Basic Concepts in Modal Logic
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Lecture – 01
What is Logic? General Introduction

Welcome back, in the last lecture that to in the brief introduction to modal logic; a five minutes lecture, we spoke about what is modal logic, why we need to do modal logic and what is going to be there in the course. So, model logic is considered to be an extension of classical logic, in a sense that we are extending the classical logic with the two modal operators, the first one it is necessary that p and the second one is possible that p, this two is considered to be duals to each other.

So, before we begin this course it is important to know something about the classical logic. Classical logic, I mean the propositional logic, so propositional logic is going to be the minimal thing that we need to know before proceeding further. So, let us talk about bit of crash course on propositional logic, then, we will move on to the basic concepts in model logics.

So, in this lecture I will be taking about some of the important differences such as differences between deduction, induction and abduction etcetera and then I will be talking about what is the difference between object language, meta language etcetera. So, all things we will be talking about and we will be talking about syntax and semantics propositional logic and then I will be introducing one important decision procedure method, so that is semantic tableaux method, semantics tableaux method occupies the central position in this course.

So, we will be talking about the semantic tableaux method with respect to modal logic little bit later, but we will be talking in this lecture we will be talking about semantic tableaux method with respect to the propositional logic.

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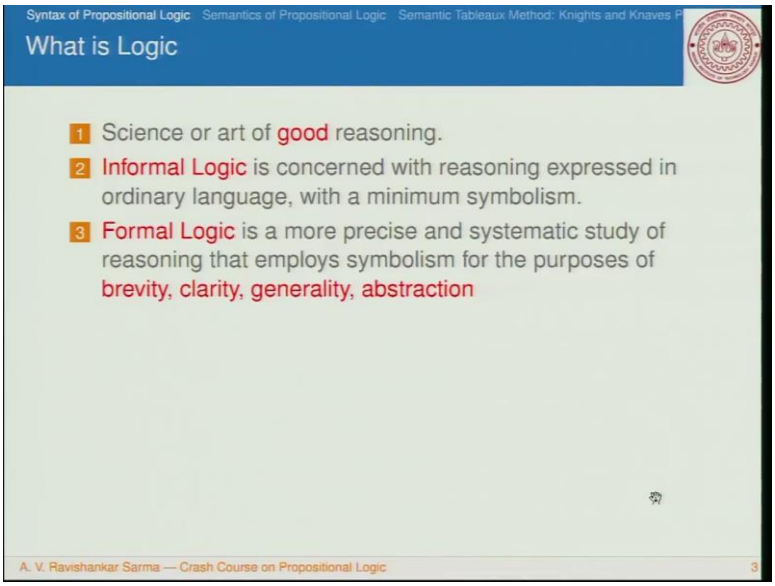

So, let us begin with this thing, so the first question that arises to us is before doing any course in logic, the first question that arises is that what is logic? So, logic is considered to be a science, it is a systematic study sometimes it is considered to be a study of argumentation and in these days particularly it is considered to be a science or it is considered to be a science of correct reasoning or you call it as good reasoning.

So, there are few things which you need to note; these of the terms of you come across whenever you are doing the any course in logic. The first thing we will be noticing is that most of the courses in logic are formal related to formal logic, then you might to come across with this question that what informal logic. Informal logic is concerned with the reasoning that it is expressed in our ordinary day to day language, day to day dispose which uses minimum symbolism and formal logic is considered to be more precise and it is considered to be systematic study of reasoning that employs symbolism for the purpose of three things at least for the brevity, for clarity, for generality and abstraction.

