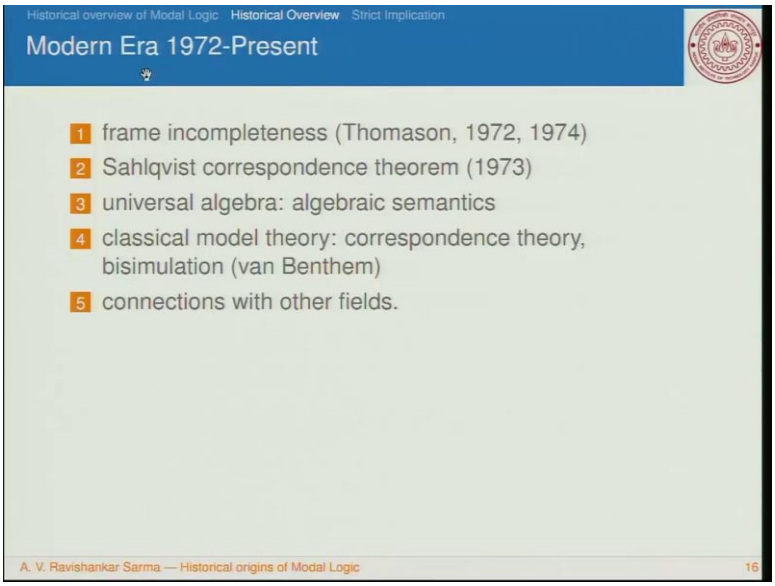
Basic Concepts in Modal Logic
Prof. A.V Ravishankar Sarma
Department of Humanities and Social Sciences
Indian Institutional of Technology, Kanpur

Lecture - 10
Strict Implication

Welcome back, in the last lecture we have seen some of the historical origins of model logic, we need to study any subject keeping it in the historical context. When we will come to know the continuity of that particular subject for that reason, I covered some other things all though this lecture may not be that interesting to those your learning model logic, but definitely it is if importance particular to those who are doing some kind of researching the area of model logic.

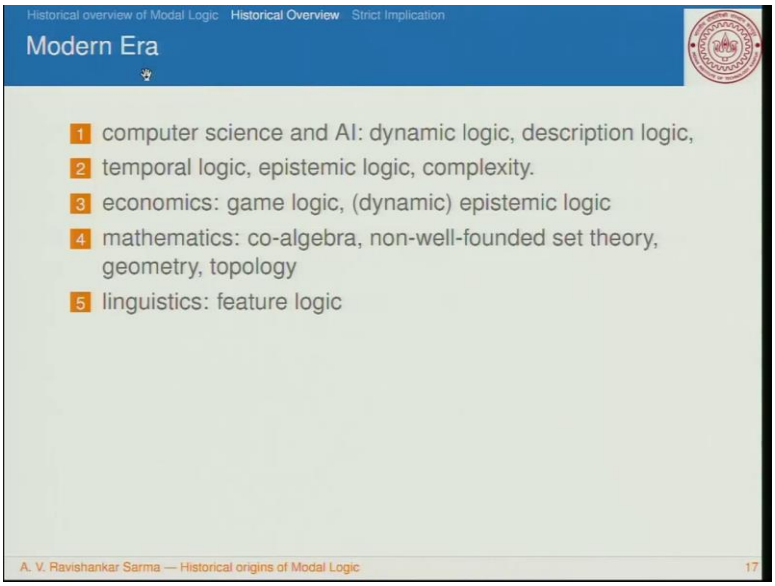
So, it is always better to divided into different periods then it is easy for as to handle things. So, where we started our journey with the pre era and I will started it to medieval period and then we spoke about lively and then we spoke about syntactically era of model logic till 1960's and form 1960's to 1970 is construct to be the classical era. Where there was excessive emphasis on the relational structures possible worlds etcetera and then comes to I missed out this modern era. So, what is what is from 1970 to present in the most of the advance logic courses you will be dealing with this modern era. So, there are certain interesting things, which you need to mentioned in our modern era. Then after mentioning it, I will be moving on to the strict implication that is of that is of interest to our course that is basic concepts of model logic. Because it is strict implication which arose out of the distance arose out of the necessity to explain the problem with respect to the material implication and that like to the development of model logics.

