

**International Economics**  
**Prof. S. K. Mathur**  
**Department of Humanities and Social Science**  
**Indian Institute of Technology, Kanpur**

**Module No. #1**

**Lecture No. # 36**

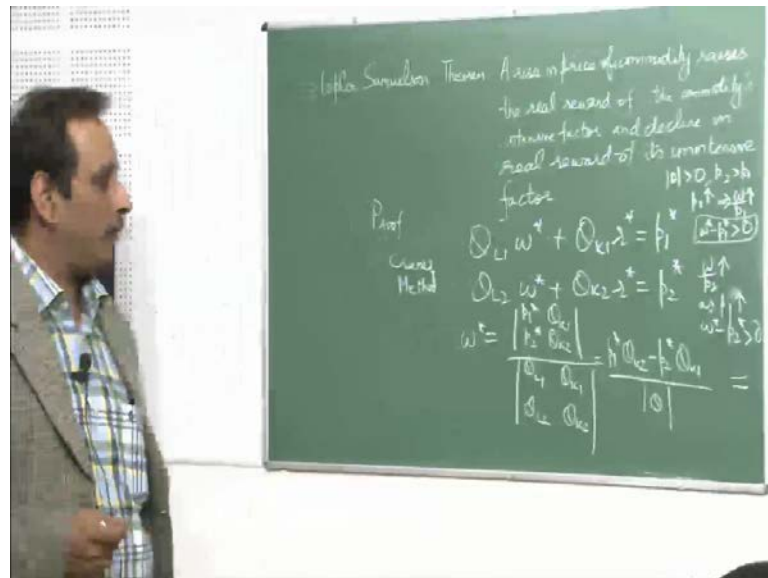
Good afternoon! We will carry on with what we have been doing for the past few lectures; that is, cover up the various theorems which are an offshoot of the Heckscher Ohlin model. Please recall, the Heckscher Ohlin model says that a country exports the good, which uses intensively the abundant factor and imports the good, which uses intensively its scarce factor. In crude terms, it would mean that a country which is rich in capital will export capital intensive product. And, a country which is rich in labour will export labour intensive product.

Now, in the real world there are more than two factors, there are more than two countries, there are more than two goods. The basic Heckscher Ohlin model is about two into two into two model. It is two factors, two goods, two countries. And, it talks about factor intensities, which are easy to define in a two into two into two model because if the capital labour ratio in industry one exceeds the capital labour ratio in industry two, you say that the first industry is capital intensive; the second industry is labour intensive.

If you have to talk about abundance, you say capital labour ratio; the overall capital labour ratio in country a, if it is greater than the capital labour ratio in country b, you say that country a is capital abundant; country b is labour abundant. Problem comes when you have more than two goods, you have more than two factors of production, you have more than two countries. Then, you need to define what do you mean, how do you define factor intensities, how do you define factor abundance.

So, after this, when we discuss the Heckscher-Ohlin-Vanek theorem, you will see how Vanek in 1968 contributed to this literature **to** on the HOV and how he related factor quantant, say labour quantant or capital quantant, in net exports to factor abundance capital abundance and labour abundance.

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But, before we do that, I want to finish up the proof of the Stolper Samuelson theorem, which is an another offshoot of the Heckscher Ohlin two into two into two model. That, a rise in price of a commodity raises the real reward **of the commodity** of the commodity's intensive factor and decline in real reward of its unintensive factor.

Now, the proof dates back to 1949. And, it can justify what u s was doing at that time. This is the time from 1930 till the end of the Second World War in 1945. It had raised protectionist barriers; it was increasing the tariff rates. But, then it had to decide on which good should it impose tariffs. And, everyone knew that U S was capital rich, at least it was human capital rich and it was scarce in labour endowments.

So, if it had to protect its labour, then what U S was doing was that, it was imposing tariffs on all labour intensive products coming from the foreign countries. And, the reason was that with tariffs, you already know that when you impose tariffs on the labour intensive products coming in, it will raise the domestic price of the labour intensive good in your country. So, this Stolper Samuelson's theorem says that if there is a raise in price of the commodity, a labour intensive commodity, it will raise the real wage rates in that country and a decline in the real reward for the capital input.