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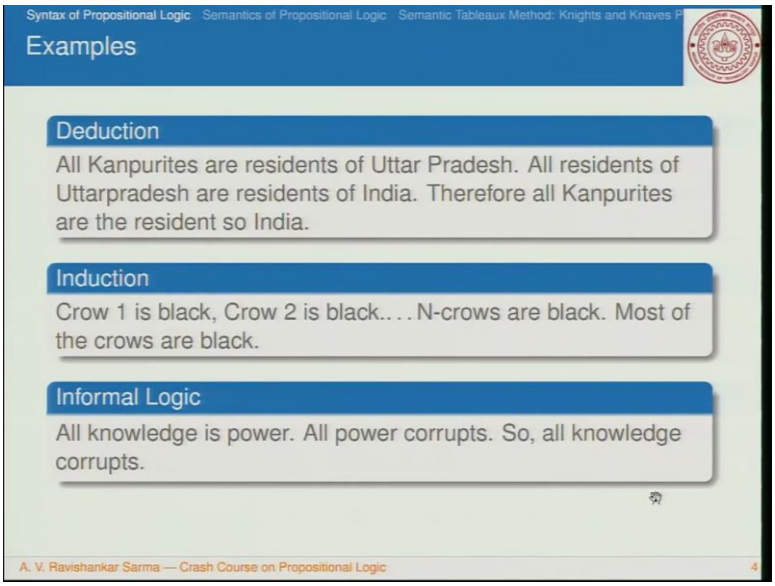

So, you want to make it very precise and you want to make it less (Refer Time: 03:25) then you will be entering into the formal logic. So, as the title suggests the formal logic means the name suggests form, so form is considered to be the important thing. The form can be like this a plus b, b plus c, so a plus c false, whereas for the informal logic what is important is that just by seeing the form we cannot analysis the given argument, so you need to invoke the content, you need to study the content properly to analyze the argument.

So, before we enter into the formal logic that is a classical logic that is the propositional logic for us which is considered to be logic of preposition. Let us distinguish these important things that is first one is deduction, the second one is induction. So, most of our course is concerned with deduction, let us consider this example all Kanpurites are residents of Uttar Pradesh. All residents of Uttar Pradesh are residents of India then; obviously, it has to be; it is necessarily follows that all Kanpurites are also residents of India.

So, here the conclusion necessarily follows from the premises, so there are three characteristics of deductive argument. The first one is this that conclusion necessarily follows from the premises and the premises are considered to be true then conclusion cannot be false that preserves the truth of the premises, truth preserving kind of

arguments and then when you say that all Kanpurites are residents of Uttar Pradesh you are certain that these things are true with 100 percent, they are 100 percent true.

So, if you subscribe to the two things premises and there is no other way then subscribing to the conclusion. So conclusion cannot be false given the premises are true and this second condition, second criteria is this that deductive arguments are monotonic in nature, monotonic in the sense that you can keep on adding new information to the premises without violating the conclusions that are derived earlier.

So, these are the two important things that we need to note about deductive arguments and then the other important thing which we need to take into consideration is that in the deductive arguments in the conclusion absolutely there is no new information present in the conclusion. Whatever is there in the premises you are trying to make it explicit by using some king of reasoning that is called as deductive reasoning, in the case of the induction on the other hand, we will be doing like this for example, it is to do with our day today observations. Suppose you if say crow one is black, crow two is black and you observed some 10000 crows and ultimately you will come to a conclusion that n crows, n is sufficiently large and n crows are considered to be black. Then obviously you will come of, with the general relation and say that most of the crows are black, but here the characteristics of inductive argument is this that, you can only talk about the strength of inductive argument.

So, the conclusion can be false given the premises are true, so all of a sudden if you observe a white crow then you need to withdraw your conclusion that you have derived earlier. Whatever the conclusion that you have derived earlier is that every crow that you have seen is considered to be black, but you need to revise your things. So, it is considered to be non-monotonic in nature exactly and other thing is that conclusion need not necessarily follow from the premises, so it happens only mostly in the area of a natural sciences, it involves statistical reasoning, probabilistic arguments etcetera all these things involves inductive arguments.

There is one thing which you need to note to separate informal logic with the formal logic, here is an example suppose if you say all knowledge is power, if you have more knowledge you are considered to be powerful person and all power corrupts and this is also true if all of a sudden if given some kind of power, then it corrupts us. Then from

this it follows that all knowledge corrupts, suppose if you follow a formal logic then it is an argument simply in this format a plus b, b plus c, so that is why a plus c, but in this case unless and until you analyze the content of the argument, you will not able to tell whether this argument is valid or invalid.