(Refer Slide Time: 01:58)



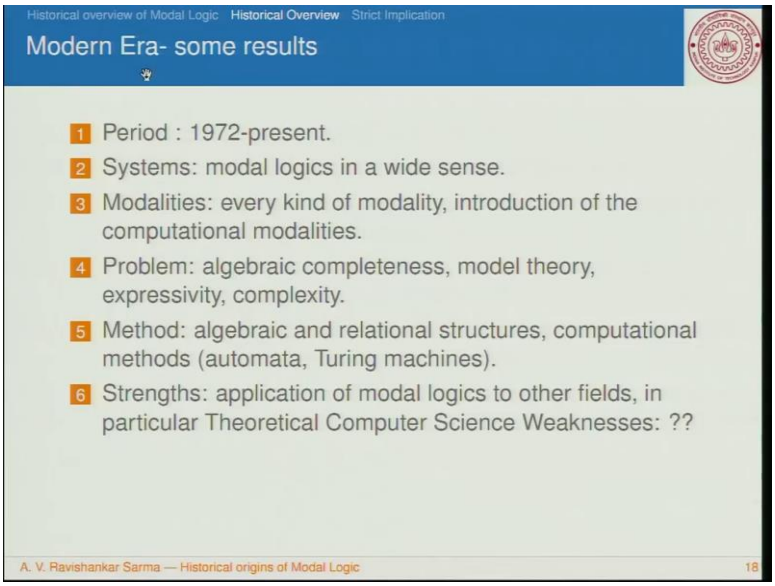
In the modern era 1972 to till to date there are many interesting results actually, you will come across in the modern era, these are like this frame incompleteness by Thomason 1972 to 1974, sahlqvist correspondence theorem these are considered most popular well known kinds of results and there was some kind of algebraic term to the model logic and then people was started studying about many valued model logics etcetera and classical model theory of model logics correspondence theory and then, there were some interesting important thing like such as Bisimulation it, was discovered by Van Bentham and they were connections of model logic to different fields like linguistic artificial intelligence computer science etcetera all these things were the recent developments. So, we see model logic in a greater context. So, most of the developments are quite easily considered to be young discipline in the sense that most of the developments took place after work although some of it is traces are there most of the ideas we borrowed, it from Aristotle's libraries and some other medieval logicians, but it is the formal kind of analysis in ever which you find it only after 1920's.

(Refer Slide Time: 03:21)



So, in the modern era much focus was on the computer science and AI in the fields of AI, for dynamic logic, description logics, are the new branches which are emerged. So, all these things if you want to know more about these thing we need to have some knowledge of the basic model logics in this course we will be talking about only Alethic model logics some necessity logic of necessity and logic of possibility and we are it is just considered to be starting point and then we are concerned with earlier propositional model logics and then we are lot of applications of temporal logic and epistemic logic, logic of complexity etcetera economics used in the area of game theory therefore, always this is. So, some kind of interplay between epistemic logic and the game theory and there were well known results like omens, inam etcetera whether we agree to disagree etcetera and the linguistics recently interesting come up that is feature logic.

(Refer Slide Time: 04:25)