So, that is the reason that they imposed tariffs to protect their labour. So, that is where the contribution of Stolper Samuelson **theorem** came; where they linked the commodity price with the factor price. And, if you work further on this, then you can prove the

factor price equalisation theorem; which says that, at the end the relative wage rates would be the same. Once trade takes place, trade is a substitute for factor movements, even if you do not have mobility of factors across countries, trade is a good substitute. Because when trade takes place, the relative price of that commodity goes up; the real reward of its intensive factor goes up; the real reward of the un-intensive factor goes down.

So, if there is a labour rich country, the abundant factor which is the labour will gain. In the other country which is capital rich, the real the rentals to the to the capital will gain, the abundant factor will gain there. Eventually, you will see that with trade when the relative prices are same, the relative wage rates would be the same.

So, if you further build on this, you can prove the factor price equalisation. But, in factor price equalisation there are certain things that need to be satisfied, which is that the cone of diversification should coincide. They should not be very far apart as far as technology is concerned. Or, at the end Heckscher Ohlin never assumes that the technology is different. Technology is same, but the relative wage rates can be very far apart.

And so, even if they, you have trade, you would not see the equalisation of the relative wage rates. So, the relative wage rates should not be very far apart and it all depends on what is your capital labour ratio, what is the total capital, total labour which is available to you. They should not be very far apart. If they are very far apart, you will not see a factor price equalisation. So, to prove this Stolper Samuelson theorem, we had already derived from the equations of change. This particular equation which said that your wages, the proportionate change in wages and the proportionate change in the rate of return on capital is related to the individual prices.

Now, there was a big proof where we first started with assuming that prices are equal to average cost. Then, we worked on the equation of change, then we replaced the values of the unit labour and unit capital requirements; which was  $C_{ij}^*$  was equal to  $A_{ij}^* - B_{ij}^*$ . So, the changes in the input, output requirement was a function of the change, which will happen in the input, output ratio due to changes in the relative wage rates and the technology. So, when we did all the... when we took into account all these changes and then at the last, we assumed that the technology does not change, we got this particular equation.

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The chalkboard contains several equations and notes:

- Left side: "ity raises commodity's line in unimpressive" with notes  $\theta_1 > 0, p_2 > k_1$ ,  $p_1 \uparrow \Rightarrow w \uparrow$ ,  $w^* - p_1^* > 0$ ,  $p_2 \uparrow \Rightarrow w \uparrow$ ,  $w^* - p_2^* > 0$ , and  $\theta_{L1} =$ .
- Top middle: 
$$= \frac{\theta_{L1} p_2^* - (\theta_{L1} - \theta_{L2}) p_1^*}{|\theta|} \lambda^* - p_2^* = \frac{\theta_{L1} p_2^* - (\theta_{L1} - \theta_{L2}) p_1^*}{|\theta|} \lambda^* - p_2^*$$
- Middle left: 
$$\lambda^* = \frac{\begin{vmatrix} \theta_{L1} & p_1^* \\ \theta_{L2} & p_2^* \end{vmatrix}}{\begin{vmatrix} \theta_{L1} & \theta_{L2} \\ \theta_{K1} & \theta_{K2} \end{vmatrix}} = \frac{\theta_{L1} p_2^* - \theta_{L2} p_1^*}{|\theta|}$$
- Middle right: 
$$w^* = \frac{p_1^* \theta_{K2} - p_2^* (\theta_{K2} - |\theta|)}{|\theta|}$$
- Bottom left: 
$$\Rightarrow w^* - p_2^* = \frac{|\theta| (\theta_{L1} p_2^* - p_2^* \theta_{L2})}{|\theta|}$$
- Bottom middle: 
$$\frac{p_1^* (|\theta| + \theta_{K1}) - p_2^* \theta_{K1}}{|\theta|} = w^* - p_1^*$$
- Bottom right: 
$$\begin{cases} \theta_{L1} + \theta_{K1} = 1 \\ \theta_{L2} + \theta_{K2} = 1 \\ |\theta| = \theta_{L1} - \theta_{L2} = \theta_{K2} - \theta_{K1} \end{cases}$$
- Far right: 
$$|\theta| = \theta_{L1} \theta_{K2} - \theta_{L2} \theta_{K1}$$
  

