Introduction to Logic

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Lecture - 30
Proofs in the Pm System

Welcome back. In the last lecture, we presented axiomatic system that is what we find it in the book famous book Principe of mathematic. So, we started with 5 axioms and then before that we have an important, you can serve as an important kind of statement that is from any primitive kind of true proposition only true proposition implies. So, that means some tautologies, you will generate only tautologies. So, now in this lecture what I would be doing is that any formal axiomatic system that; we come off with by using some simple kind of axioms etcetera and all. So that axiomatic system I mean we should be in a position to derive. It is some of the important loss of logic that are law of identity that is p implies p law of excluded middle p or not p are I mean there is another mean law which is called law of contradiction. It is not the case that both p and not p is a case.

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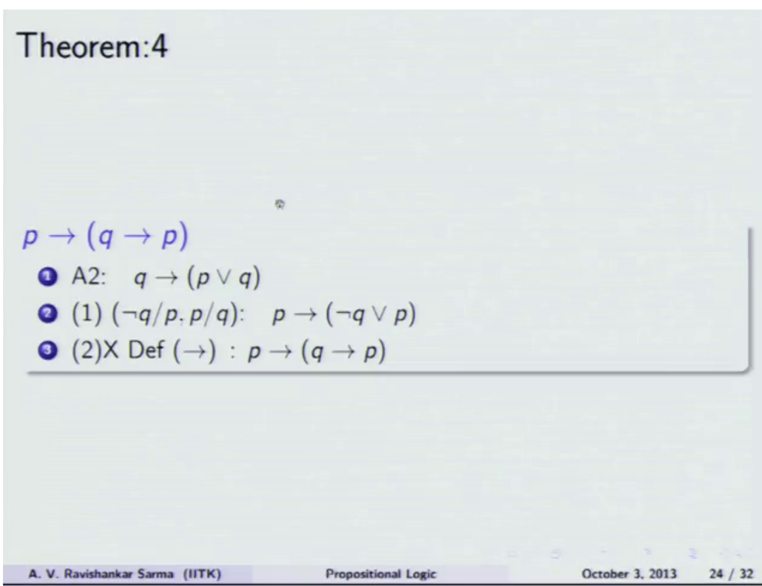

So, how do you derived all this valued formulas that occurs in a given formal axiomatic system. So, now for this 1 can start with by selecting some kind of axioms. Now, will be talking about Russell whitehead axiomatic system, where it has on the 5 axioms.

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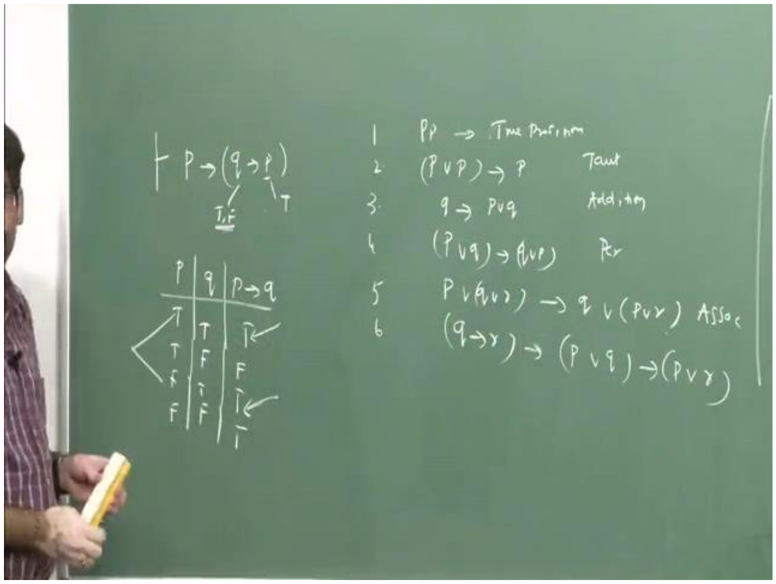

To start with we have this things suppose, if anything which is a true proposition that implies only true proposition that means; from true proposition, you will not get contradictions. So, there is a first thing so that already there; so this is called as the axiom related to tautology and second 1 is q plus p r q. So, this is called as addition and fourth 1 is off course, you can say third 1 it is p r q implies q r p. This is either p or q, in the case or q r p is the case; which is called has permutation and the fifth 1 is law of association q r p r r. This is what is called association know and then sixth 1 is summation q implies r is p r p r q implies p r q implies p r r p r q implies p r r.

So, now to start with we have this axioms and then we have some kind of transformation rules, if you apply this transformation rules or any one of this things; which will retain is tautology, would and other important rule is that if you have p and if you have p implies q; where this implication is consider to be material implication then this p gets detach and then what you get is q. So, that is all we have to begin with and from this particular kind of thing, we can derive all the valid formulas. So, all the valid formulas means; so all the true proposition it any given formal axiomatic system.

There are different ways to check whether, given formula is valid or given formula is tautology. So, 2 important methods that; we have already discuss that is we can check it semantic tabular method are 1 can use truth table method then; we can find out whether a given formula is consider to be valued other mis tautologies not. So, now what I will be doing is I will be deriving some theorems some important theorems, in these are theorems in a any given axiomatic system. So, the first theorem that I will be deriving is this thing. So, once you right this thing in this way at means; something is a theorem p implies q implies p. So, this statement says that something is true then the truth is obtained from many kind of proposition. And all this is also called as famously put it afterwards has paradox of material implication, it is consider to be a theorem any in any given axiomatic system, which is considered to be a theorem.

So, the only problem with this theorem is that; if this proposition is true. So, that means; using the semantics of implication p implies q. So, this is the way we define the material implication T T F and F alternative T and alternative F. So, this is going to become false only in this case; when p is T q is false in all other cases it becomes T. So, now in that sense if the consequent; this is the antecedent and this is the consequent if the consequent is true irrespective of whether the antecedent is true or false that means; the antecedent part is this 1 whether or not the antecedent. We need to consider only those cases, in which you have true consequent that means; these 2 cases.

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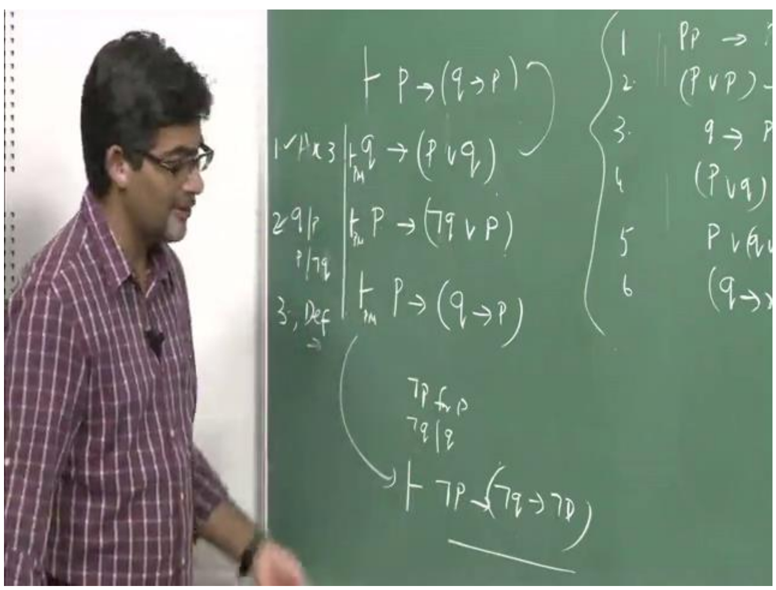

Now, irrespective of whether p is T whether p is false. This is going to be true only. So, in that sense true preposition is implied by any kind strange preposition that is even if this proposition is true proposition or its non sense or it is a false; any kind of word a true preposition should not be implied by any kind of strange kind preposition, that is what we discussed at this moment when it these theorem supplied to day to day discos here are some problem which you talk about it as a limitation of this. This particular kind of axiomatic system especially when it applied to day to day discos but as per as analyzing the digital switching circuit circumstance as per mathematical reasoning is concerned these are thing which perfectly works all right.

