International Economics

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Lecture No. #35

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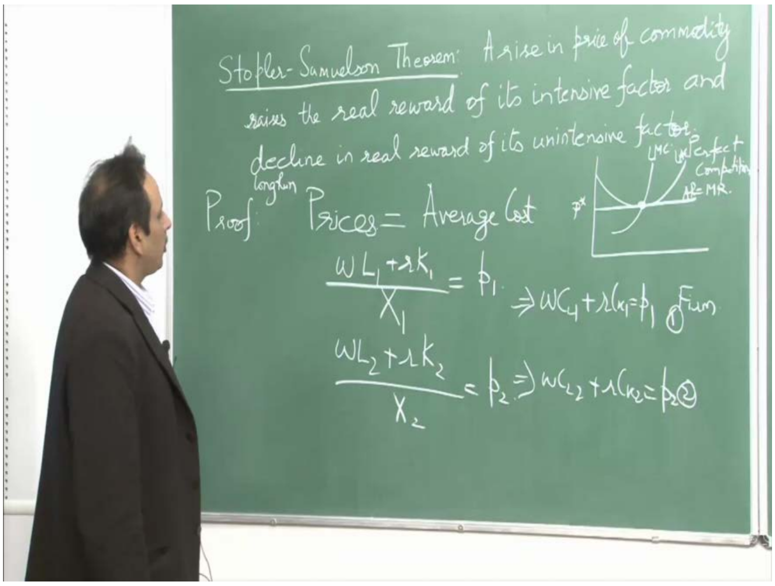

Good afternoon. Today, we going to prove the Stopler-Samuelson theorem. The Stopler-Samuelson theorem says that a rise in price of a commodity raises the real reward of its intensive factor, and decline in real reward of its un intensive factor. This theorem dates back to the year 1949, and the relevance of this is in context of the tariffs, which are imposed by a country.

The US being unskilled labor intensive country wanted to protect its unskilled sector. So, it imposed tariffs on all imports coming to its country, which were unskilled labor intensive. And the idea was that if it imposes tariffs on such products, it will raise the returns of the unskilled labor in the united states. The theorem which is working behind this relationship, which the US believe is the Stopler-Samuelson theorem.

So, say for example, you want to protect, India wants to protect its r and d industry. So, it can impose tariffs on all the r and d imports, which are coming from outside, because then it can it can protect the interests of the workers working in the r and d industry.

Basically, they will be the skilled skilled labor, which will be protected by the imposition of the tariffs on the r and d imports from outside. So, the it has its relevance, and the Stopler-Samuelson theorem is an offshoot of the of the Heckscher-Ohlin theorem, because one can also understand Stopler-Samuelson in context of the trade, which takes place among countries, the trade is Heckscher-Ohlin type, where you exchange products, which have similar factor intensities.

So, according to the Heckscher-Ohlin trade theory, a country exports the good, which uses intensively. The abundant factor and imports the good, which uses un-intensively. The scars factor correction Heckscher-Ohlin trade theory says that a country exports the good, which uses intensively its abundant factor and imports the good, which uses intensively its scars factor, not un-intensively, intensively its scars factor.

So, if a country is rich in labor, it will export labor intensive product, a country which is rich in capital, will export capital intensive product. So, if the Stopler-Samuelson theorem is applied to such trade taking place among countries, it will be the abundant factor, which will gain from such trade. So, that abundant factor in India will be the labor, which will which will whose earnings will rise, with this inter-industry trade and in the foreign country, the labor will lose because the foreign countries rich in capital. So, the rentals which accrue to the capital will be the one who will gain. So, the abundant factor there would be the capital, here the abundant factor would be labor.

So, eventually you would see that the abundant factor in each of the country gains. So, if you further work on it, you will see a factor price equalization, where the relative wage rates would be the same, the real wages would be the same, the real rate of returns would be the same after this trade, at least theoretically. Empirically, whether factor price equalization theorem holds or not that is a question, which is posed by many researchers, it is to be tested and generally, this factor price equalization does not hold for various reasons. The If we go into it, that will be another set of another set of lecture.