$$= \frac{\omega_{L1} \lambda_{K2} - \omega_{L2} \lambda_{K1}}{p_1^* p_2^*}$$
  

$$= \frac{\omega_{L1} \lambda_{L2} - \omega_{L2} \lambda_{L1}}{p_1^* p_2^*} \left( \frac{k_2}{L_2} - \frac{k_1}{L_1} \right)$$
  

$$= \frac{\omega_{L1} \lambda_{L2}}{p_1^* p_2^*} (k_2 - k_1)$$
  

$$\begin{cases} \text{If } k_2 > k_1 \Rightarrow |\theta| > 0 \\ \text{If } k_2 < k_1 \Rightarrow |\theta| < 0 \end{cases}$$
  

$$|\theta| = (1 - \theta_{K1}) (1 - \theta_{K2}) - \theta_{L2} \theta_{K1}$$
  

$$= 1 - \theta_{L2} - \theta_{K1} + \theta_{L2} \theta_{K1}$$
  

$$= 1 - \theta_{L2} - \theta_{K1} - \theta_{L2} \theta_{K1}$$
  

$$= \theta_{L1} \theta_{K2} - \theta_{L2} \theta_{K1}$$

Now, once we have this equation, you can always solve for  $w^*$  using the Cramer method. And,  $r^*$  to be equal to.... So, this was  $\theta_{L1} p_2^*$ . And, you have determinant of theta. Now, here you should note what determinant theta means. It means... and  $\theta_{L1}$ ... So, **so** if you take  $w^* - p_1^* > 0$  and  $w^* - p_2^* > 0$  to be common, so you will get  $k_2$  by  $L_2$  minus  $k_1$  by  $L_1$ .

So, this is  $w^* - p_1^* > 0$  and  $w^* - p_2^* > 0$ . Now, you can see that if  $k_2$  is greater than  $k_1$ , this implies determinant theta to be greater than 0. If  $k_2$  is less than  $k_1$  this would imply determinant theta less than 0. Further, if you work on determinant theta, then you can write  $\theta_{L1}$  as  $1 - \theta_{L2}$  because  $\theta_{L1} + \theta_{L2} = 1$ .

So, you get  $1 - \theta_{L2} - \theta_{K1}$ . Please recall that,  $\theta_{L1} + \theta_{K1} = 1$ ;  $\theta_{L2} + \theta_{K2} = 1$ . Why because the total output totally exhausts the factor payments, which are done towards the labour and the capital. So,  $1$  if it is replaced by the first  $\theta_{L1} + \theta_{K1} - \theta_{L2} - \theta_{K1}$ , then determinant theta is equal to  $\theta_{L1} - \theta_{L2}$ . And, if  $1$  is replaced by  $\theta_{L2} + \theta_{K2}$ , **sorry**, then  $\theta_{L2}$  and  $\theta_{L2}$  cancels. So, you get  $\theta_{K2} - \theta_{K1}$ . So, please note these two set of results and determinant theta to be equal to  $\theta_{L1} - \theta_{L2}$  and  $\theta_{K2} - \theta_{K1}$ .

Now, put  $w^* - p_1^* > 0$  and  $\theta_{K2} - \theta_{K1}$  is determinant theta plus  $\theta_{K1} - p_2^* \theta_{K1}$  divided by determinant theta. Now this, you can write this to be equal to  $w^* - p_1^* > 0$

star is equal to  $\theta_{k1} p_1^* - p_2^*$ . So, you get  $w^* - p_1^* \theta_{k1} p_1^* - p_2^*$ .