So, now this is the 1 which we are trying to derive p implies q implies p by using only this 5 axioms. So, now 1 can use any 1 of this axioms to begin with and then ultimately our begins, with this axioms all this axioms are absolutely true. The truth the proofs of this axiom cannot be questioned because already they are self evident truths. So, some how you need to use 1 of this axioms; so that you will generate this particular kind of thing. So, remember what we are trying what we essentially trying to do is we take up 1 particular kind of axiom we in such a way; so that you will get this p implies q implies p as a outcome.

So, let us see let us take into consideration this particular kind thing axiom three that is p r q somehow definitely; this is not this particular kind of format somehow we need to manipulate this 1, that means; you need transform this particular kind of things which we already know it is true all axiom are obliviously, considered to be true. So, from the true preposition you need to get another true kind of preposition. So, now if you can replace this thing; let us say q p wherever, q is there you replace it with p and wherever, you have p you replace it with p. These are the 2 substitution which are uniformly making it this particular kind of axiom 3.

So, you have to note that whatever, substitution you make in the given axiom; if your substitution is uniform it is according to the transformation rule, which we have discussed in the last class; than its transformation is also corresponding to tautology. So, that means what we are trying to do instead of q we are putting p and wherever, we have p we you put it as not q and than q stands for; so we put not q. So, then this will become not q wherever, q is there putting p. So, that what is essentially what we done here you replace q with p and p with not q; so that is what here written.

This justification for this particular kind of thing since, the substitution is uniform if this tautology this is true any substitution, which if it done here is uniform manner should also be true bracket is to be closed here; so this the second step. So, now using the definition of material implication that is a implies b p q s here, that is not a r b. So, that means; this not a or b is same as a implies b. So, now this as it is a p and than not q or p is nothing but q implies p. So, now this is what we are suppose to prove each step each stage. We write particular kind of thing and if you want to be most specific, we can write p m p m stands for that particular kind of ex amity system. So, now since we got this thing as theorem; now, we can substitute uniformly anything into it that will also become a theorem.

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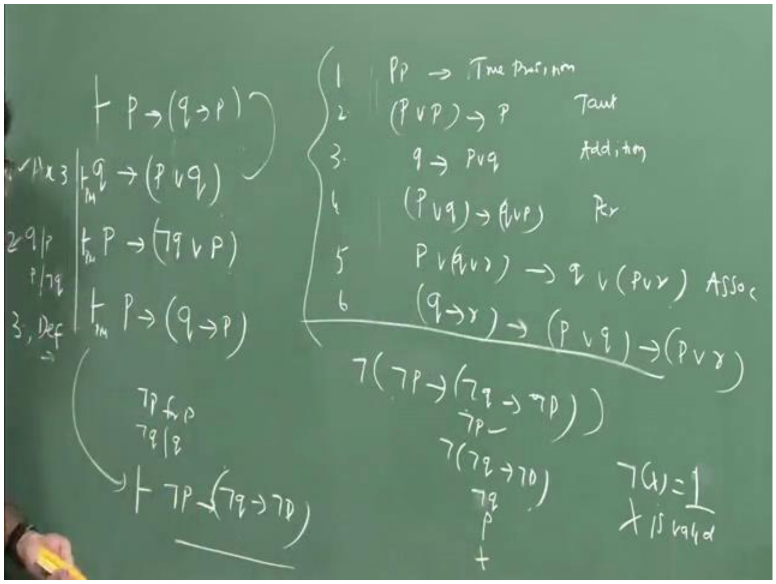

For example, if you substitute sorry not p for p wherever, p is there we substitute not p and wherever, q is there we substitute with not q. So, now this kind of prorogation will become not p implies not q implies not p; if this is consider to be a theorem everything, which if you substitute anything to it uniformly, for p we substitute not p for q we substitute not q s than; this is what is going to what we going to get. This should also be theorem if you are in doubt 1 can check it with a method or any particular kind of methods. So, now let us try to check this particular kind of formula using simple methods.

So, this not q or not p; so now you have taken into consideration and again formula and than we are consideration a 3. Now, this will become not of not q implies not p; so now taken into consideration and against of formula and than we are consideration a 3. So, this will become not of not q implies not p so this will become not q not, not p is p. So, now you will see here clearly not p here p here that is branch close that means; what we showed or simply is that a negation of x is unsatisfied, so that mean; x has x is value. So, in prorogation logic validity tautology there are 1 another same; so that is is formula as to be true prorogation and all the true prorogation are also basic consideration to be a theorem in the prorogation logic.

So, in that sense this all consider to be theorem of prorogation logic. If this is consider to be of material implication 1 which I written just now, should also be consider as one of the instants of material implication.

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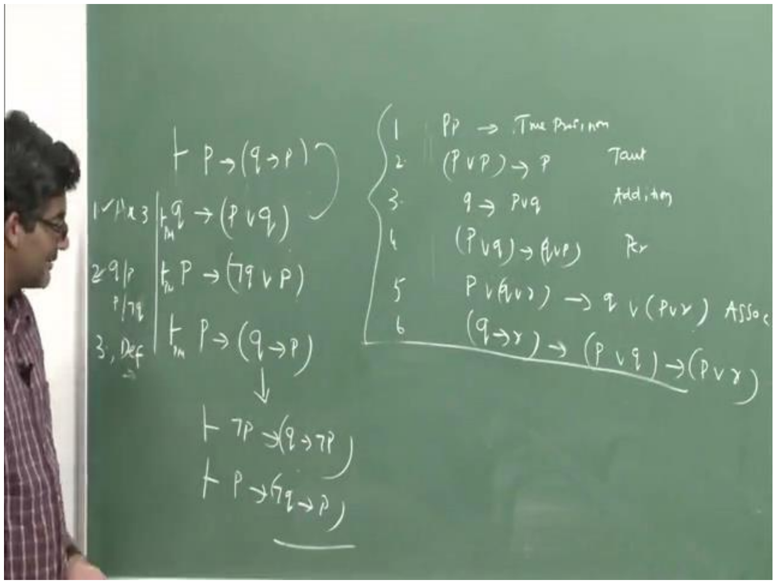

So, when we discus about of material implication and I will talk about this things and greater detail another instants of this 1 can be; for example for p just you substitute not p an all. So, this will become q implies not p so there are thousands of instant like this so you keep p as it is instead of q, we substituted not q and you keep it as it is, we should also become a theorem; what all the beautiful thing you see here. This is a something is a theorem something is a true prorogation and it imp lice only true prorogation.

So, tautology will led to tautology only, it is no way in which we begin with tautology and we will end up with that should not be the case at all; that is the reason why legislation will be continuously insisting on tautology rather than dealing with contradiction. We can we will get any particular kind of prorogation; so that the reason why we are not insisting on considerations unit lionization on insist on only tautology.

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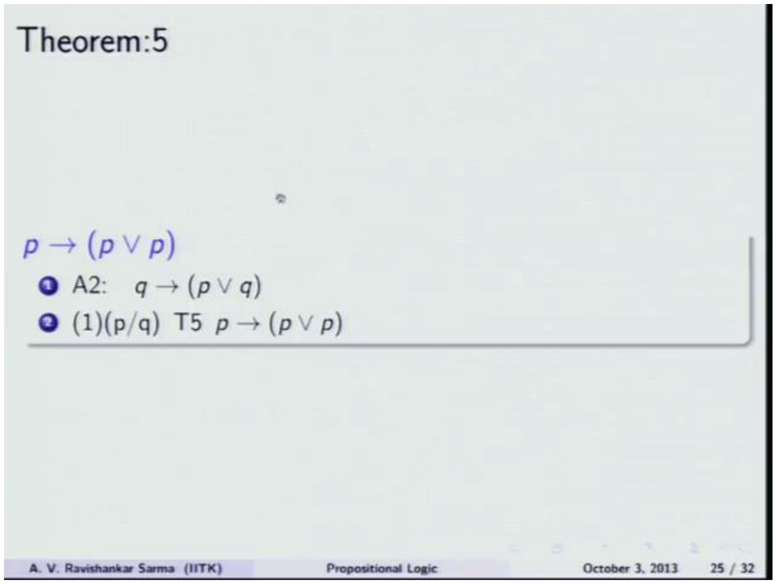

So, now let seen on to derive p impress p r p.