So, in this argument the problem arises that in all the deductive arguments, it is presupposed that there is no shift in the meaning of the words that you have used in your argument. So, in the first premise all knowledge is power, power is used in certain sense and in the second argument all the power corrupts means something else, when the shifting meaning of the word that you have used that is power here. So all deductive argument presumes that there is no such kind of shift in the meaning of the word; meaning of this words in the given premises.

But in this case the shift in the meaning of given argument, this argument can be only analyzed with respect to the content of the argument. So, this is considered to be a kind of fallacy which we studied it in the informal logic. So, there is another way in which we famous philosopher and logician Charles Sanders Peirce is considered to be a pragmatist, he distinguished deduction, induction and there is one more reasoning which is considered to be the one which is these days are very popular, that is abductive reason.

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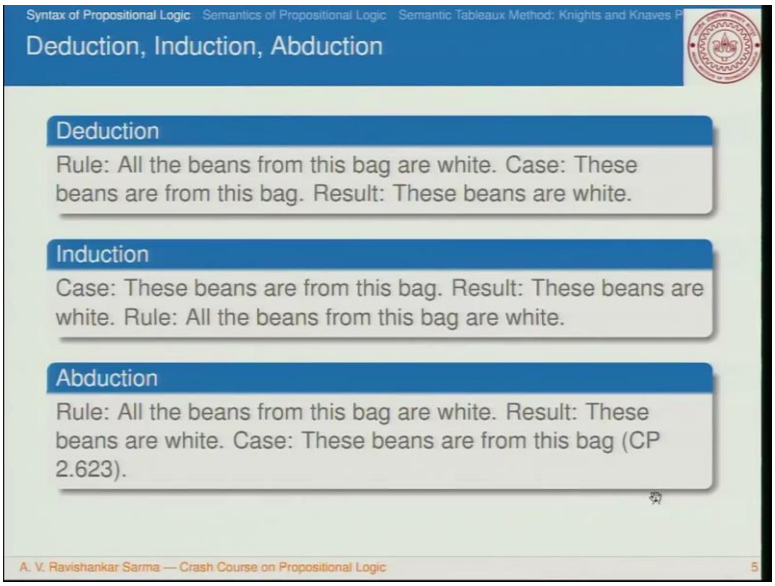

So according to Charles Sanders Peirce, it is like this deduction; deduction involves a particular kind of rule for example, if you say that all the beans from this bag are white.

So, you have a bean bag and all the beans that are there in the bean bag are considered to be white.

So, now you picked up some beans from that particular kind of bag and it turned out of the case that they are from this particular kind of bag. So, then obviously, these beans have to be white, there is no way in which you can pick something from that particular kind of bag and turns out to be certain color, so conclusion necessarily follows from the premises. In the case of the induction, he is trying to distinguish these three kinds of reason so we require these; we need to know something about these things in the beginning of this course.

So most of the time we will be focusing our attention on deductive logic that is a first part. In the case of the induction that we employ it in the natural sciences, it begins with the case that these beans from a particular kind of bag and the result is this that these beans are; obviously, white turned out it is the case I so happened white in color. So, then you formulate a rule and you say that after observation you will formulate a rule and it becomes like this, all the beans from this bag are white.

So, if you do like this then it is considered to be induction, so in a way in deduction we move form general to particular this is not a correct kind of definition but in our most of the case it works. So, we move from general to particular in the case of deductive arguments, in the case of induction we move from particulars to general. For example in the case of crow one is black, crow two is black they are all particulars, we move to a general statement that most of the crows are black.