Further,  $w^*$ , if you had written this as  $\theta_{k2}$  and this you had replaced  $\theta_{k1}$ ;  $\theta_{k1}$  is determinant  $\theta_{k2} - \text{determinant } \theta$ . So, you would have got  $w^* - p_2^* \theta_{k2} p_1^* - p_2^*$  divided by determinant  $\theta$ . So, you get  $\theta_{k2}$  and you have  $- p_2^*$ . And, **in place of  $\theta_{k1}$** , in place of  $\theta_{k1}$ , you have put  $\theta_{k2} - \text{determinant } \theta$ .

So, you get  $+ p_2^* \text{determinant } \theta$  **plus  $p_2^* \text{determinant } \theta$** . So, that goes here. And then, you have  $- p_2^* \theta_{k2}$ , you have  $p_1^* \theta_{k1}$ . **all right**. So, you take  $\theta_{k2}$  common  $p_1^* - p_2^*$  divided by determinant  $\theta$ . So, you have, you have this, you have this.  $r^* \theta_{L1} p_2^*$ ;  $\theta_{L1}$  is determinant  $\theta + \theta_{L2} p_2^* - \theta_{L2} p_1^*$  divided by determinant  $\theta$ . So, it becomes  $r^* p_2^* + \theta_{L2} p_2^* - p_1^*$ , whole to the determinant  $\theta$ .

So, from here, you get  $r^* - p_2^*$  is equal to  $\theta_{L2} p_2^* - p_1^*$  determinant  $\theta$ . Then, if you had replaced  $\theta_{L1} p_2^* -$ , instead of  $\theta_{L2}$ ,  **$\theta_{L2}$**  is  $\theta_{L1} - \text{determinant } \theta p_1^*$  divided by determinant  $\theta$ . You would have got  $w^* r^* - p_1^* \theta_{L1} p_2^* - p_1^*$  determinant  $\theta$ .

So, we have one, we have two, we have three, four. So, 1 and 2 and 3 and 4. So, we have these four equations.  $w^* - p_1^*$  is equal to  $\theta_{k1} p_1^* - p_2^*$  divided by determinant  $\theta$ . We have  $w^* - p_2^* \theta_{k2} p_1^* - p_2^*$  determinant  $\theta$ ;  $p_1^*$  is the proportionate change. We have  $r^* - p_2^* \theta_{L2} p_2^* - p_1^*$  determinant  $\theta$ . We have  $r^* - p_1^* \theta_{L1} p_2^* - p_1^*$  determinant  $\theta$ .

Now, come back to this Stolper Samuelson theorem which says a rise in price of a commodity raises the real reward of its intensive factor and a decline in real reward of its un-intensive factor. So, assume that determinant  $\theta$ , say if  $k_2$  is greater than  $k_1$ , then determinant  $\theta$  is greater than 0. So, good one is labour intensive; good two is capital intensive. So, what happens if the price of the labour intensive product goes up? That is,  $p_1$  goes up, and there is no change in  $p_2$ , if  $p_1$  goes up. So, look at this equation this is

the unit capital requirements, it is always positive. This determinant we have assumed to be positive,  $p_1^*$  is positive;  $p_2^*$  there is no change. So, the entire thing is positive. it is greater than 0.

So,  $w^*$  is greater than  $p_1^*$ . So, what it means is, if determinant  $\theta$  is greater than 0,  $k_2$  is greater than  $k_1$ . If  $p_1$  increases this implies  $w$  by  $p_1$  ratio goes up. Why because  $w^* - p_1^*$  is greater than 0. This follows because  $w^* - p_1^*$  is greater than 0. This implies that  $w$  by  $p_1$  ratio goes up with an increase in prices. So, the real wage rate, in terms of the price of the first commodity goes up.