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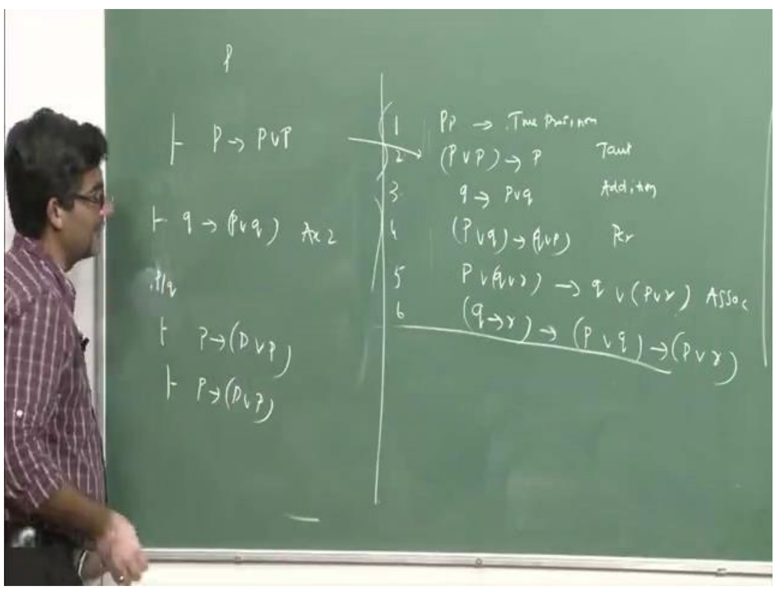

So, this another kind of theorem p implies p r p this is different from what we have here. So, this is axiom p r p implies p but, we are we would like to derive p implies p r p; so again you can begin with the axiom 2 that is q q implies p r q. This is axiom 2 this is somehow you need to change this thing into this particular kind of format. So, now what substitution you make into this particular kind of thing so that it will lead to this particular kind of thing, which we required so what exactly we are trying to do in all this theorem this is the a, we take this things axiom as ideal kind of situations. And then; this all are insistent particular kind of axiom.

So, now want to prove any one of this theorem an all we begin with one of this axiom and applied transmission rule and you trim this axiom in such a way until till such a way that you will get whatever, you deserve. So, this is the axiom which we begin with this is axiom number 2. So, this not a particular kind of format somehow need to translate change it into its corresponding form that is for example, if you substitute p for q than it will become q will become p and p is as it is and q is this 1. So, now it is converted into p impress p r p with 1 substitute; we got this particular kind of formula this also consider to be theorem.

So, in this way one can prove value formulas suppose whatever, is consider to be kind of true prorogation or tautology it should be proof a give axiom system sometime the prove by the very lengthy or sometime you might get proof in simple 4 or 5 steps for insistent.

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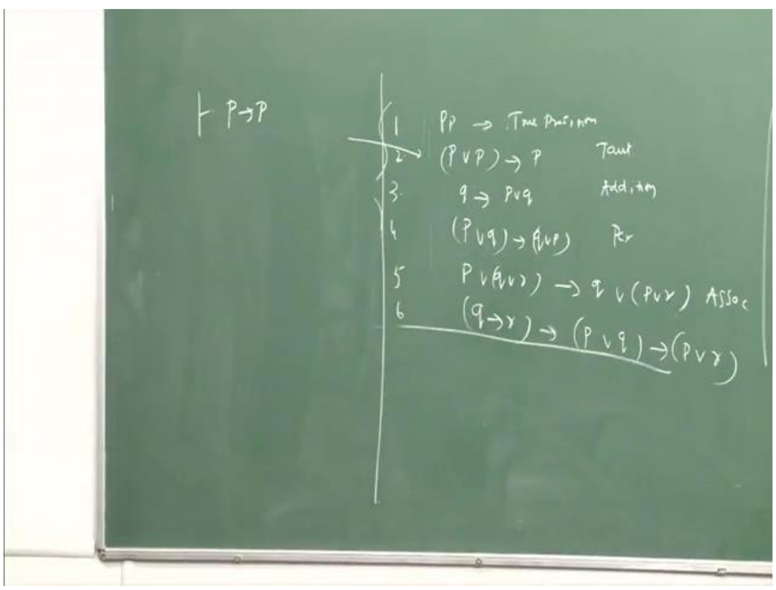

Let us try to prove particular thing, which we commonly know in logic that is law of identity so using 1 of this a axiom particular; so you will be proving this particular kind of thing; how do we prove the last intent y using a given kind axiom.

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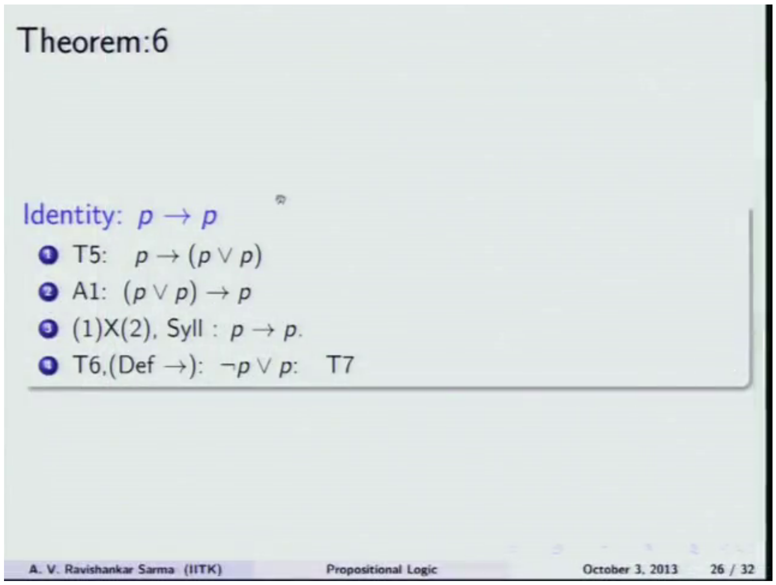

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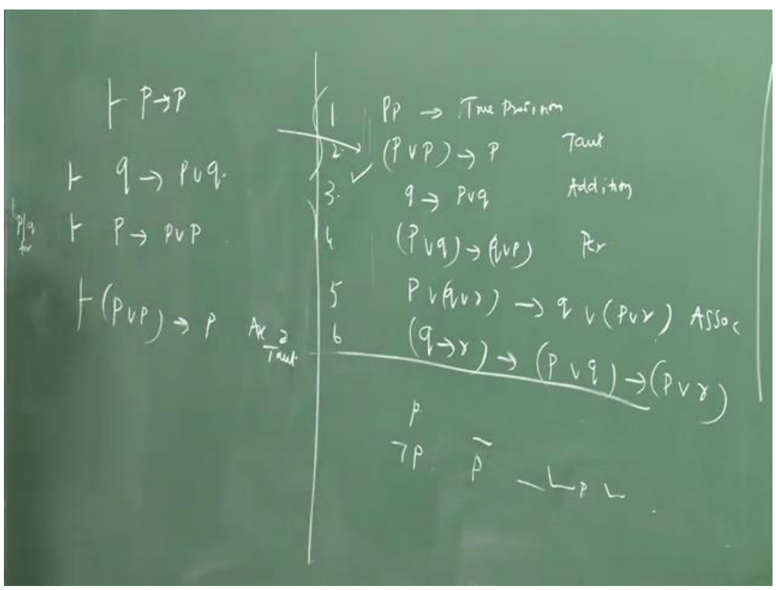

So, now; just now, we proved this particular kind of thing. So, q implies p r q this already theorem substitute q for p than; you will get p implies p r p how did you get this 1 we substituted p for q you get this particular kind T. This we already shown it the last slide. Now, this is the first 1 to begin with the now second thing is that we now that ah axiom a states that p r p impress p, we know that this is obese sly consider to be theorem an all axiom. So, that is axiom number to which is tautology axiom of course in the broad with this p q etcetera an all.

