Basic Concepts in Modal Logic
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Lecture – 11
Strict Implication: Examples

Welcome back. In the last lecture we discussed little bit about strict implication and more on logic as A merged with thorough dissatisfaction with respect to the material implication. So, we just when it concerned with the material implication, we have two issues that are the first issue is it that A two proposition is implied by any strange kind of proposition and A first proposition implies anything.

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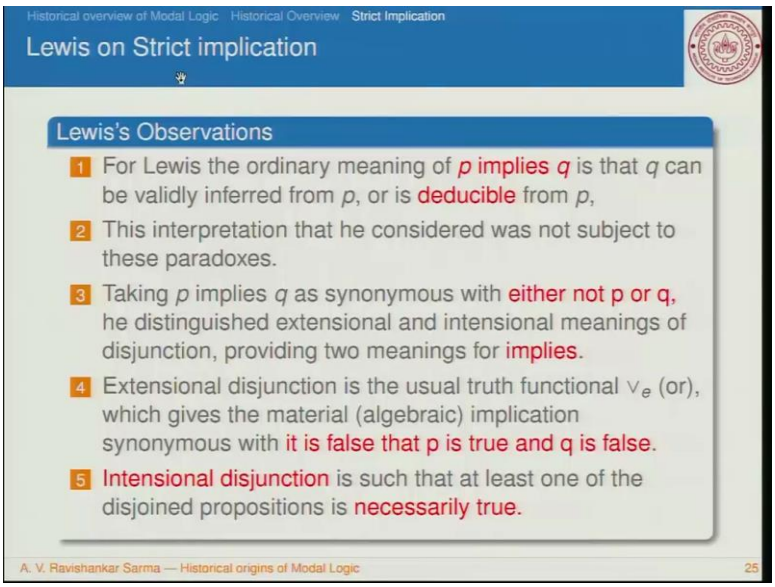

And Lewis has come up with another kind type of implication, which he calls it as strict implication. So, what is different in this strict implication some of the things, which we are discussed already in the last class, but we will be we will be continued with the things that we have seen in the last class.

So, here are some of the Lewis's, C I Lewis's important observations on strict implication. First of all he is not happy with happy especially when they talked about material implication. According to Lewis he does not imply that you know q deduced from p. So, q deduced from p is not just material implication. But it is it as to be p that

the anticipated of the condition as to be strictly hook to the consequent that is q. So, in the sense A implies B if it is material implication.

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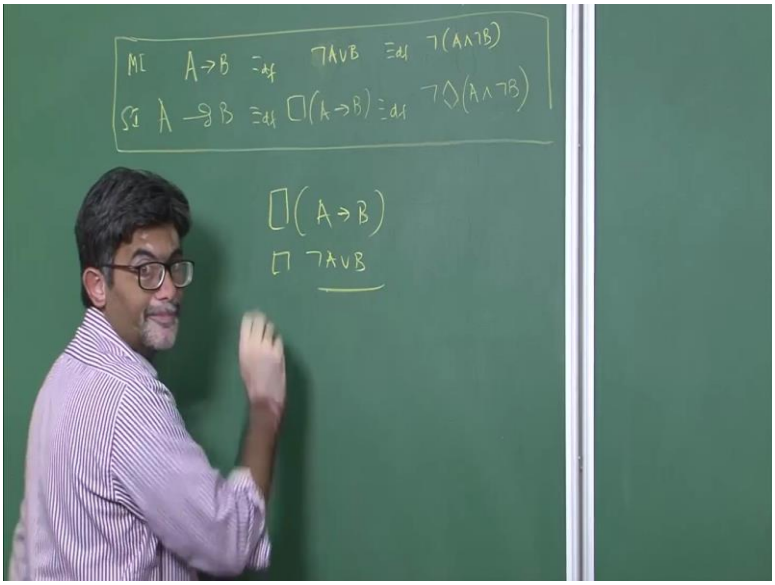

The definition is like this it is not the case that A false or B is the case or we can read it like this it is not the case that is true and B is false, strict implication A strictly implies B this is symbol that is used there A is strictly hooks something B it is defined as A implies B is considered to be necessary or we can view it in this sense that it is not possible that A is true and B is false. So, here if you write A implies B. Of course, this can be written like this not A or B. So, these are some of the important observations of C I Lewis with respect to strict implication, the already meaning of p implies q is that q can be validly inferred from p that is q is deduced from p. For example, if you have p all the write down p will be deducing q. So, q is deducible from p. So, this is where Lewis is unhappy with this particular kind of thing whether deduction is captured by material implication or not.