There is another interesting kind of reasoning that we do use now day to day this course that is abductive reason. Here instead of having this thing we begin with the rule that in all the beans from this bag are white and we have also a result that these beans are white then these beans are from the particular bag. So, this is the example which is quoted in most of the text books in logic, just to make this three kinds of reasoning little bit different, but we will be focusing our attention on although induction and abduction are considered to be very important, we do use it in the natural sciences and abductive reasoning is one we most of the time we employ in our day to this course, but we will be focusing our attention on the first part that is the deductive reason. So, here is the remark

that Charles Sanders Peirce makes, so that is like this; deduction infers a result that is a conclusion that is considered to be certain.

For example, if you say all men are mortals of criticize man, so criticize is mortal that is that follows necessarily from the certain kind of premises that in all men are mortal, so criticize the man etcetera.

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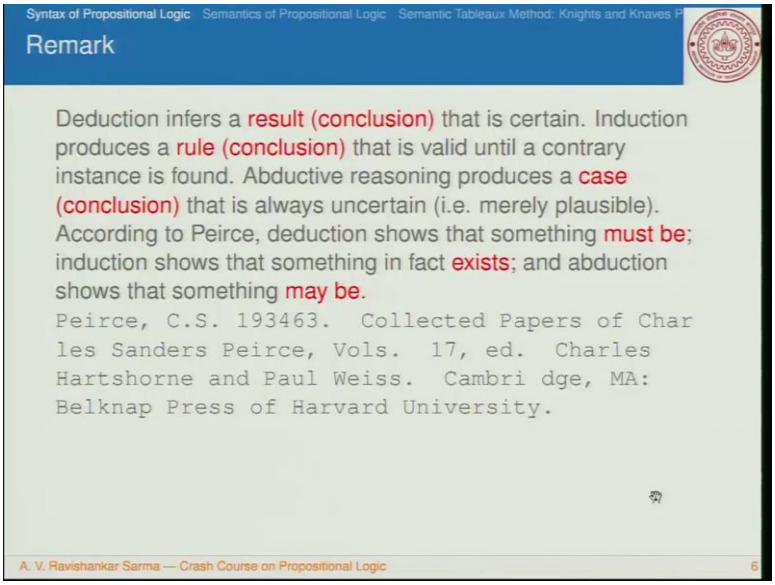

So, in the induction produces a rule, again that is considered to be conclusion that is valid until a contrary instance is found for example, in the case of crow one is black crow two is black and most of the crows are black, as long as you do not find, you will not observe a white crow that inference is still considered to be OK for us.

In the case of abductive reasoning, it produces a case usually hypothesis kind of that is always uncertain. Deductive logic such as certainty abductive logics we will be talking about uncertainty, so it produces the case that is always considered to be uncertain. It is also considered to be a fallacy in classical logic particularly when you have a plus b and you have b here and then from that a follows, so that is a fallacy of more responses.

So, now Peirce's talks about these distinction in this way; deduction shows that something must be the case, it should be the case must be the case etcetera, it means conclusion necessarily follows from the premises. Induction only shows that something in fact exists as long as you do not find a new observation which violates that thing, then

rule then you accept it. In the case of abduction that shows that something may be the case, in most of the cases in our day to day argumentation in particular, we do make use of abduction and induction in the sense that you know no matter how much knowledge that you attend, so we reason with incomplete information, uncertainty etcetera.

So, these are things which are used in some other context, but in our course we will be focusing our attention on deduction. So, after all why we need to do formal logic, so formal logic means the definition it has to do something with form.