We can leave this p’s r’s q’s etcetera as some kind of prorogation as been the in the branch of in the field of our or any other kind field which you can think of or you can simply treat this p’s q’s etcetera as some kind of switching switches in particular simple switching in the simple switching circuits; if p’s r’s q’s, which represents some kind of switches that means.

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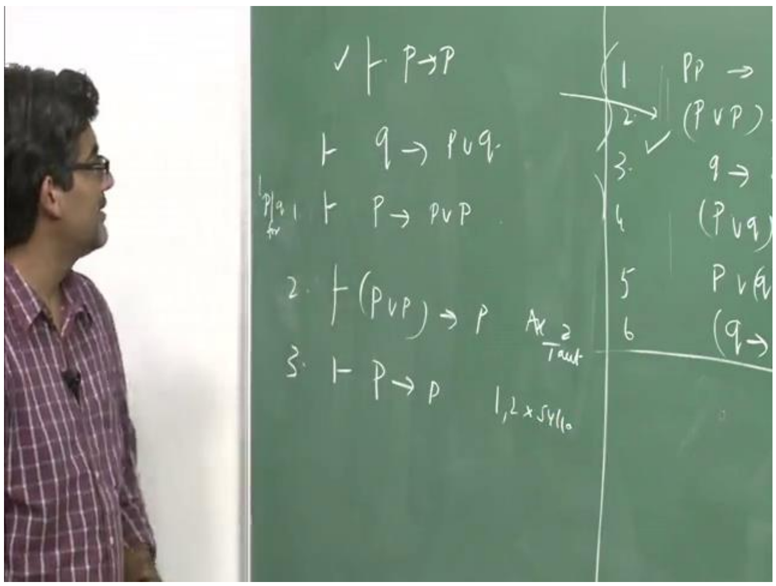

If p is if I write simply p we can interpret it as switches on if I write not p usually it is written in this way p bar that means; which is off. In this way when switches are on for example, when both switches are on and off the current passes through this particular kind of thing. So, there is and gate if it is in if it is arranged in a parallel kind of connection, it can be in all kind of all this thing which, we can visualize in this same thing in the context of simple analysis of simple digital switching circuits. So, now coming back to our particular kind of thing; we are trying to show that law of identity will come as outcome of this 5 axiom.

So, the first 1 which we have shown is substituted p for q than you got p impress p r p. So, now we have this is the first step and second step is this and third step. So, we have a rule which says that rule of which is obviously cases an all suppose, if x implies y and y implies z than x implies z that sense; so here, p impress p r p and p r p impress p. So, that means; simply p impress p in the justification for this 1 is 1 2 you get p implies p. So, you can come out at least 2 or 3 steps to prove this particular kind of thing something is equivalent to itself you want to show it than this is what this is a case this is a law of identity.

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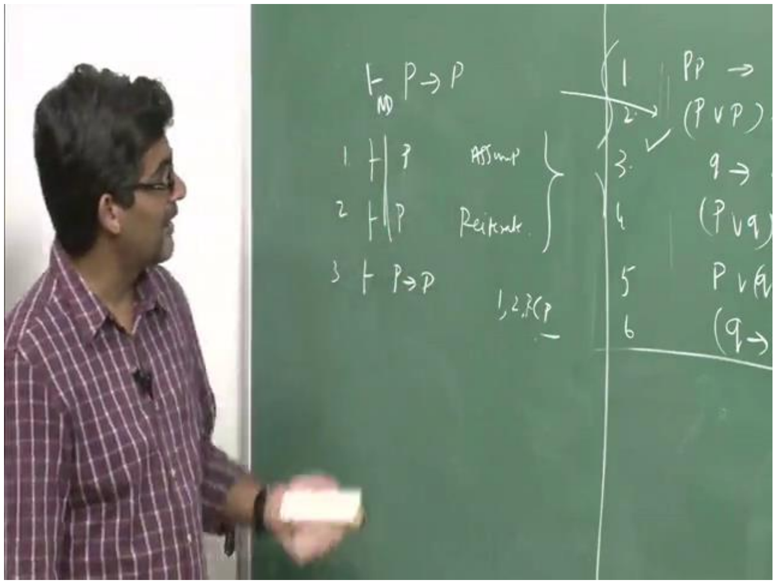

So, now for example for a same thing the context of natural dedication for number, it when we about discussed about, this particular kind of method natural dedication we showed the same thing how to prove this things using natural dedication. So, this involves may be 1 may be less than 2 steps an all. So, in this you take first 1 antecedent of this 1 as assumption so now what you do here this is already true you retread in 1 retread the same prorogation again and than that means; we got p in that means draw a line like this and then you say that p impress p in this natural dedication method this will come as come in just 2 step an all.

But, same kind of thing in may be it involves 3 or 4 steps are may be in the other aximotion, in which are thinking of F which will be studying which is system it might involves more than 5 steps an all to prove simply p implies p. But here, what we have used is just principle natural principles of logic that is you used principles of reiteration. So, we just reiteration that same thing and then you draw a line like this and than 1 and 2 3 1 and 2 conditional proof than we get p impress p. So, what you will do here is a is a discharge your assumption p and then we talk about p impress p here this is in conditional proof. It is also called as rule conditional proof. So, so far we did used law of identity. Now, let us consider the law of excluded middle and how to reduce law of excluded middle.

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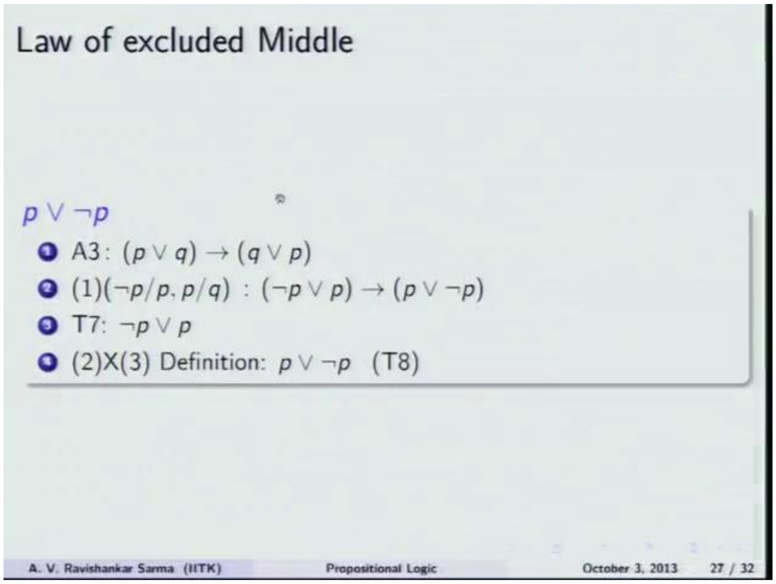

So, law of excluded middle states that: either the sentence proposition is p or it is not p but, not the other 1 off not p or p is not consider to be law of excluded middle. So, this hold in classical logic but this may not hold in many other situations; when it comes to day to day discourse for examples related to discourse this may not hold simply this there to be applicable this is going to be applicable in only those cases, in which you can draw a clear boundary between x and not x example we are talking about mortal and non mortal. It is easy to draw a line between whichever is considered to be mortal for living beings and the dead beings can easily draw a line in the same way you draw a line between black and white.

So, everything is not in this particular kind of situations when this comes to day to day discourse, you can very well have both p true and not p is also true this is also contested, in the context of mathematicians like he argues against this law of excluded middle he is of the law of excluded middle may not be may not come as a theorem in a given logical system. So, how it is a case because suppose if you are accepts p or not pm only when you are able to deduce able to prove not p and all. So suppose, if you are not able to prove both of them p and not p and all then that can be questioned some leads to a fact that, you did not have to have this law of excluded middle as 1 of the theorems in the axiomatic system.