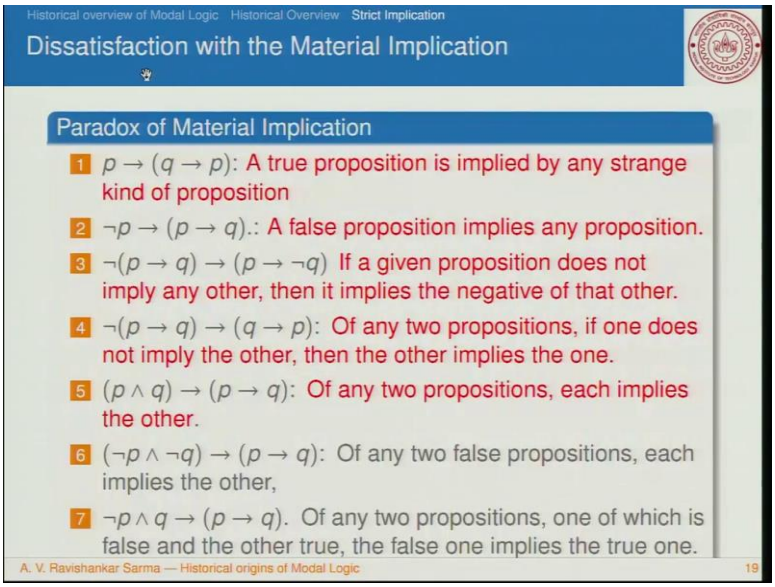


So, these are some of the results of modern era that is 1972 to present the different systems which are developed model logics. We are studied in a wide sense and modalities it is every kind of modality introduction of the computational modalities on the ones, which you see it in this era and one of the problems of the modern era is that each and every era has it is own problems Aristotle Grafil with future contingence and the process of expanding non logical proof for the existence of God etcetera.

You have to come up with there is possible worlds and within the formal analysis which you find it in the libraries and then the problem is, in the modern era is that algebraic competences and model theory expressivity complexity etcetera. All these are problems in the modern era. The method that was used in the modern era is a combination of algebraic and relational structures, which you find it in the classical era and several logicians are also started using the computational methods such as automata and Turing machines. So, in a model logic is discussed in philosophy departments, computer science, mathematics etcetera, but these days you will find it many interesting courses you will find it in the computer science and mathematics departments; early it is in philosophy, but you will study this things in the computer science department get of the obligations one of the strengths of this modern era is that application of model logics to various other fields in particular to the theoretical computer science, weaknesses is the one which you need to come off with because this some going kind of research in this area.

So, what we are seen so far is we have seen development of modern logic in a very crude manner, at least recover some of the most important things that are there in this four periods. So, you will be focusing our attention on the first three periods on will be ignoring the fourth period all though it is considered to be important, it is difficult to cover everything in the course, that is why we will not be dealing with the modern era for definitely we will be talking about some of the important things such as a well being knowing some of the interesting things such as relation structures etcetera, which is considered to be important to understand the other things.

(Refer Slide Time: 06:56)



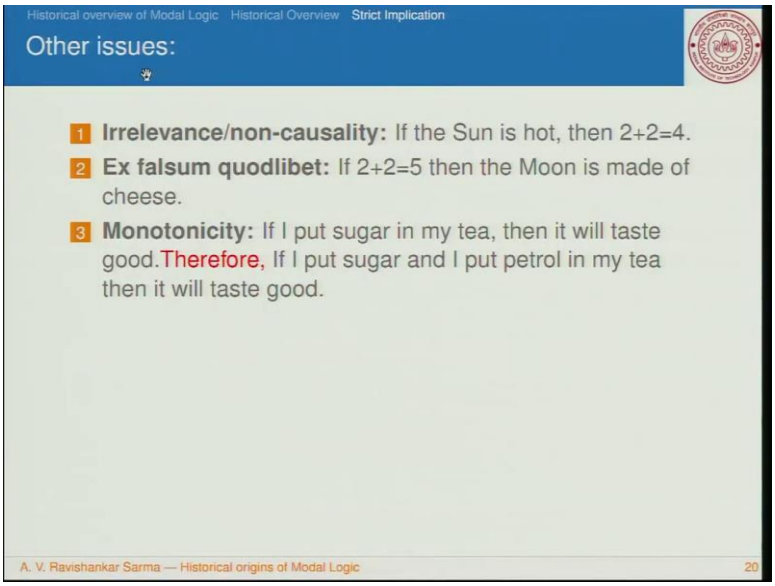
So, now I will be focusing my attention on the strict implication, how this strict implication did has coming into existence? There was thorough dissatisfaction with respect to the material implication. So, material implication is defined in the sense p implies q means is defined as not p or q or it is not the case that p is true and q is false. With that you know, we can also talk about the validity it is not possible for the premises to be true and conclusion to be false severe Luis is this of the view that implication that is used by in their book it is not sufficient enough to captured what we call it as deduction.