So, this interpretation when not p or q implies q is defined as not p or q he constrain was subject to paradox, if you view it in this way it leads to paradox what are the paradox is A two proposition is implied by any strange proposition; that means, all tautology should come as an outcome of any contingent or any kind of strange proposition and A false proposition implies anything and there are several instances of paradox of material implication. So, taking p implies q synonymous with either not p or q, he distinguished extensional and intentional meanings of disjunction. So, for any logical connective apart

from negation we have conjunction disjunction implication and by implication, each and every logical connective as is corresponding intentional connection like in intentional disjunction, intentional conjunctions, and intentional implication that is strict implication circle.

So, he is trying to provide A sort of two different kinds of meanings of the word implies p, we have to we have to this way extensional disjunction is construct to be the usual truth functional connective. For example, if you say p or q the truth value of p or q is solely determined truth value of p and truth value of q. Let is simply viewed as it is false that p true and q is false q is true that is A implies B is not A or B in intensive disjunction is suggest at least one of the disjoined proposition has to necessarily true.

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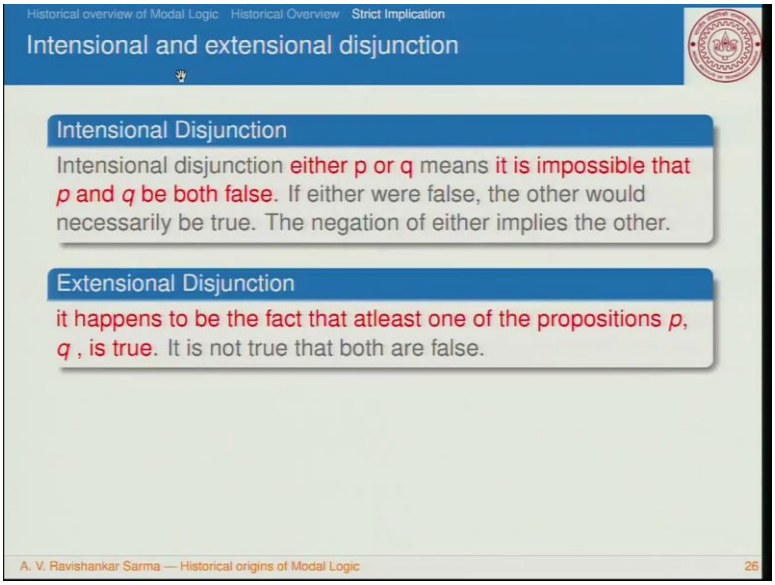

For example, this is A difference between intentional and extensional disjunction, when intensional disjunction either p or q means it is impossible that p and q can to be both false. If either were false the other would be automatically be necessarily true, but the negation of either implies the other one.

Whereas, in the case of extensional disjunction, it happens to be the fact that at least one of the propositions p is true and q is true it is not true that both are considered to be false, but in the case of intensional disjunction. If you say p intensional disjunction q, it means that either p or q is A case p or q means it is impossible that p and q be both be false there will be that the possibility that p and q is true.

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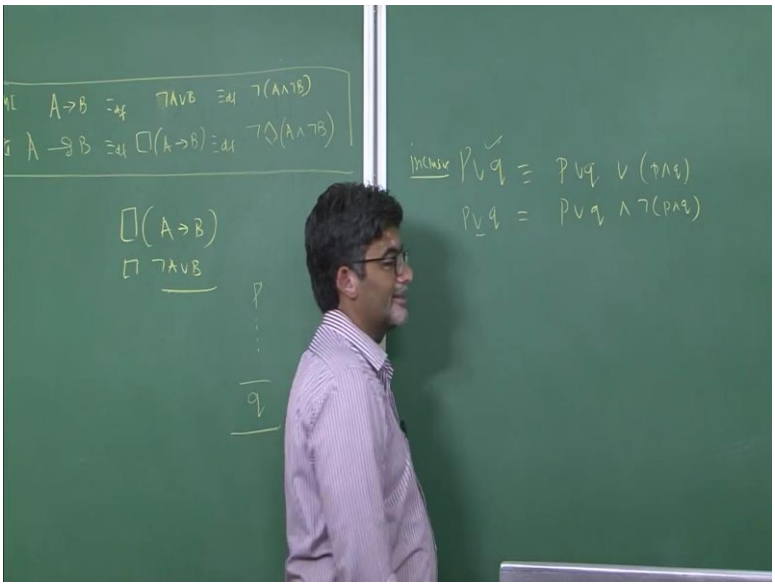