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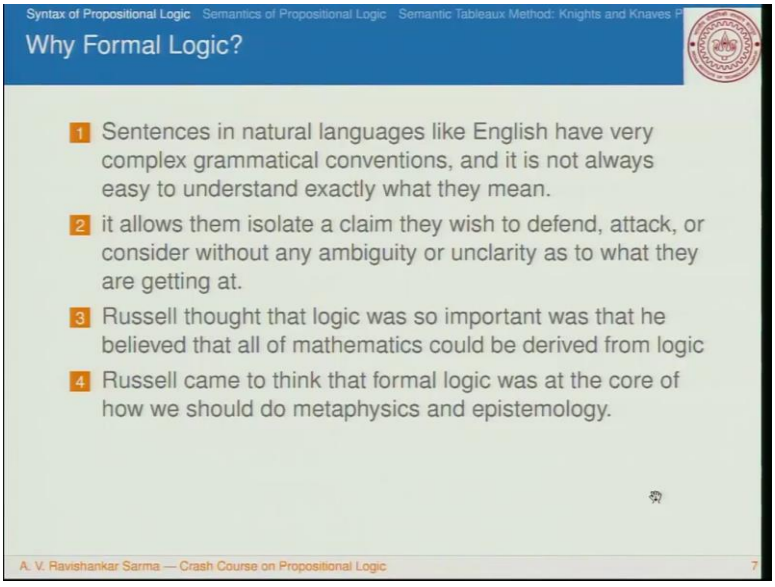

So, the sentences in natural language like English have very complex grammatical conventions and it is not always easy to understand exactly what they mean. For example, if you say Ravi is tall, so you will not able to figure out what exactly mean by tall, is this the case that 6 inches is to be considered to be tall or 5.8 inches to be considered to be tall or what exactly is the case that predicate that is involved in that kind of thing is weight predicate; that is tallness.

So, it depends upon culture, context etcetera, cultural background etcetera, the second is that it allows them to isolate a claim they wish to defend or attack or consider without any ambiguity or unclarity has to what they are getting it, for that reason you want avoid ambiguity, you want to achieve precision, rigor etcetera, we will be following formal logic. Famous philosopher and logician Russell, Metan Russell thought that logic was so important was that he believed that all mathematics could be derived from logic.

So, the idea is he has come up with a view that which is considered to be view which is called logicism which mean; which is of the view that entire mathematics; mathematics in a sense arithmetic and geometric can be reduced to simply logic that means all the statements of geometric, arithmetic etcetera can be appropriately translated into simple axioms; four, five axioms in the transformation rules, substitution rules etcetera and more (Refer Time: 16:50) more (Refer Time: 16:51) with that you can explain the entire formal systems. So, it can be reduced to a capsule then he is of the view that mathematics can be reduced to logic.

Russell came to think that formal logic was at the core of how we should do even metaphysics and even epistemology, in philosophy these are the other branches ethics a epistemology and metaphysics and formal logic plays an important rule, according to him plays an important role even it understanding some of the concepts of metaphysics and epistemology.

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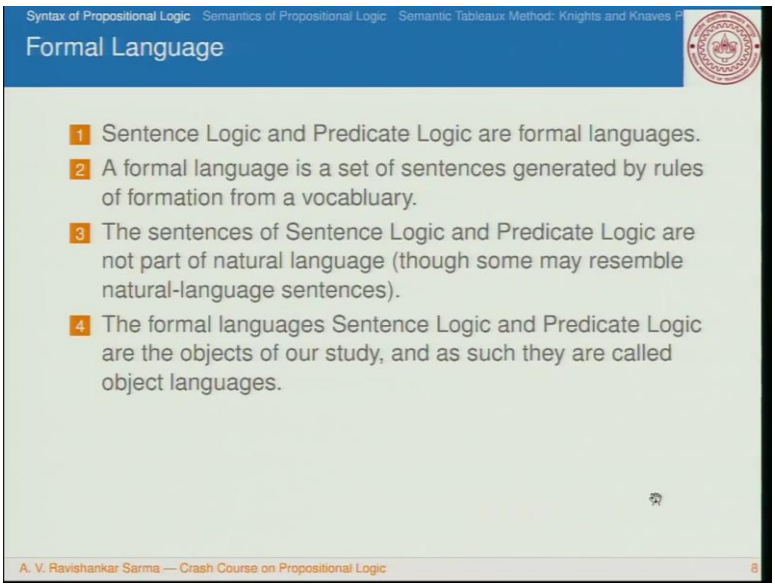

What do you mean formal language and what kind in what sense it is different from natural languages like English etcetera. So, we need to note that logic needs to be viewed as a language and every language as syntax and semantics that is what we are going to talk about in a while from now. Sentential logic and predicate are considered to be formal languages, a formal language is considered to be a set of sentences generated by some kind of rules of formation from a given vocabulary. These are considered to be well

formed formulas etcetera they are rules to generate is well formed formulas, just like in the case of English, we have some alphabets and then there are some words these words combine together it will form a meaningful sentence.