So, some of the problems with respect to material implication is this thing, a true proposition is implied by any strange kind of proposition that is a case in the first one p plus p plus p particularly when, p is true a true proposition is implied by any strange

proposition here the strange proposition is q and if the statement is false, there is not p then false proposition implies any kind of proposition if 2 plus 2 is equal 5 when, you can show that move is made of move is, made about all those there is no connection between two plus is equal to five and move is made about, but material implication does not take care of in the relevance relation etcetera or you can have several instances of this material implication it leads to counter and other instance is this thing of any true propositions there is the fourth one I am talking about of any two propositions if one does not imply and other then the other implies the one that is not of p implies q means implies q implies p. So, all the things which are listed out here are considered to be some kind of instance of paradox of material implication.

But the first 2 things are the once which are widely studied, that is we do not expect it a true proposition to be implied two proposition is an tautologies, logicians are absence to with tautologies why there absence with tautologies because all tautologies are considered to be valid formulas. If all valid formulas are generated by any strange kind of proposition then there is something wrongs way or a false proposition, if you begin with a false proposition. So, they logicians treat this kind of inconsistency as a hell kind of situation. If you have a falsity it leads to anything.

(Refer Slide Time: 09:32)

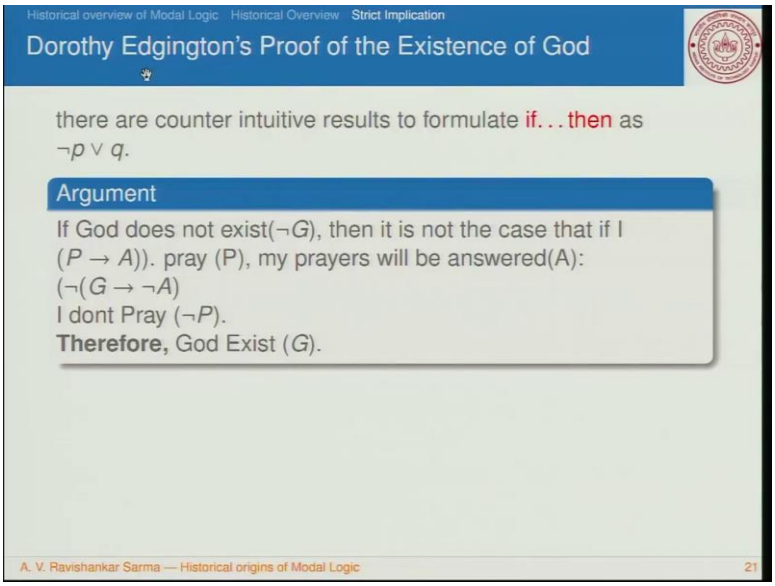


Other issues with respect to material implication is one which we are discussed in last few lectures that is irrelevance and there is no material implication is viewed in such a

way that there is no causal implication between these two and the consequent of the condition. Suppose if you say that if the sun is hot then 2 plus is equal to four sun is hot has a nothing do it with the 2 is equal to 4, but in the case of classical logic sun is hot is represented as p and 2 plus 2 is equal to represented as q and then, p plus q has is meaning that is you come off with the truth conditions of p plus q based on the definition not p are q. So, it is going to be false only when sun is hot and two plus is equal four is false and means the premises are true the antecedent is true and the consequent is false then only the conditional if the sun is hot and 2 plus 2 is equal four is going to be false there is no kind of relevance between antecedent and the consequent here another problem of material implication phases is this thing. If you have if you start with the false proposition 2 plus 2 is equal to 5 when, you can have any strange kind of proposition like in the moon then the moon is made of cheese or you can even say that if 2 plus 2 is equal 5, then moon is not made up of that is also going to be true. So, we do not want such kind of things we want to have some kind of meaningful relationship between p and q are exist in a conditional sentence p plus q.

Another problem which is there is there is a monotonicity problem, which led to the development of non modern logics then example could be like this, if I put sugar in my coffee, it will be tasty of course, sugar is there it will be tasty therefore, if I put sugar and you add some more new information to it add petrol or kerosene or diesel whatever it is in the tea then it will still be tasty or if you still you feel it as good it does not seem to be intuitive to us. So, there is seems to be some problem with the property of monotonicity. So, our day today reasoning or common sense reasoning, is construct to be non monotonic in nature and then we need to reason with the incomplete information.