Usually when, you write extensional disjunction, it is going to be like this p or q it is either p or q or it can be both also. So, usually we separated with another kind of thing which is considered to be inclusive or I will write it here inclusive or. So, it is like either I take coffee or I take tea. If I will take coffee and even I will be taking tea also, but inclusive or means p or q, but not both of the thing; that means, if you take tea you will he would be to take coffee. So, that is not the one which I am talking about here extensional and intensional disjunction is one which we are talking about, if you take intensional disjunction to consideration, Lewis is up the view that out of these 2 disjunction p q 1 as to be necessary other one can be actually true its, but at least one of the disjunction have to be necessary.

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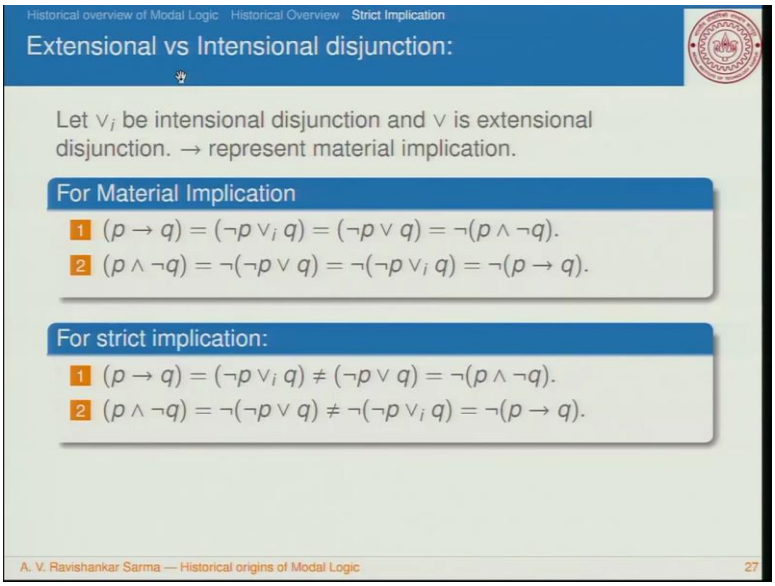

So, here is the distinction which is trying to make if extensional and intensional disjunction once we thoroughly study the disjunction of course, negation and with negation and disjunction you can easily come up with implication because A implies B, A can be defined as not A or B as you are see it here for the case of material implication p implies q is equated as A c not p intensional disjunction q are it is equivalent to not p or q. It is not such kind of distinction between intensional and extensional etcetera. Everything same and that is same as it is not the case that p is true and q is false in the same way, if you take p and not q that is A conjunction. So, that is equivalent to it is not the case that not p or q it same as that using De Morgan's laws and it is same as it is not the case that not p or q and this is same as it is not in the case p implies q. So, first we talked about p implies q now we talk about not of p implies q as you are seen it here all are equivalent to each other.

So, this is the main interesting observation that Lewis had made and then using this observation we could come up with strict implication and the idea of coming up with A strict implication is that, he wants to avoid some kind of paradoxes. Paradoxes that will have seen are true proposition is implied by any strange proposition falsity implies the anything they some of the most important things that we need to address there. In the case of strict implication p implies, q definitely it is not p inclusive or q sorry, it is not inclusive or intensional disjunction q, but this is not equivalent to not p extensional disjunction q this is way on the difference rise of course, this not p or q is equivalent to

not of p and not q. So, these 2 are anyways not equivalent to the first one that is not p intense intensional disjunction q. So, this kind of equality will not hold for the sake of purple strict implication. So, this is for this reason strict implication is different from material implication and with this while differentiating this intensional and extensional disjunction. Lewis has come up with strict implication and his we are hoping that you know this strict implication can handle at least paradoxes sums of the paradoxes of material implication.

So, how we can handle the paradoxes of material implication, I now right now he is talking about this that, if you one of the instances of paradox of material implication is this.