In the case of a formal language each and every sentence is represented by some kind of propositional variable p q r's etcetera and these p q r etcetera combined together with the help of some logical connectives and are implies another connectives and then we formulate some kind of compound sentences. So, the sentences of sentential logic or predicate logic are not part of natural language though some may resemble natural language.

For example if you say are, end implies negation etcetera the resemble actual words like not are etcetera but they not part of our language, the formal languages like sentence logic, predicate language etcetera are the objects of our study and as such they are also called as object languages.

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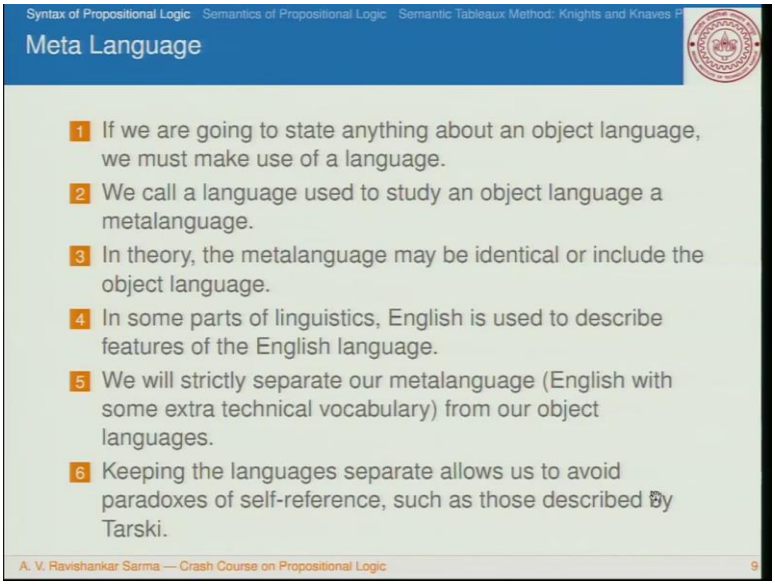

So, here is the difference we need to understand that is the difference between object language and the meta language. This distinction is considered to be very important and later we will use it in the analyzing some kind of paradoxes like lire paradoxes etcetera.

What is meta language? Meta means above beyond language, so if you are going to state anything about the object language, we must make use of a language which is considered

to be meta language and we call a language used to study an object language that is what is called as meta language. So, in theory the meta language may be identical or include, it might include object languages; in some parts of our linguistics English is used to describe the features of our English language itself.

Sometimes it might be the case, but otherwise this is clear distinction between object language and the meta language. Suppose if you talk about a particular kind of sentence the sunrises in the east; that is considered to be a something related to be the object language. When you talk about truth of that particular kind of thing that preposition you are talking about meta language either negation or something like that. So this strictly separate our meta language English with some extra technical vocabulary from our objective sorry not objective; object language.

So, keeping the languages separate allows us to avoid some paradoxes of self-references such as lire paradoxes etcetera. Suppose if you have a sentence like this sentence is false; that means the sentence is referring to itself, so that sentence is neither true and or false. So, when you make this kind of distinction object language and meta language, when you are talking about the truth of that particular sentence that this sentence is false; you are talking about meta language or truth of that particular kind of preposition, you are talking about meta languages.