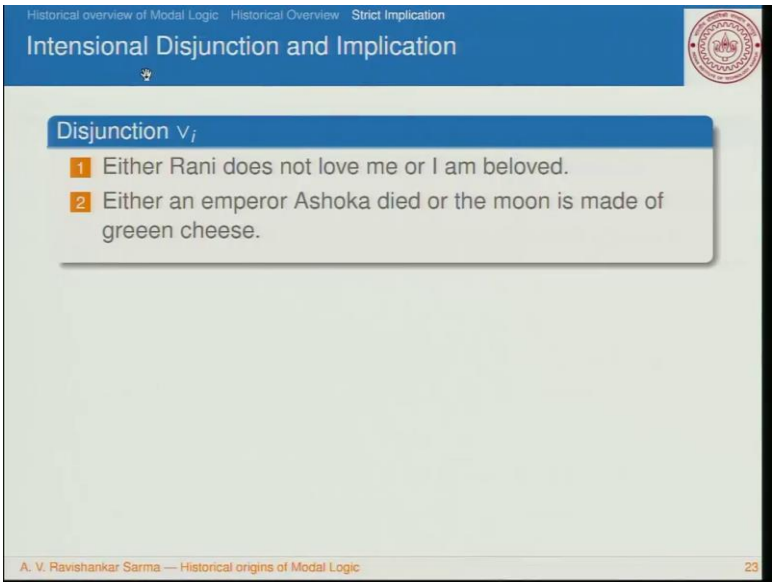
(Refer Slide Time: 11:52)



So, when is the interesting example which is given by Dorothy Edgington a leading logician he is of the he has come up with the funny kind of proof for the existence of god like this if god does not exit then it is not the case that you pay, you pray and then your prayers will be answered that is why you do not pray, but see god exist even if we do not pray god will exit nor has to exist here.

So, now it is in this context Luis has come up with distinction between suppose if we take this logical connectives conjunction disjunction implication etcetera for every such kind of logical operator conjunction disjunction etcetera we have something called as intensional disjunction, intensional conjunction, intensional implication. Suppose if you say that here is the statement which is little bit funny, but it is interesting to note for example, if we say that I became sick.

(Refer Slide Time: 12:53).



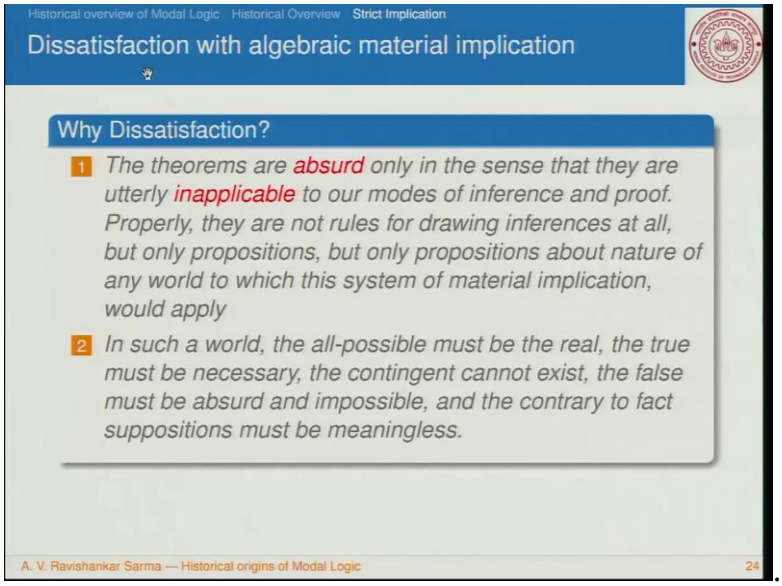
And I went to see the doctor whenever your move become sick they will go to see the doctor suppose if you reverse the sentence that is p m q is the one which has taken earlier. So, now, if is q and p I went to see the doctor and I become sick. So, this is second thing does not seem to be acceptable to us nobody goes to our doctor to become sick. So, I went to see the doctor and I becomes. So, I becomes sick and I went to see the doctor that is represented as p and p and q and second thing is there are I went to see a doctor and I become a sick that is q and p logic these two remains a same things q and p is same as p and q.