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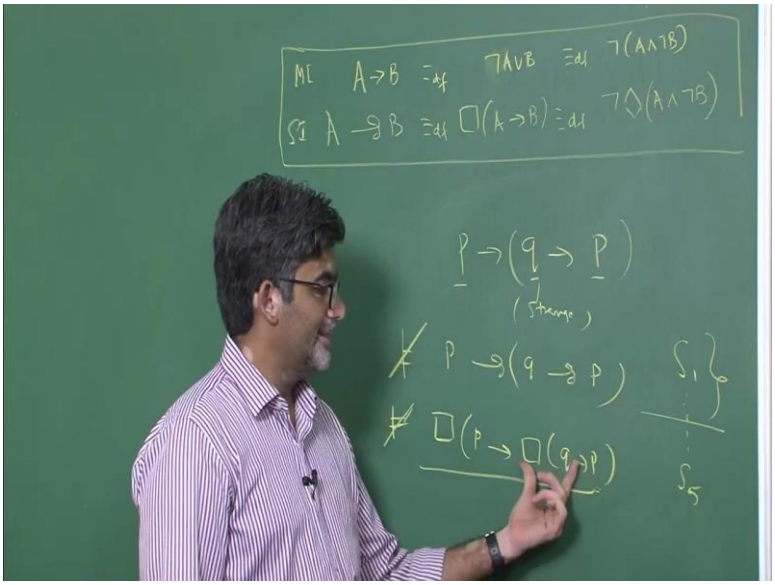

For example, p implies q implies p. So, if p is true now this true proposition is implied by any strange proposition it is considered to be any strange proposition it can be contingent it can be any other irrelevant kind of proposition I easy. So, new knowing the implication as strict implication may be you can write like this p strictly, implies q strictly implies p now according to Lewis this is not A theorem the logical system that he is trying to come up with yes come up with 5 logical system S1 to S5 and in some of the papers he mentioned it that the first two logical systems seem to be capturing the strict implication in the better way than the other systems. So, this is an interesting book written by c I

Lewis survey of symbolic logic this book is still used as still serving as A classic text book for as six exhaustive survey of symbolic logic.

So, it will view it like this will become p implies q implies p now some where way to show that this is not theorem or it is not valid in your logical system, then you are achieving your purpose because this is leading to paradox of material implication; that means, truth is implied by any strange of proposition. We do not want tautology is to be inferred from stranger kind of proposition. So, if we can someone show that in A logical system wish where going to talk about little bit later this does not come as an outcome of out coming your logical system then you have achieved your purpose A purpose is that you want to avoid paradoxes of material implication. So, these are some of the things such use some more observations.

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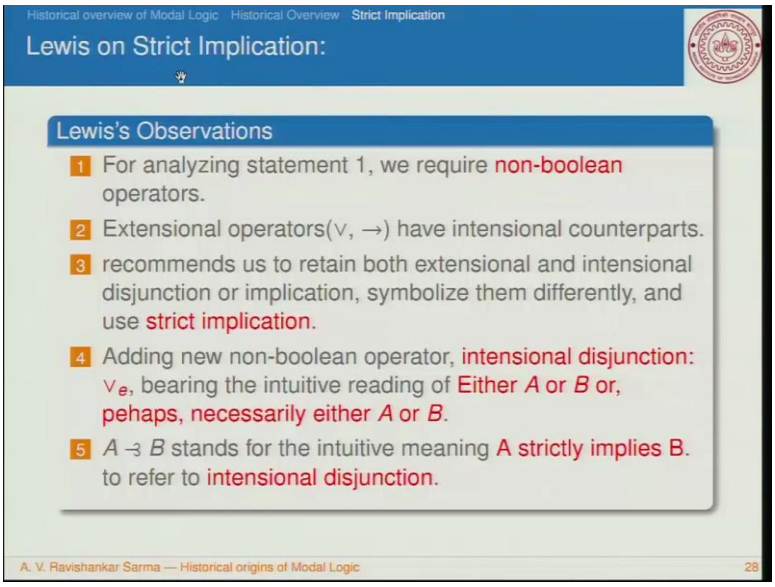

So, for analyzing the statement any statement which includes intensional disjunction or implication etcetera; we require non-boolean operations like necessity possibility etcetera. There all non-boolean operators what is the non-boolean operators and negation and conjunction implication by implication etcetera. So, we have extensional operators like or implies etcetera, and it as his corresponding counterparts their intensional disjunction intensional implication that is strict implication.