So, that is logic and many logic as deviation logic, logic etcetera and all. You do not have this law of excluded middle as an outcome of it will not come as an bi product, in particular from using your assets that means; you know you are talking about some deviant logics. In the beginning of this classical propositional logic, we already we clearly stated that in all the classical logics at least this law of law of excluded middle and law of contradiction are necessarily 1 of the rules of 1 the important theorems in any given arithmetic system which follows classical logic; so forget about this thing.

Let us try to show this law of excluded middle using the axiomatic system. So, now in the same way while proving this thing 1 can begin with any 1 of these axioms. So, it is not hard and fast that you have to begin with the any 1 of these things, but you have to use little bit of creativity if while proving theorems also; so if proof is considered to be an effective proof only when it ends in finite steps and in finite intervals of time. Suppose, if proof involves come 550 steps and all and some other proof somebody else came up with simple proof it involves only 5 or 6 steps; then the second thing is considered to be a better proof that the first proof.

So, what constitutes a effective proof is the 1 in which has less than that is number of steps are less and it ends in finite intervals of time your proof should not variety time.

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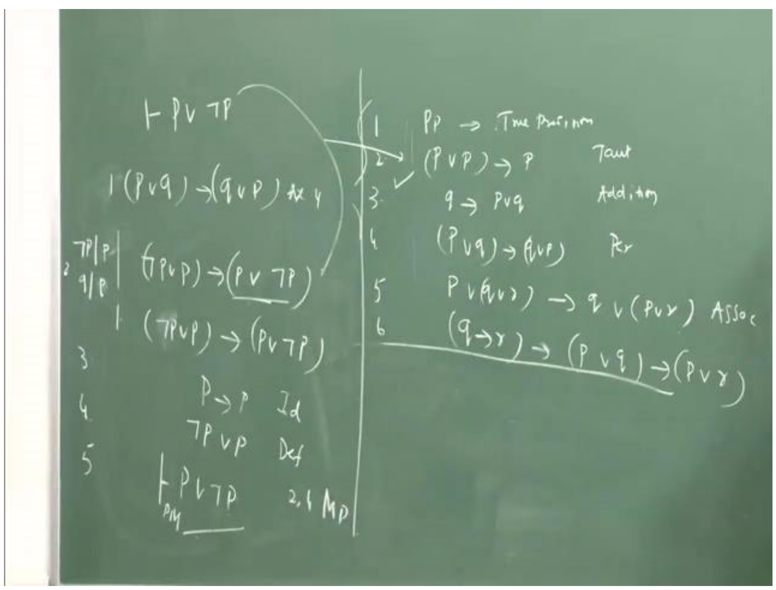

So, now what is that we are trying to prove; we are trying to prove this thing. So, now taking to consideration the fourth axiom p r q implies q r p. So, now we somehow we need to transform this thing in such a way that, you will have only the letters p, because in our final result that is p or not p only 2 only 1 literal applies that is p that means; you need to get over these q s and all. So, now what you have done here is this somehow this q needs to be eliminated and all. So, it has to be substituted with p. So, now what you will do here is this is to begin with axiom 4. So, now we substitute not p for p and then not q for q; so then this will this will become this is the first step and second step is this thing this is the justification, which we are trying to provide for this 1.

Now, this will become not p r q. So, now this we will change it as; so what exactly you have done here, is this not p and for q where ever p is there you substitute with p. So, now this will become p r p for p what we have substituted not p; so that is what we have. So, now this transforms to by definition this goes to not p r p implies p r not p. Now, somehow we have to use something so that this gets detached and you will get this thing as your outcome. So, now for that you can take into consideration this 1 p r q not 1 second see somehow, we need to get over from this particular kind of thing example; if you substitute in this 1.

Let us see do it and all in this 1. So, earlier we proved not p r; so earlier we have proved this particular kind of thing. So just now, we have proved this particular kind of statement so this is I so that is written as id so this is the third step. So, now by definition p implies p can be written as not p r p by definition so what is the definition the definition material implicates. So, now p not p r p is the 1 which has come as an outcome. So, now here you should note that; so till here its fie and all but in a way to prove this thing p implies p all the steps which are there in p implies p has to be there before that since, we have already proved it.

So, we are just straight away inserting that particular kind of thing in this proof, there are many proofs which we have already proved; so that can conceive as a starting point for proving other kinds of theorems. So, in that sense this since we have already proved that this is to be true as a theorem; so from this by definition you will get this 1. So, now somehow you need to detach this particular kind of so now, observe this 2 things not p r p here not p r p here. So, these 2 that means 2 and 4 are rule up detached with you can use. So, this is what you got p r not p. So, how did you get this p r not p. So, we started with axiom number 4 1 can start with any one of these axioms but ultimately, it can be very simpler if you can start with this particular kind of thing.

So, what exactly we have done here the strategy is that the last step of your this thing. So, this will come as particular kind of thing; so somehow you need to transform this thing into this particular kind of formula; so that is what we have here and rest of the things what we are trying to do here is that we are trying to detach whatever, is there in the part then you will get this particular kind of thing. So, how did you get this 1 using role of transformation mode of etcetera plays an important role here. So, there are 2 things, which are important in the quotation that we have seen, in the last class. The thing, which is central to the derivation is the material implication not only material implication but, also this particular kind of rule of detachment that is; if you have p and if you have p plus q then you will get p. So, this is the way of showing p r p as a theorem in principia mathematical that means we used axioms and then what we got is this law of excluded middle. So, in the same way 1 can show whether or not law of non contradiction is holds or not.

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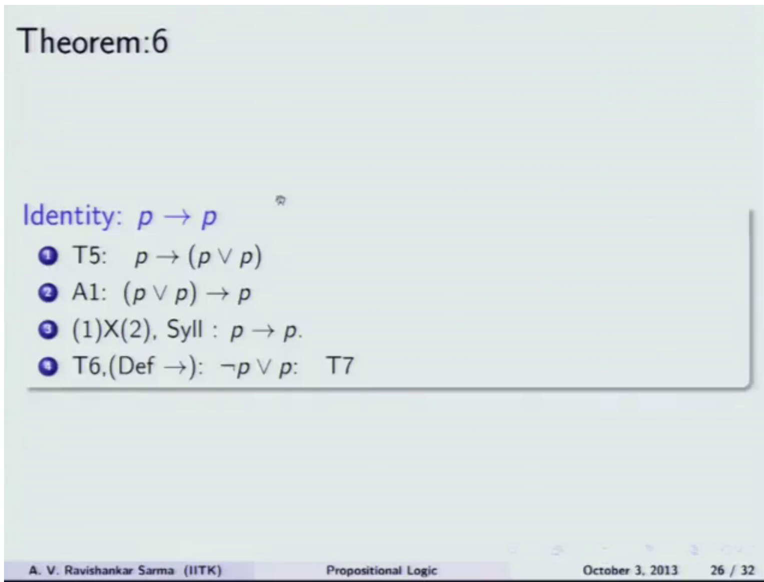

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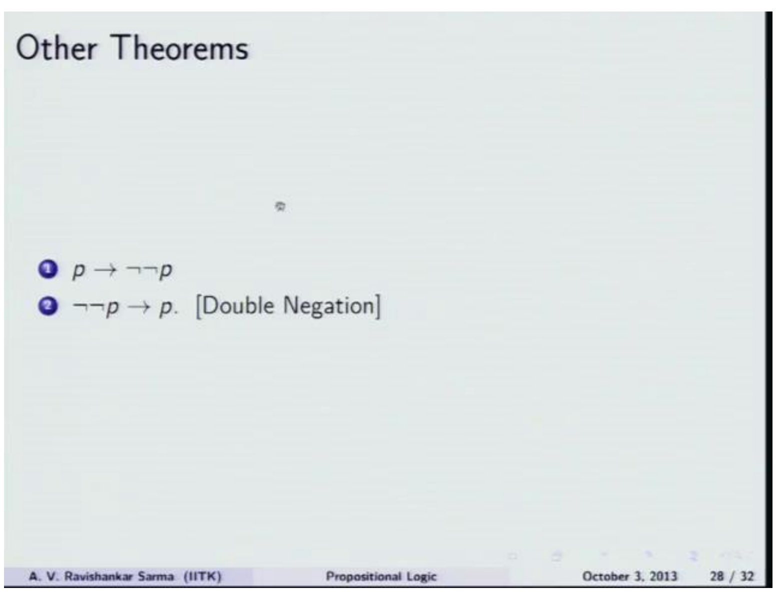

So, now there is a important thing that is the law of double negation. Let us try to show whether, I mean; how we can prove this double negation kind of thing this p implies not not p and not not p implies p is considered to be the rule of double negation. So, now since we got a p r not p that is law of excluded middle. So, now from that p r p implies not not p we will get it as an outcome.