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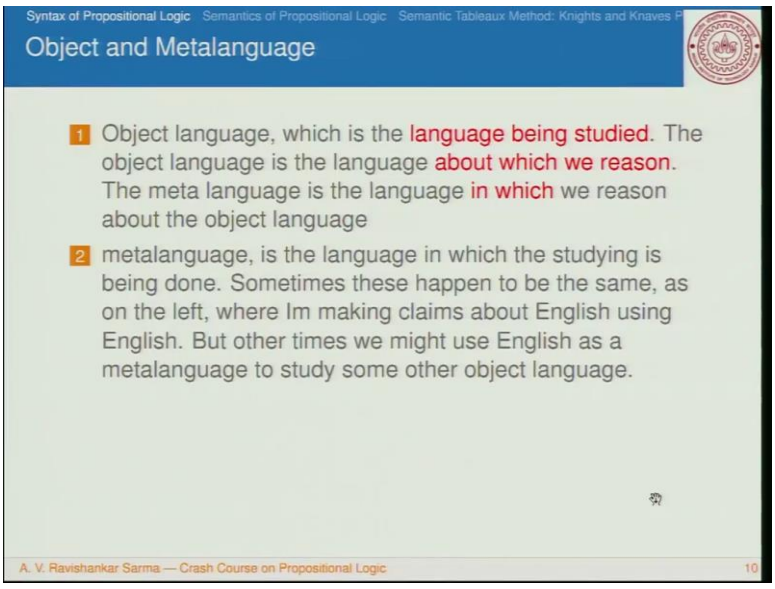

So, object language is the language which is the language being studied English etcetera. Object language is the language about which we reason and meta language is the language in which we reason about the object language, so this is the difference between object and meta language. Meta language is the language in which the studying is being done, sometimes this happen to be the same in some cases on the left for example, where I am; we are making some claims about English using I am speaking English to make some claims about English or I am talking about truths of English using illusion.

But other times we might use English as a meta language to study some other object language.

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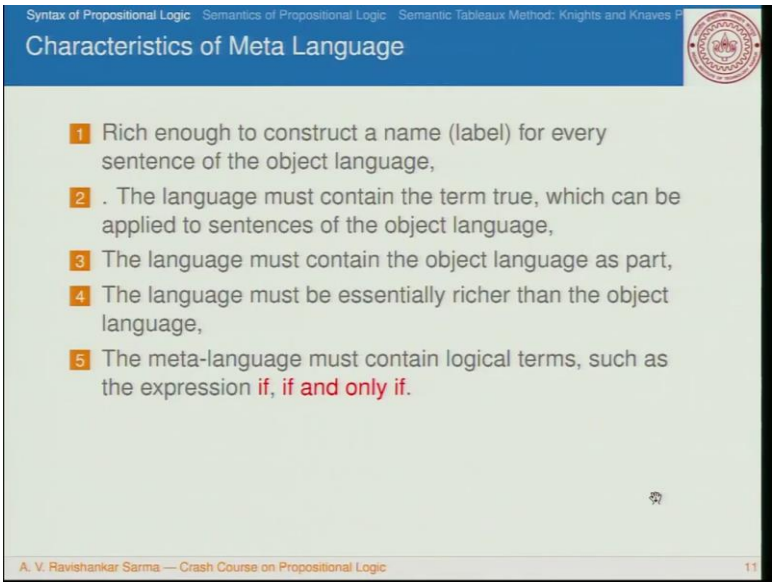

So, what are the characteristics of meta language; it is rich enough to construct a name for every sentence of the object language, the language must contain the term truth, which can be applied to sentences of the object language for example, if you say sunrise in the east, the truth of that one we are talking about the meta language it is applied to the sentence that sunrises in the east. So, the languages must contain the object language as a part and the language must be essentially richer than the object language; that means, English is not sufficient is weak etcetera, so the meta language must contain some kind of logical terms such as expression such as if and only if, negation, are, end of all these things.