We recommend as to retain both extensional and intensional disjunction implication A material implication is seem to be serving our purpose in many number of situations. Say

it captures mathematical reasoning we can capture mathematical reasoning in A better way based on this material implication, but when it comes to day today reasoning and other kinds of reasoning that you come across usually, we call it as common sense reason it may not help us much, handling A new non-boolean operator. So, that is intensional disjunction which is represented as or with A letters v e bearing the intuitive reading of either A or B or perhaps, necessarily either A or B that is what we will be mean by A or B it is A simply like A or B what it is A or B or perhaps even necessarily one of the things have to be necessary.

So, that is what we mean by intensional disjunction we talk about intensional disjunction of anything A or B at least one of the disjunction has to be necessarily true. So, the movement you bring in necessity of it. It is intensional kind of operator. So, it becomes non-boolean any way. So, A strictly implies B stands for the intuitive meaning the intuitive meaning of this one is this A strictly implies B and it is referring to intensional disjunction if you simply it then it will becomes not A or B not A or B not A or exclusive sorry not intensional disjunction b.

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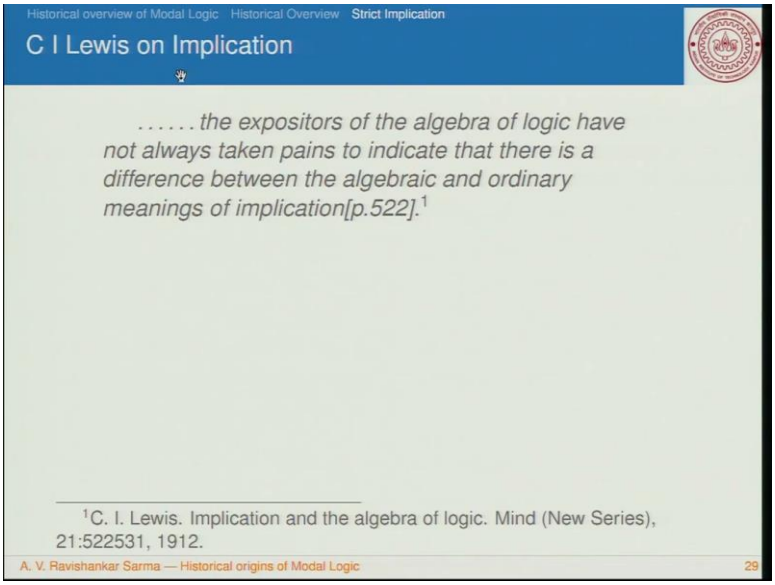

So, Lewis here is A quotation which is taken from the book I think C I Lewis, implication and algebra of logic thus published in mind he is say like this the expositors of the algebra of logic have not always taken pains, to indicate that there is A difference between algebraic and ordinary meaning of implication algebra implication is simply

seems to be material implication. Whereas, ordinary meaning of implication missing to be little bit different.

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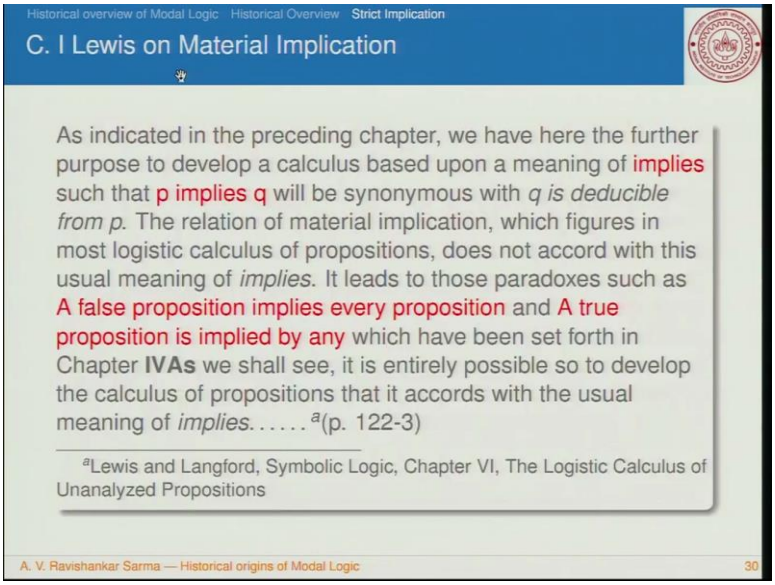

So, these at the way we have here the further presuppose that further purpose is to develop A calculus based upon the meaning of implies such that p implies q will be synonymous to q is deducible from p material implication will not serve our purpose, but p strictly implies q according to be serving our purpose.

