that there is water on mars, and It is necessary that 2 plus 2 is equal to 4 it is consider to be necessary kind of statement, and it is known to us any Indian most of the Indian there should be able to at least know, who is the chief prime minister of India? etcetera and it is known to us, that mister Narendra Modi is consider to be the current Prime Minister of India, and you can also say that I believe that, suppose their some people who are living

in some kind of (Refer Time: 16:59) so something like that, they firmly believe that someone is the Prime Minister of India they view that, Rahul Gandhi is the Prime Minister of India, you can believe many things, but when you say that I know that Rahul Gandhi is a Prime Minister of India nobody is going to accept, but it was the current Prime Minister is Narendra Modi.

I can say it is obligatory that doctors needs to address emergency cases. It is the deontic logic we use this particular kind of sentence, a Proposition P is not possible if and only if the negation of the P is considered to be necessary. So, the idea here is that each and every Modal operator comes up with some kind of duels, they are considered to be duel operators for example, if you say a Necessity of P this can be define in terms of Possibility like this, it is not possible that not P. So, you can read it like this, not P is not possible or not P is consider to be impossible, if you rule out not P and what is left is P only, that P has to be necessarily true. So, that is what you mean by a necessity of P, Possibility of P is defined as it, it is not necessary that not P is the case so; that means, not P need not have to be necessary.

So, in this way we translate these things in to appropriately like this, and each and every modal operator comes in comes in (Refer Time: 18:39). So, we translated in this way the same if we take the negation of this one, it is like negation, goes inside this one and it is possible that not P and then negation of possibility of P, by the definition is necessity of not P, you push the negation inside and then negation of possibility will become necessity operator.

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What is Modal Logic?
Syntax of Modal Logic
Proof Theory of Modal Logic

What could have been.....

Modal statements are about **what could have been** and they occur for example in the following examples.

- 1 Hitler **could have won** World War II;
- 2 I could have been a fisherman or farmer.
- 3 The speed of light could have been twice as fast as it actually is;
- 4 Swans could have been black; It is impossible for there to be round squares;
- 5 Necessarily, $2+2=4$.

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So, Modal statements are ultimately about what could have been, and what and they occur for example, in various kinds of examples. So, various example which you can think of they are there are like this Hitler could have won the world war history would be very interesting when you take in to consideration various kinds of possibilities and counterfactual, or you can say that I could have been a fisherman or farmer (Refer Time: 19:30) being a teacher something like that, the speed of light could have been twice as fast as it actually is and the calamities would be more because if we substitute E is equal to MG square, it will be too much are all swans are white is the case which you commonly observe and you might be wondering, you might be interested to talk about these kinds of sentences swans could have been black or it is impossible that there are round squares its up ward kind of statement are necessarily 2 plus 2 is equal to 4

So, in this class what we have seen is that we have just started with what exactly is the one which we just maintain the distinction between "intention" and "extension", and we said that Modal Logic's are intentional in nature. So, any kind of modal sentence it cannot be analyzed by Boolean kind of operators, that is if, if you have a modal sentence Necessity of P implies Q it cannot be evaluated simply if you know just know the truth value of P and truth value of Q . We need something more. So, there is a one which will be going to talk about in the next class, in the next class will be dealing, with will be continuing with this language of Modal Logic, where will be covering the language of

Modal Logic first we will be starting with the syntax, and then we will be taking about the semantics.

Thank you.