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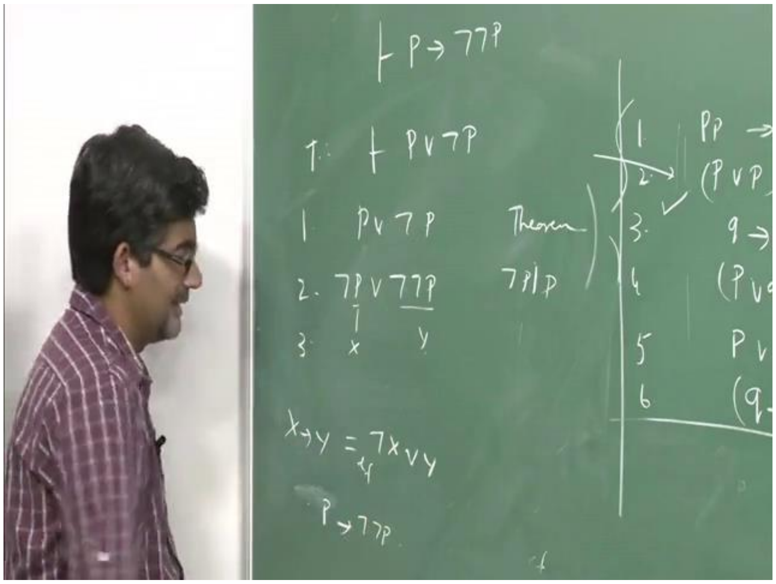

So, this is what you have already shown to be true that is p r not p just now, we proved this particular kind of thing p r not p. So, what is that we are trying to prove p implies not not p. So, now this is already a theorem which we have shown just now.

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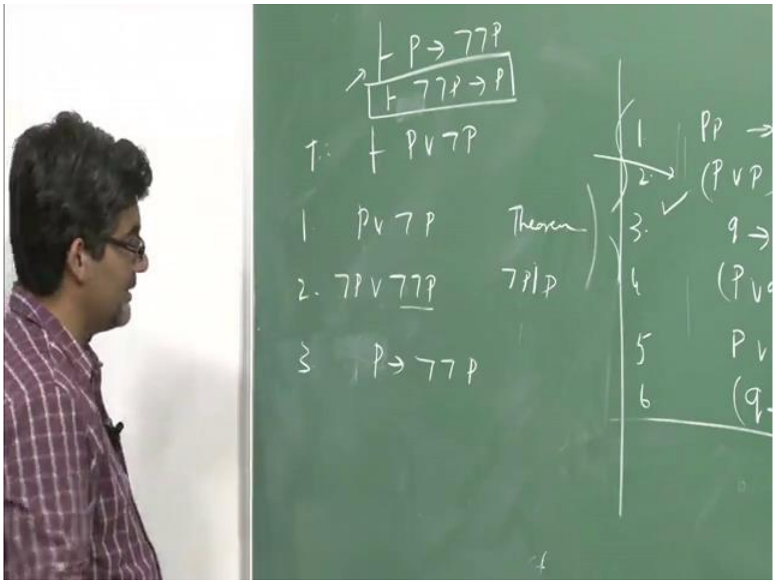

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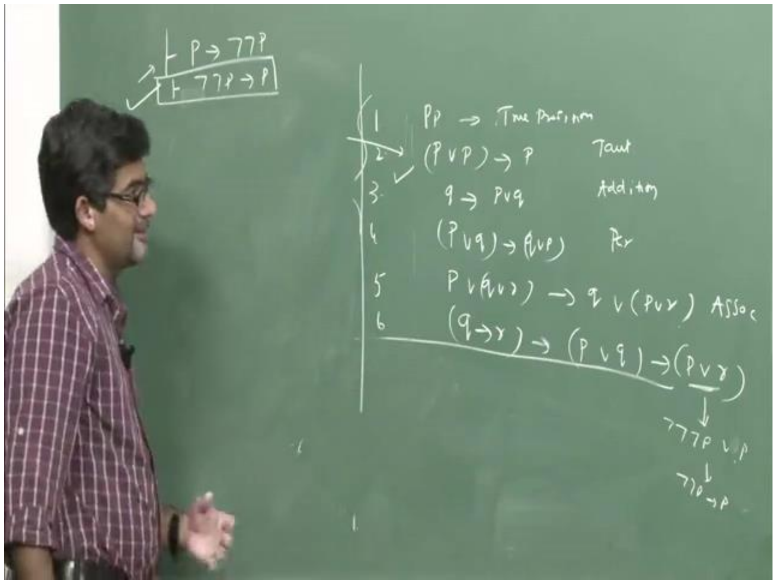

So, now what you do here is this is to begin with this is the theorem just now we have proved. So, now substitute not p for p; so that is if you substitute where ever p is there you substituted with not p. So, now this will become not p and this will become not not p. So, now this by definition suppose, if you root as p as x and this whole thing as y; so then this is in the form not x r y. This is the same as by definition x implies y. So, now what is x here for us this not sorry p and y is not not; so that means, what you have got here is the third step this by definition leads to p implies; this is by definition as same as p implies not not p. But that is not what we want to get in all but, we want to prove the double negation the double negation rule that is not not p implies p.

So, 1 will be tempted to say that if x implies y and y implies x and all that is not the case here, so x implies y and y implies x are totally different things. So, this is now what we have already proved shown to be the case, So, now we are trying to show this particular kind of thing not not p implies p we will make use of this particular kind of thing little bit later. So, now let us try to prove this particular kind of thing what we are trying to prove we are trying to prove not not p implies p, so 1 needs to start with any one of these axioms and all.

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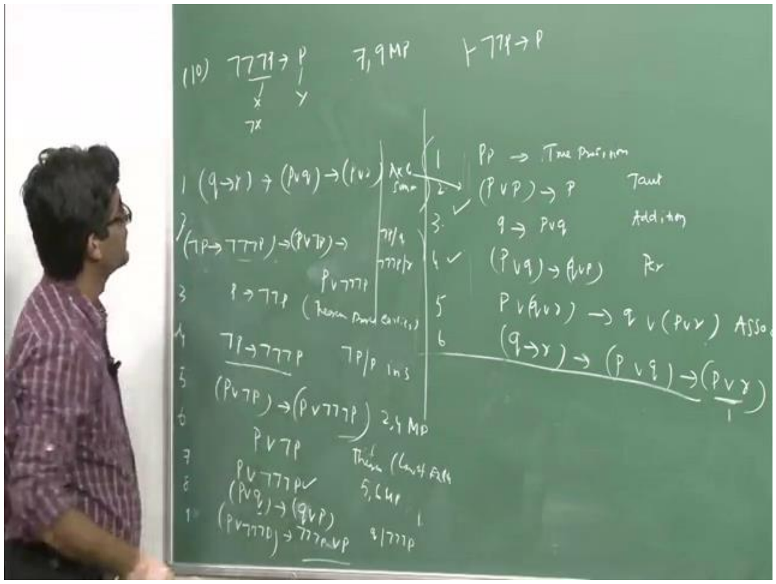

So, now somehow if you can use this particular kind of axiom and then somehow you replace this in such a way that; so this will be like somehow if you substitute not not p, and then if you substitute r for p and all r for not p r for p itself, then this is what you get no not p implies p. So, with this somehow you will get some kind of clue for what to substitute for p and what to substitute for r in this whole formula. So, if the last this consequent of the conditional will have this particular kind of what and the so this what you will be doing is you will be trying to detach whatever, is there before that by using rule of or may be through other transformations etcetera; so that is what we will be doing here.