What about...? If you look at the second, **if we look at**, we were looking at the second. Look at the first. If  $p_1$  goes up, there is no change in  $p_2^*$ . Assuming determinant  $\theta$  to be greater than 0; if  $p_1$  increases,  $p_1^*$  positive, so **all this** positive. So,  $w^* - p_2^*$  greater than 0,  $w^*$  greater than  $p_2^*$ . If the increased proportionate change in wage is greater than the proportionate change in the price of the second commodity, it means that  $w$  by  $p_2$  ratio goes up as  $p_1$  increases, as  $p_1$  goes up because  $w^* - p_2^*$  is greater than 0. So, a rise in price of a commodity raises the real reward of its intensive factor. So, the price of the labour intensive product had gone up. So, it **rise**, it increase the real reward of its intensive factor. It increases **s** the real wage rate in terms of price  $p_1$  and  $p_2$ .

So, this is what you need to see. This is what has happened. But, there is something in addition to this. There is a decline in real reward of its uninventive factor. Now, you need to look at equation 3 and equation 4. Now, what happens if  $p_1$  goes up? If  $p_1$  goes up there is no change in  $p_2$ , this is negative. So,  $r^*$  is less than  $p_1^*$ . So, what does it mean?  $r$  by  $p_1$  ratio goes down. So, this implies  $r$  by  $p_1$  ratio going down as  $p_1$  goes up. What about  $r$  by  $p_2$ ? You can see if  $p_2$ , if  $p_1$  goes up there is no change in  $p_2$ . This is a negative amount. So,  $r^*$  is less than  $p_2^*$ . So,  $r$  by  $p_2$  ratio goes down with an increase in price of the first commodity.

Now, this is the Mathematics. What is the economics intuitive thing which is going on? When the price of a commodity goes up and it is a labour intensive product, then it will force producers to produce more of that commodity; because now they are getting higher price for that. And, because it is a labour intensive commodity there will be an increase in relative demand for labour. So, when it increases the relative demand for labour, the

wage rates go up. That is the reason that you see that the wage rates go up. And, because of this thing which leads to an increase in labour intensive product, there is a decline in price of the capital intensive product. So, when there is a decline in the production of the capital intensive product, the relative demand for capital goes down. When the relative demand for capital goes down, the  $r$  by... the rental it is also goes down.

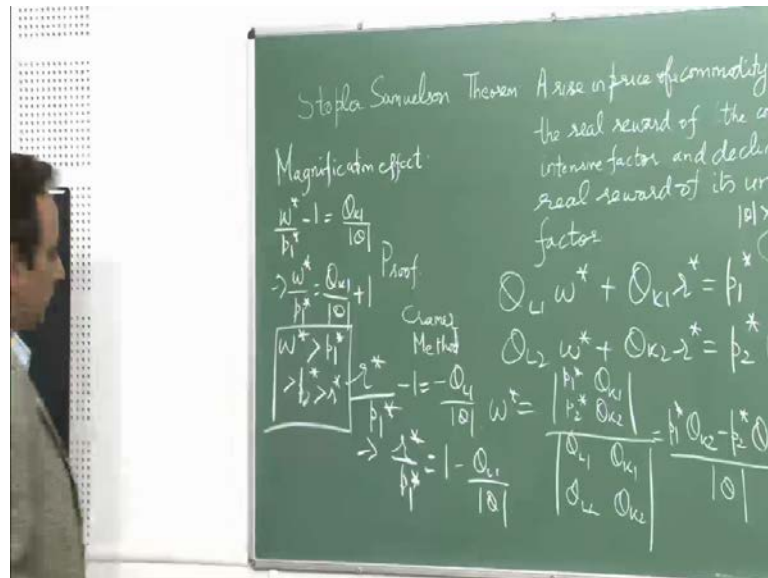
So, you see this happens. Now, yeah decrease in price of capital intensive good is because the people are shifting from capital intensive to labour intensive. No. because  $p_1$  by  $p_2$  ratio has gone up, right, so it is equivalent to saying that  $p_2$  by  $p_1$  ratio goes down. Or, when if you do not want to take into account the relative prices, say  $p_1$  goes up. Then you write that as the production of labour intensive good goes up because you are on the  $ppf$  as the production of labour intensive product goes up, there has to be a decline in the production of the other commodity. That has lead to a decline in the prices and the real reward for the capital. right. So, here we are not even talking of the relative prices. We are talking of one price, which is changed. That is  $p_1$ .