The relational material implication which figures in most logistic calculus of proposition that is not you; find it in the princity of mathematical and followed by the particular that I v A s axiomatic system etcetera. It does not acquired with this usual meaning of implies, implies means according to Lewis is not material implication it is strict implication which is defined as it is it is not possible that p true and q false.

So, q to the base paradox such as A false proposition implies every proposition and true proposition is implied by any strange kind of proposition which have been set forth in 7 such chapter we are going to see is entirely possible. So, to develop calculus of propositions that it across with the usual meaning of implies usual meaning of implies is strict implication according to c I Lewis is of the view that material implication fail to capture what he calls it as deduction when we say if q is deduced from p material implication fails to capture it.

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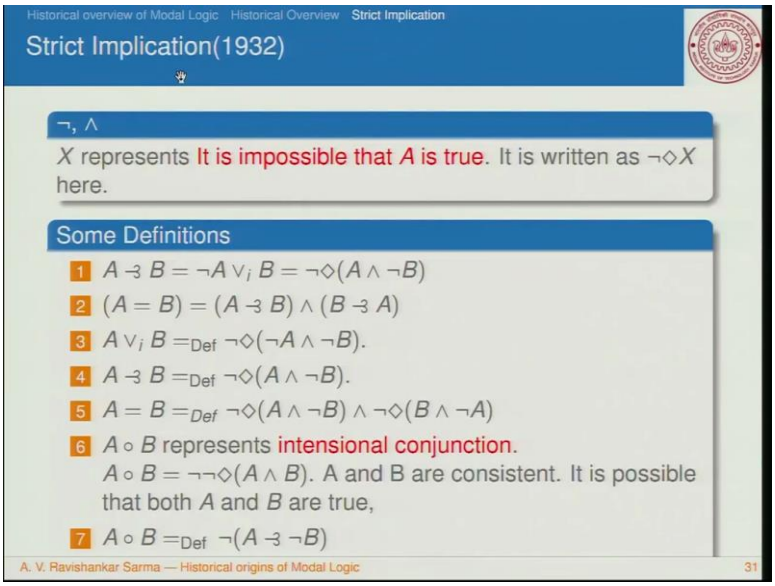

Now, strict implication is like say X represents this thing it is impossible that A is true then we write it like this not of something is impossible that X is A case then, you write it like this not followed by diamond operator and X . So, now, there are these are the some of the definitions that we comes up with in his axiomatic systems from S1 to S5 here strictly implies B means not A intensional disjunction been you are saying that it is impossible that A is true and B is false in the second d 1 A is strictly equivalent to B provided A strictly implies B and B strictly implies q B strictly implies A and A intensional disjunction B means by definition it is not possible that both A and B are false. Now A strictly implies B we can be defines as another way that is it is impossible that A is true and B is false in the same way A disconjunction, B that is A that what you are finding it in 6. So, that is nothing g, but naught of possibility of A and B. A and B are set to be consistence naught, naught is naught, naught possibilities possibility of the possibility of A and b. So, A and B are said to be consistence it is possibility that both A and B are true.

So, like this you know conjunction can be defined in terms of strict implication intensional disjunction is defined in terms of strict implication like this.

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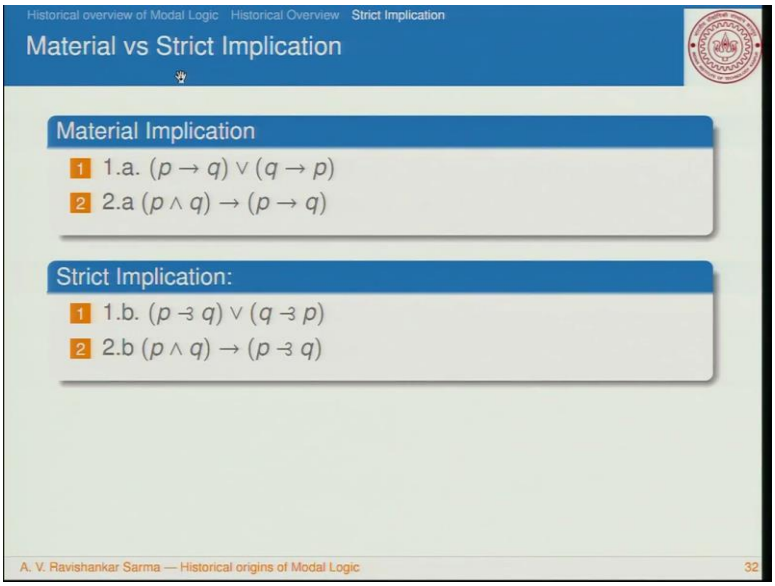

So, this what is the case p implies q are q implies p that is p strictly implies q and p and q implies p this is A two theorems in material implication in the case of strict implication p strictly implies q or it as to be the case, that q strictly implies q means q strictly implies p and p and q strictly implies p strictly implies q all are considered to be. So, one interesting example you try to considered.