So, now you take into consideration the first step q implies r that is what we have taken into consideration p implies q implies p r so this is what axiom number 6 r summation axiom, but you need to note that this summation axiom later it was shown to be the case that it is no longer an axiom in the because, this will come as an outcome of one of this axioms and all. So, it loses its axiomatic it lost its axiomatic status but at this moment Russell in invited in their book principia mathematic proves by considering; this also this also is an axiom, but later student of I think he showed that this is no longer an axiom an axiom is a 1 which is considered to be a self provident proof which does not fit or any proof.

So, now we start with this particular kind of thing; so now, we substitute not p for q and not not p for r. So, why you have substituted this thing, because somehow we need to generate this particular kind if thing; so now, this will become so wherever q is there we have not p here. So, this will become not p r plus this not of not of not p 3 not s are there as in p and now p r q p means; as it is q for q we have substituted not p implies p r r so we have substituted not p for r sorry. So, this is the same as p and r for r we are substituted not of not p. So, this is what well get from this 1.

So, now we have already proved this particular kind of thing this is what you have already shown that p implies not not p that is what you are using here; theorem proved earlier just now, we have shown this is the case so how did you get your this 1 your p r not p. So, you substitute not p into this 1; so this will become not not p. So, this is the same as p implies not p. So, in that way you prove this particular kind of thing; so you make use of this particular kind of thing; So, now you substitute not p into this 1 not p for p in 3. So, now this you l l get not of not of not p.

So, now observe this 2 and 4 not p and plus not pan d this is same as this 1. So, now these 2 you will get whatever, is remained here this gets detached and then you will get this thing p r not p implies p r not not not p. So, this is considered to be the fifth step; so till now we did not get to this particular kind of thing. So, now how do we get this 1 2 and 4 is what this on this portion and this portion is same; so that’s why it gets detached it is like x and x implies y. So, you will get y so this is the fifth step till now, we could not trim this 1 in such a way that it is translated into this particular kind of thing. So, now we showed that p r not p is the case this is the theorem which we have proved law of excluded middle.

So, there are so many things which we are embedding into this particular kind of theorem and all by proving this particular kind of thing; we made use of all the things which we have already shown to be true earlier this is shown to be theorems earlier. So, now this is what the case is so this theorem number 8. Now, so why we are using these things, because somehow we need to detach these things we have to get to the last 1 which is there in the consequent who occupies the consequent of your condition. So, now these 2 5 and 6 you will get is p r not not not p.

So, that is what you get so till now this dint get over. So, now in the 8th step we use axiom number 4 that is p r q implies q r p. So, now somehow this should be in this particular kind of format; so that it gets detached so that means, for q we need to substitute this not not not p. So, now this with the transformation you will get p r. Now, q is so i need to write like this q is nothing but not not p. So, now this you will get not not 3 not s p q is this not not not not p r p. So, now observe this 8 and 9 sorry 7 and 9 this is what I am writing it here.

So, now the tenth step is here from 7 and 9 because, this p r not not not p is same as this 1 p r not not not p. So, these 2 gets detached and then this is what you get not not not p r p implies p. So, now till now we did not get what we wanted we what we wanted is not not p implies p. So, this is what we are trying to get. So, now this by definition suppose, if you root as this thing the whole thing as x and this thing as y. So, this is in this form not x

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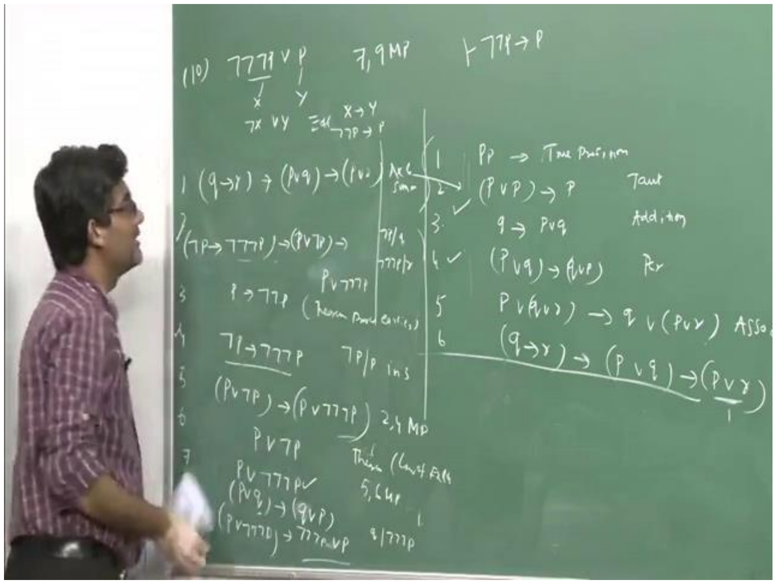

So, I will just so not not not p r sorry r p sorry this is not. So, this is r p; so this is like a not x of y, so that means the same as by definition x implies y what is x here not not p and what is y here its p here. So, that is what you get by your definition.

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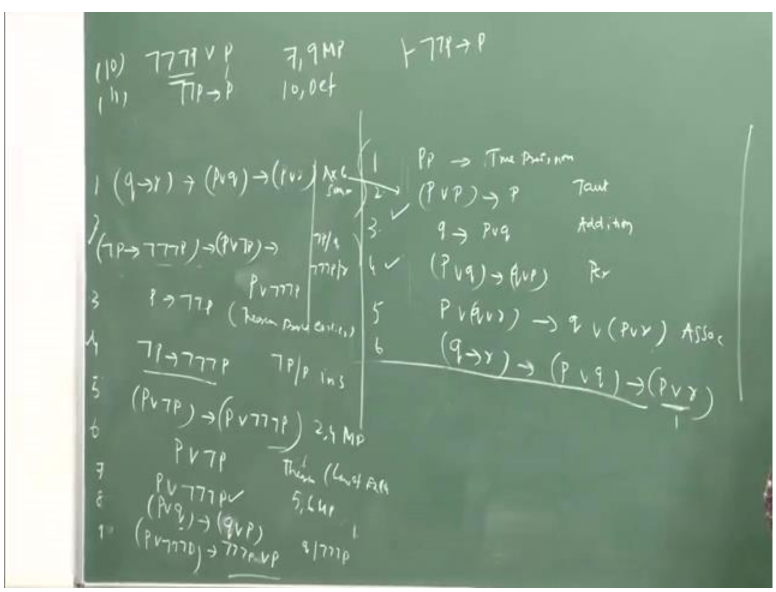

So, now in the eleventh step so this by definition 10 by definition you I will get not not p implies p. So, now what is that we have achieved in this particular kind of thing; we showed that for example we want to show that it is not the case that it is not the case that this is duster means; that it is a duster. So, that is what you 2 times you negated that 1 double negation leads to the same thing so now why this proof is sometimes it make saver silly to talk about proofs like p plus p or p r not p etcetera. And all but 1 thing 1 should note in all here in all these proofs our proofs are considered to be very rigorous in the sense that we started with the axioms which are considered to be obviously true.

Then transformation rules which possess the true and then the rule of detachment that is also considered to be true preserving kind of rules and everything is stated explicitly on the board and all from that; we got this not not not p implies p there is no step in this proof which can be that can be questioned in enough just like, in the case of proof of 2 is equal to 1 is 1 of the friendliest proof that we have see, in the last class. We clearly have seen that after following some 6 or 7 steps in the eighth step there was some problem and all that is the cancellation of a square minus a b both sides. And all that means 0 by 0 is not permitted in that particular kind of proof that means; the proof is considered to be was considered to be defective in that kind of case.