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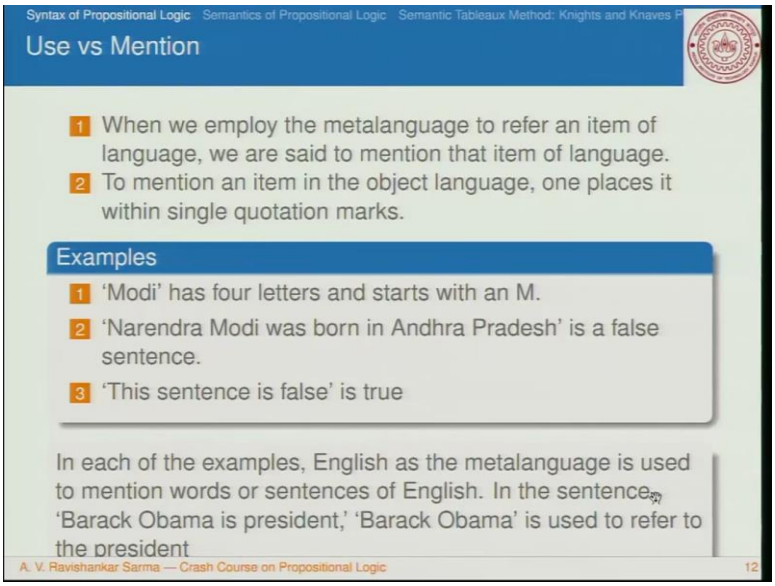

Whenever you are referring to some sentences with these things then; obviously, it is considered to be a meta language. There is another important distinction we need to; so far we spoke about object language and the meta language, now there is another distinction which you commonly come across which is called as use versus mention. You are mentioning something and you are using the sentences, so when we employ the meta language to refer to an item of a language like truth of some particular kind of sentence, we are said to mention that item; item of that particular kind of language. To mention an item in the object language one places it within single quotation, this is the some of the convenience that we follow.

For example here the example that we have Modi in the quotation marks has four letters and it starts with letter m, so item in the object language is Modi, Narendra Modi was born in Andhra Pradesh is considered to be a false sentence, he was born in Gujarat somewhere else, so this sentence is false is true. So, in each of these example English has a meta language is used to mention words or sentences of English say in the sentences for example, if you say Barak Obama is the president, Barak Obama is used to refer to the president. Narendra Modi was born in Andhra Pradesh is referring to the person called Narendra Modi.

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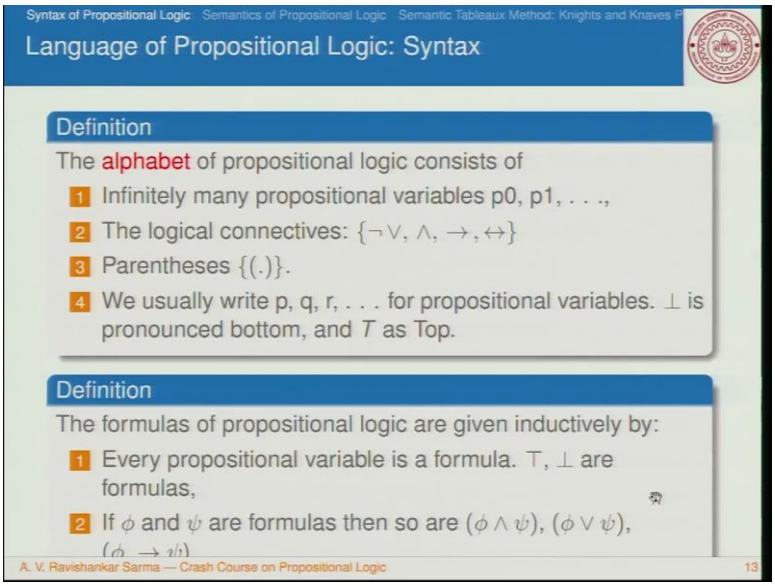

So, we will have in the next lecture we will be talking about the syntax of propositional logic. In this lecture, I briefly talked about what are the three kinds of reasoning that we employ in our day to day this course, that is deduction, induction and abduction and we made an important distinction between object and meta language and there is also another distinction of using something and then mentioning something.

So, in the next lecture we will be talking about the syntax of propositional logic and then we will move on to semantics of propositional logic.