So, that is like this to distinguish material implication and the strict implication. So, the example goes like this now, here is A debate between the prosecutor and the defense lawyer in A court it is A very simple kind of argument, but let us try to see or feel particular effort this argument in particular why you require strict implication.

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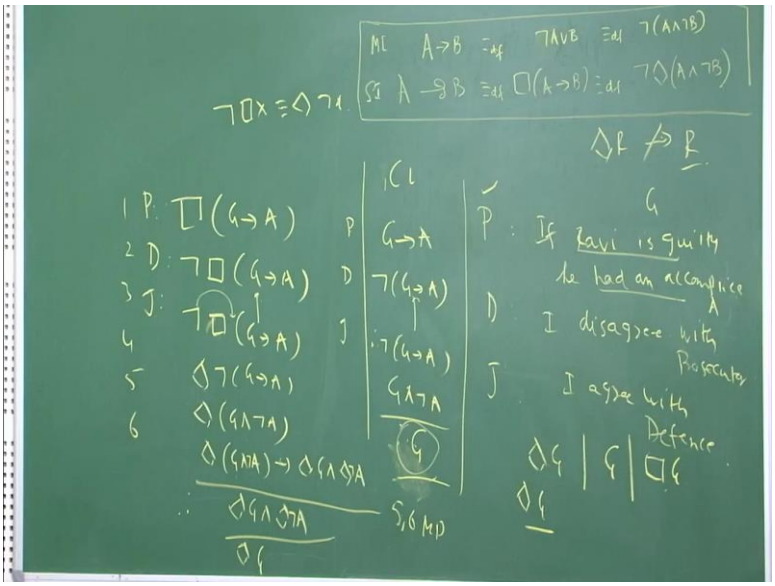

So, it is like this. So, the prosecutor is saying like this if, Ravi is guilty is guilty he had an accomplice; that means. Whenever Ravi commits some kind of whenever is he said to be guilty he always accompanied by someone else or other he never commit crimes without others help alone he cannot commit crime.

So, we have not analyzing this particular kind of sentence this simple kind of argument which goes in the court let us says. So, now, the defense lawyer; obviously, they will be A defense lawyer and they will be A prosecutor. So, you will be going to the defense lawyer and then prosecutor will be arguing argued against the defense lawyer. So, he is trying to defend the Calphater. Whatever it is and prosecutor will be going against what the defense lawyer says. So, the defense lawyer says I disagree; I disagree with prosecutor it looks like. So, simple that is the argument is simple, but you know that assume that he has provided enough evidence and with that evidence he could come off with these kind of conclusion that I disagree with the persecutor; that means, whatever he saying is denying.

So, now, ultimately there will be a debate between prosecutor and the defense lawyer and ultimately judge as to take a decision. So, judge either he has to defend upon the argument of prosecutor and defense lawyer judge make his own conclusion, and his conclusion is that I agree with A defense whatever defense lawyer says saying to be convincing for the judge that is why he is he is saying this particular kind of sentence.

So, now if you just follow classical logic and this is simply translated as this g Ravi is guilty his represented as g and he had an accomplice this represented as a. So, this is first sentence is this thing. So, this is what is the prosecutor is trying to do now the defense lawyer says that I disagree; that means, whatever he is saying yield disagree not of g implies a.

So, judge ultimately judge passes his conclusion I agree with the defense means this is the conclusion that is d f d not of G implies A. So, from this we can conclude this thing G and not A because G implies A is not G or A and not of not G or A is G and not A. So, from this; whenever you have p and q you can always deduce q. So, from this you can say that g is considered to be guilty.

So, now, this is not of instance towards we have following the definition of material implication and then you are come up with the view that Ravi is considered to be guilty there is no way which you can escape. When I say when I just say simple letter G; that means, he is stands for G is considered to be actually guilty. Now if you take the same argument in the case of model logic in particular this sentence is represented as it is necessary that G implies A. So, this is what is prosecutor is saying.