Now, you can always see these equations in terms of, what would have happened if determinant  $\theta$  is less than 0. If determinant  $\theta$  would have been less than 0, then it would have implied that good one is capital intensive; good two is labour intensive. So, if good one is capital intensive and if the price of the capital intensive product goes up, if the price of the capital intensive product goes up, then you can see from this equation, this  $r^*$  equation. Now, good one is the capital intensive product, but then the determinant  $\theta$  is less than 0.

So, even if  $p_1$  goes up, you would see an increase in  $r$  by  $p_1$  ratio; because  $r^* - p_1$  is greater than 0; because determinant  $\theta$  is less than 0. And, if you look at these equations; if  $p_2^*$  goes up, see what happens to... sorry. Now,  $p_1^*$  goes up and determinant  $\theta$  is negative. You would see that  $w^* - p_2^*$  would be less than zero. So, the relative wage rates would go down. So, you would always see that a rise in price of a commodity raises the real reward of its intensive factor and a decline in a real reward of its un-intensive factor.



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Further a small point on the magnification effect. Now, you can see, you got this equation. And, if there is no change in say  $p_2$ , then you can always express this as  $w^*$  by  $p_1^*$  minus 1 equal to  $\theta_{k1}$  determinant  $\theta$ . So,  $w$  by  $p_1^*$  is equal to  $\theta_{k1}$  determinant  $\theta$  plus 1. And then,  $r^*$  **r star**, say by  $p_1^*$ ,  $r^*$  by  $p_1^*$  minus 1.

Now, look at these two equations. If determinant  $\theta$  is greater than 0; that means good one is labour intensive, good two is capital intensive. You would have this relative wage rate to be equal to 1 plus this. So,  $w^*$  is greater than  $p_1^*$ , is greater than  $p_2^*$ , is greater than  $r^*$ . This is the magnification effect.

That means, if the price of first commodity increases wages also go up, but the increase in wage is greater than the increase in prices. Increase in price is greater than the increase in price of the second commodity. This is greater than the change in the rate of return on capital. I say change because in  $r$ , you see a decline, but the proportionate decline this is the least. So, magnification is price increases, wages go up, but the increase in wage is greater than the increase in prices. Increase in price is greater than the increase in the price of the second commodity. And, this is greater than the proportionate change in the rate of return. This is the famous magnification effect, which follows from the Stolper Samuelson theorem.



So, then if this is the Stolper Samuelson theorem, if we want to test this Stolper Samuelson theorem, we should have an **econometric** equation. And, that **econometric** equation will follow from the Heckscher-Ohlin-Vanek model. And, there we can, we can test for the Stolper Samuelson hypothesis; that a change in price of a commodity, changes the returns on returns of the factors.

So, in the next lecture we will discuss the empirical testing of the Heckscher Ohlin model. We will start by discussing the famous Leontief's paradox; Leontief, who first tested the Heckscher Ohlin model in 1953, with the data **which is available for 1947**. And, he found a paradox. We will give some explanations of the Leontief's paradox and then subsequently discuss **leamer's** work, who questioned the methodology of Leontief and proved that, what Leontief was doing was not correct. He suggested an alternative methodology. And then, once you have the Heckscher-Ohlin-Vanek equation which relates factor quantity with factor abundance, then you can see that you can test... theorem, you can also test Stolper Samuelson. And, once you have this equation you will, you **you** can test the Heckscher-Ohlin-Vanek theorem by using the partial and the complete tests. So, my next lecture will be about the Leontief's paradox and the Heckscher-Ohlin-Vanek theorem and many offshoots, which come out of the HOV model. **Thank you so much.**