But the meaning of the formula p and q is slowly determined by the meaning of it is individual constituents p q then your miss or many things. So, these two should remain the same thing I went to see the doctor and I became sick and I become sick and I went to see the doctor these two should remain the same thing go something wrong them. So, the meaning of the formula is not slowly depending on the truth failure of it is individual constants suppose, if you say that either Rani does not love me or I am beloved see is a distant disjunction kind of thing. So, this disjunction needs to be understood as intensional kind of disjunction. So, this statement either Rani does not love me or I am beloved I am be loving loved by everyone. So, this needs to the truth failure of this one these not solely determined by the true value of it is individual constants. So, that lead to what is called as intensional disjunction. So, these was working on this examples and then he has come off with for every logical operator or logical connective that, we have

he has come up with corresponding intensional disjunction intensional conjunction and intensional implication.

In tensional implication is on which will be talking about it and that is what we calling it has strict implication. So, we have to come up with an adequate notion of implication which captures what we call it as deduction what it means to say that q is deduced from p Lewis is of view that material implication failed to capture that. So, in these two examples extension disjunction will not serve our purpose.

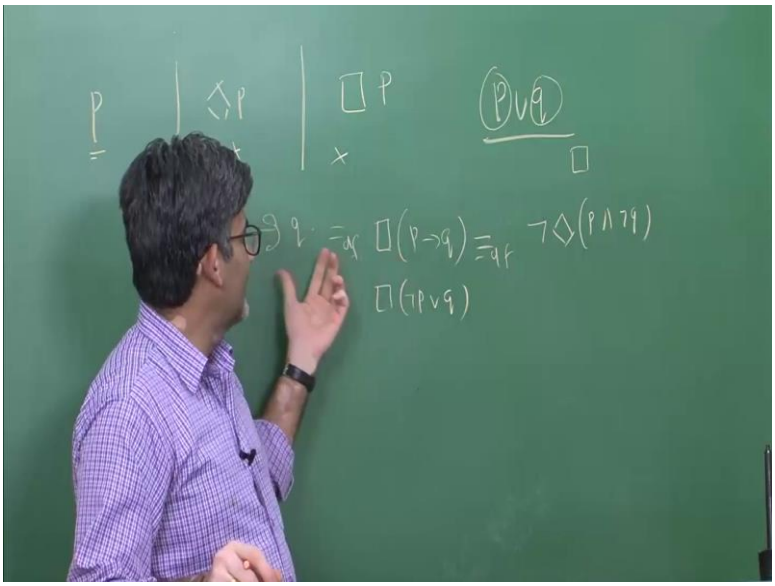
(Refer Slide Time: 15:28)



So, that is why we need to view it in a totally different way that is you need to consider intentional disjunction. So, why there is a disjunction with respect to algebraic material implication that is defined as knot p or q, one reason is that theorems are considered to be absurd only in the sense that they are utterly inapplicable to our modes of inference and proof properly they are not rules for drawing inferences at all this is according to Lewis, but only propositions, but only propositions about nature of any world to which this system of material implication would apply another reason, why there is a dissatisfaction is that in such a world the all possible must be the real the true must be necessary and the contingent cannot exist.

As you clearly see here we have Lewis was talking about this particular thing. So, that is this. So, there is no distinction between classical logic a particularly material implication is gained up to explain only the actual truths.