Now, defense lawyer he is negating this g implies a. So, now, judge he passes his judgment and then he is exactly accepting this particular kind of G. So, now, this is same as this one. So, this is kind of judgment that he is passing. So, which is different from this one let us try to see in what way it is different this. So, this by definition we have this thing whenever we have not of not X that is means it is possible that not X is a case. So, now, you put the negation inside and negation of necessity will become diamond operator that is possibility and G implies A. So, this becomes now G and not A. So, now, there is A difference between g and not A and it is possible that G and not A.

So, now, we have A theorem which we need to do it in the next class in particular, we need to prove this particular theorem when the context comes we have A theorem which tells us that possibility of G and A implies possibility of g and possibility of A suppose, if this is considered to be A theorem then individually it is distributed kind of property the possibility of G and possibility of A. Now substitute not A for A this will become this G and not A and then possibility not A now these two more despondence, 1, 2, 3, 4, 5 and 6. 5 and 6 more despondence you will get this one it is possible that G and it is

possible that not a. So, ultimately not a are trying to conclude is this thing not G, not G it is possible that g is the case.

So, there is A difference between possibility of G and necessity of g that is what we are all to trying to make it and trying to make this particular kind of point and this triggers as to do model logic. Otherwise if there is no distinction between possibility of G. G and necessity of G all the model distinctions collapses and we are just talking about proposition logic only.

So, now, it is possible that G means it is possible that Ravi is considered to be guilty. So, there is a difference in stating that it is possible that it is possible that it is raining outside and it is actually raining it is possible that it is raining it does not imply that it is actually raining it is possible that A may be something else, but that is A mean that I am actually that particular kind of thing. So, possible that p does not imply p.

So, we want to make this particular kind of distinction and then if you if you view this thing without referring to intensional and extensional disjunction etcetera. Then in one set of argument you are reaching to a conclusion that it is actually G and in other set of conclusion here is to be the conclusion possibility G. So, there is a way in which you can escape from the thing it is possible that G automatically implies that it is also possibility not G the case is not also said to be guilty, always some kind of element of doubt it leads us some kind of doubt if you argue in this way if you argue in this way there is no way to escape from it. So, everything is meaning G is considered to be actually true. So with Lewis has come up with various axiomatic systems. So, that they are like this you know he has come up with various axioms system which are S1, S2, S3 etcetera.

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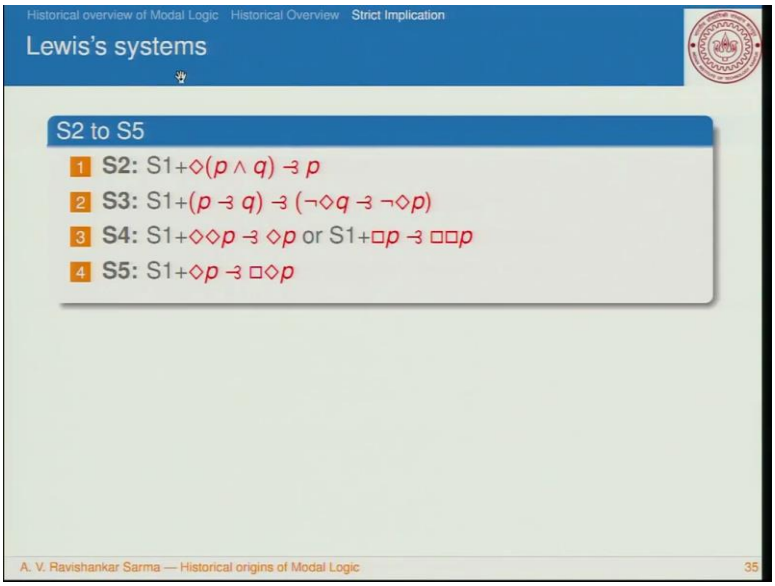

So, ultimately what a four important is towards is for the sake of strict implication. We view it in such A way that S1 and S2 seems to be capturing the strict implication in A much better way. So, out of this S1, S5 these are considered to be the most important systems. First two systems we called it first three system we called it as non normal model logical system and S4 and S5 he use it as normal model logical systems in the next class. We will be talking about the language of model logic where we will be talking about syntax and the symmetric.

Thank you.