So, you cannot move further because, that step is wrong. So, but here in this case each step is we are making our journey in such way that we started with the truths and the next step is also going to be true and then we are moving to moving closer to whatever, we wanted to derive. In this case we assure that not not p and plus p by following some kind of rigorous kind of methods that is; so will be rigorous method is used implying here, why because everything is stated explicitly and from that you have derived is no not p implies p.

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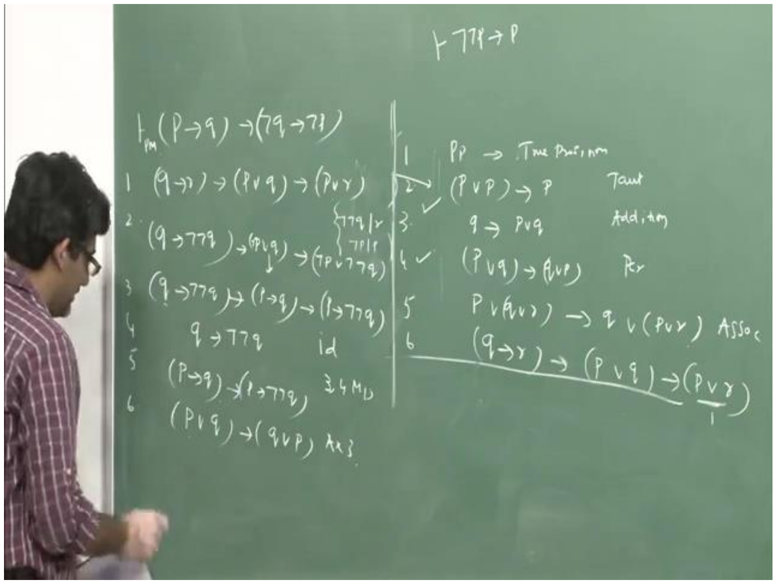

So, finally we 1 can also derive some of the important other theorems and all like 1 of the theorems that you employ in classical logic is law of contra position that is if p implies q is the case where not q implies not p is the case. So, how do we get this law of non contra position law of contra position within this axiomatic system. So, for that also you begin with some axioms that means; they are oblivious proofs and all then you trim those axioms in such a way and then using transformation rules and rule of rule and then we will generate. So, you will generate law of contra position so again the axioms remain the same.

So, what essentially we are trying to prove is this so from p implies q not q implies not p since it is a theorem you write it in this way. So, since it is a theorem in principia mathematic you write it as pm or write it as or even r w also right. So, now again you start with the summation axiom; so that is the 1 which you state here, with the summation axiom. So, that is the 1 which you have stated here clearly p implies r. Now, in this in this particular kind of thing somehow, we need to generate this thing suppose you can substitute p for not p, you will get p implies q.

If you substitute for p or p or if you substitute something else maybe you may get closer to this 1 it is not the same formula. So, now what substitutions 1 can make is like this so you substitute not not q per r. So, now this you get; if you substitute not not q per r this you will get not not q. So, p r q is and what else 1 can substitute here is this not p per p this the 3 2 things. We are substituting here so wherever p is there we are substituting with not p and wherever, r is there we are substituting with wherever r is there not not q. So, this will become not p r q implies p means; not p r r means not not q.

So, why we have done this thing because somehow we will transform this thing as much as possible closer to our destination our destination is this 1. So, now a 3 we already; so we write this 1 first of all q implies not not q. So, this by definition is same as p implies q this is the reason why we have transformed this thing into this particular kind of format and this is nothing but p implies not not q. So, this is what is the case? So, now we already showed that p implies not not p; it is same as q implies not not q. So, this is what is called as id means law of identity; so now this 3 and 4. So, you will get this particular kind of thing this 2 gets detached and you will get p implies q implies not not q.

So, now till now it is not transformed completely into this 1 we should trimmed little bit more; so that in so this will become not q impress not p for that what you will do is we will be making use of another axiom. So, each step is consider kind of true an all either you can use axiom or use theorem, which you showed earlier or it should be or it should come as outcome of some kind of transmission rule that mean in the uniform substitution, we should get this particular kind of thing. So, now we have p implies q implies q r p this what we already have here axiom number 3 somehow this we need to transform this axiom in this way.

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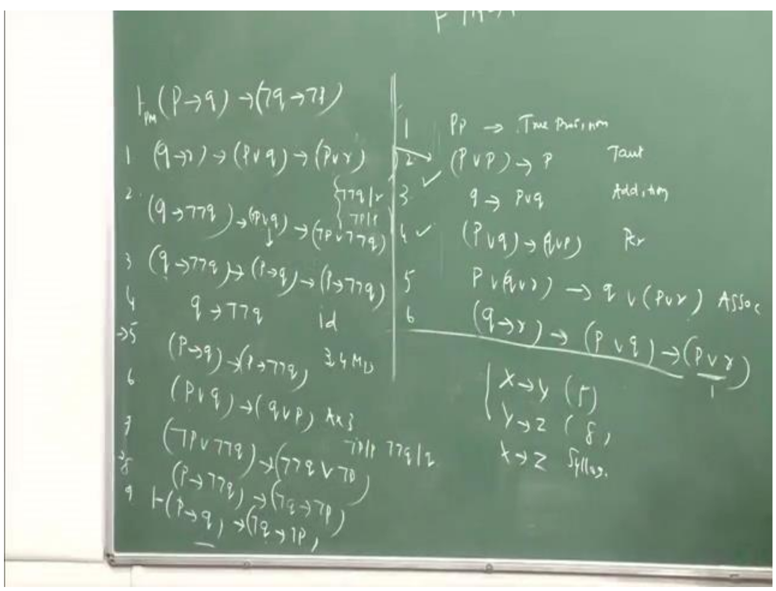

So, now this axiom we suppose you substitute not p for p and not not q for q. So, this is what you have done. So, now this text transformed to p will become not p or q means; not not implies q for q you substituted not not q and for p you substituted not not p. So, now this by definition you will get p implies not not q and this this same not q implies not p. So, now this is what we have; so now p implies not not q implies q or not p, so 6 and p p implies p implies q means 1 second. So, now observe this particular kind of thing 5 and 8 p p impress q impress p impress not not q this same p impress not not q implies not q impress not p this like: x implies y which there in the step number 5 and y implies z in step number 8 to find similar kind of thing like this x implies z by using properties called as.

Now, can again once again see here p implies not not q at is 5 p implies not not q is this 1 sorry p implies q is this 1 and p implies not not q is this 1 that means; p impress q should go to this 1 p impress q implies not q implies not p. So, this is what we have trying to get that means; in some kind of we got whatever we wanted to prove in this way 1 can derive many theorems whatever, to consider to be kind of valid formula are tautology in that should find the proof in this principle system. So, in the next class what we are doing is: whether all the valid formulas find the prove or not there are 3 important properties that, we should talk about now, the question immediate question that comes to your mind is that is this system consist in the sense that.

Suppose, if you derive both x and not x from within the given axiomatic system, that means; the system is consideration to be in consist in that sense consideration to be consist. And another important properties is that all the provable thing; that you have just now, proved have have to be true that means; it has to support the properties of soundness all the provable theorems are true and in the same way the other were on, if all the valid formula are also find prove an all the system is going to be complete. So, we are going see, in the next class that the system; which was in the introduce by in that many valid formulas can be derive an all this things corresponds to some kind of statements and arithmetic.

So, it is in that sense all the statements of arithmetic are translated into one of these axioms an all. And then all this axioms can farther, be transformed into corresponding theorem an all that; also corresponds to all the statements in arithmetic, in this class what essentially, we have seen is this we have started with axiomatic system the axiomatic system were it as only it has and negation although in the axiom, you have implication sign here. But, actually it should read as actual things, should have only and negation. And from that the transformation rules and rule of detachment, we derive many theorem, in the next class. We are going to see whether, principles is consist or complete, sound always important properties; which we discuss in it the next class.