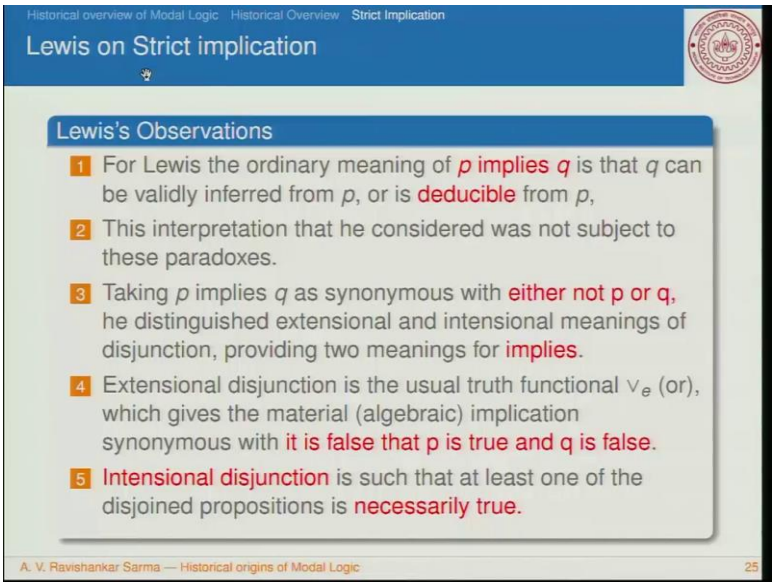
(Refer Slide Time: 16:32)



So, there is some nothing called as contingent sentence like this are something which is considered to be necessarily true, all these things are ruled out here in such a world that is the actual world all worlds must be a real they are considered to be actual the true must be considered to be necessary something is actually is necessarily true. And the contingent; obviously, cannot exist the false some statement is false and it has to be absurd and impossible and the contrary to fact suppositions must be meaningless he has nicely summarized it in such a way that, you know for example, the last statement contrary to fact suppositions must be meaningless for example, if you take a example such as this thing counter factual conditionals where the antecedent is always false, if you take material implication into consideration if the antecedent is false irrespective of your consequent the conditional is always going to be true in that sense.

Suppose if I had dropped this chalk piece it would have fallen it would have fallen on the ground that is going to be true suppose another counter factual is that if, I dropped this chalk piece it would have you turned into some donkey or cat or something like that or it would have flown up or something like that it flies etcetera. So, these counterfactual is also turned out to be true. So, it makes all the counterfactuals true, but we know that some counterfactuals true some are considered to be false.

(Refer Slide Time: 18:21)



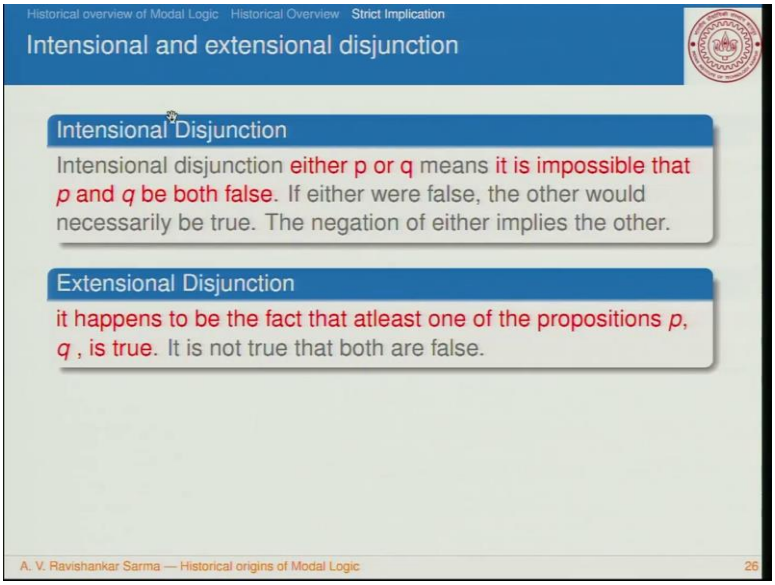
So, these are some of the observations of Lewis it is this for c I Lewis the ordinary meaning of p implies q that is q can be validly inferred from p or q is deduced from p this interpretation. That he considered was not subject to the so, called paradoxes of material implication. So, he has come up with another kind of connective which he calls it as hook. So, this is defined in this sense p hooks q or q is the deduced from p in the sense that it is necessary that p plus q or it is also defined as it is not possible that p is true and q is false. So, this is the definition that he has come up with according to him.

If you have this particular kind of definition then you can avoid the paradoxes of material implication. So, taking p implies q needs to be synonymous with synonymous with either knot p or that is the definition of material implication he distinguished between the distinguished extensional and intensional meanings of that particular kind of disjunction without disturbing the things that already working there that is material implication he extended it a little bit and then he has introduced the intensional disjunction. So, he has provided two meanings of implies extensional meaning and intensional meaning.

So, we will be talking about what is the difference extension and intension sense difference all these things little bit later in the next class. So, intensional disjunction is such that at least one of the disjoined propositions has to be necessarily true. Suppose if you write like this p or q either Rani loves me and I am below for example, if you say that thing it implicitly means that one of the either this or this has to be necessary. But in

classical logic everything is actually the case you know everything is represented as something which is actually the case. So, there is no way which can talk above contingent sentences necessary sentences like, this if you invoke intensional disjunction then you will have this kind of flexibility intensional disjunction means is such that at least one of the disjoined proposition is considered to be necessarily true.

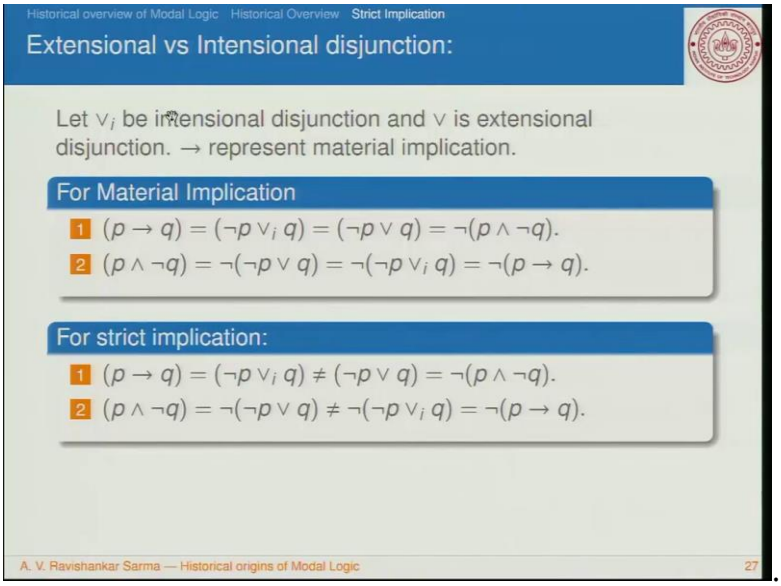
(Refer Slide Time: 20:50)



So, intensional disjunction defines it like this either p or q means this thing it is impossible that p and q to be both false in a similar way he has come up with the definition of the intense implication that is it is impossible that p is true and q is false.

So, while maintaining this distinction intensional disjunction and extensional disjunction Lewis has come up with the notion of strict implication. So, when you talk about p implies q is not p or q this is what he means by this. So, this is not p or q it is necessary that knot p or q is the case it is same as it is impossible that p is true and q is false ah.

(Refer Slide Time: 21:48)



So, these are the sum of things which holds for if you maintains this kind of distinction intensional and intensional disjunction and there are certain things which holds for material implication there are certain things which holds for strict implication for example, in this case p implies q not p it is a it is a relationship between extensional intensional disjunction this like this p implies q is billed as p intensional disjunction q it is same as not p or q and it is also same as not of p, p is true q is false in the case of strict implication p implies q means this same knot p intensional disjunction q is not equal into not p or q.

So, these 2 are considered to be totally two different thing because if take any in case of intensional disjunction in any intensional disjunction which connects to propositions one of the thing has to be necessarily it is for this given not p r I i stands for intensional disjunction is not equal into not p or q. So, maintaining such kind of disjunction Lewis as come up with in the portion of strict implication and he defines strict implication in this way.

So, in the next class we going to see how Lewis has come up with his modal examtic systems from S1 to S5 and then we will be talking about the language of modal logic in this class in this lecture past two lectures, what we have seen is this thing we have put modal logic into the historical context and we have seen various phases and then for our course we will be dealing with the developments some of the developments from

Aristotle to the classical era you will be in a way skipping the modern era that is from 1972 to till today although this considered to be very important, but because of the time constancy we will be skipping some of the important things developments after 1972 so, but our emphasizes will be two important things that is the reputational structures and the second important thing is the method that is schematic method which we will be using it to determine a it is also tradition method to know the validity totality of any given modal logical formula with this I will end this lecture.

Thank you.