So, this Stopler-Samuelson theorem can explain the workings of the Heckscher-Ohlin model and eventually this result is needed to prove the factor price equalization theorem. So, we will prove this theorem the and it will be a long proof, but eventually we will prove the Stopler-Samuelson theorem and then in addition, which is the magnification effect, that it the rise in price of the of the commodity magnifies the impact on the factor returns. So, we will prove both by by the proof that we will we will do here. So, the proof starts with the proof starts with the assumption that there is a perfect competition in product and factor markets. So, that in the long run in the long run prices is equal to the average cost.

So, please recall for the firm in perfect competition, each firm is a price taker and the firm operates at the minimum of the long run average cost. This is the long run marginal cost, which cuts L A C from below at its minimum point, this is the demand curve. So, the firms operates at the minimum of the average cost curve. So, prices are equal to the average cost. So, you have w L 1 plus r K 1 divided by X 1, which will be equal to P 1, because there are two commodities that we are considering and we are also considering two factors of production, which are labor and capital. So, the average cost in the first industry is equal to P 1 and the average cost in the second industry is also equal to the prices.

Now, you can express this in terms of the unit labor requirements, you have w C L 1 plus r C K 1 is equal to P 1 and you have w C L 2 plus r C K 2 is equal to p 2. So, this is 1, this is 2.

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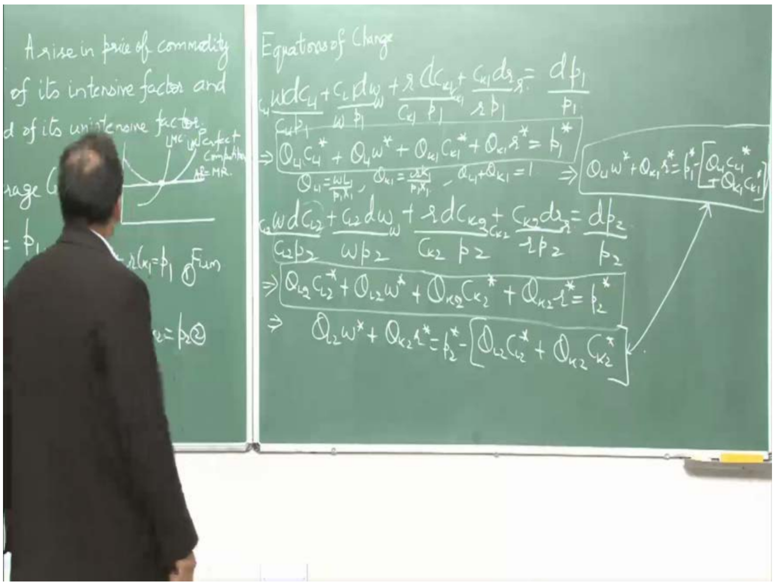

Now, if you have to work on the proof of the Stopler-Samuelson theorem, then you need to work out the equations of change and the equations of change would be similarly for the second. Now, C L 1, C K 1 C L 2 C K 2, they are the unit labor and capital requirements to produce one unit of good 1 and good 2 respectively. For example, C L 1 is the amount of labor required to produce one unit of good 1, C L 2 is the amount required to produce one unit of good 2. So, these are unit labor requirements, we have unit capital requirements. So, we have the equation of change. Now, we can further work on these equations of change, divide this by p 1. So, we get on the right hand side the proportionate change in the prices and then here we multiply with C L 1, multiply with C L 1, w, w, C K 1, C K 1, r and r.

So, then if we open this up, you will get this to be theta L 1 C L 1, star proportionate change in the unit labor requirement, this would be theta L 1 w star, this would be theta K 1 C K 1 star, this will be theta K 1 r star is equal to P 1 star. This theta L 1 is w L 1 by P 1 X 1, theta K 1 is r K 1 P 1 X 1. It is the theta L 1 is the nature in the total output, theta K 1 is the capital share in the total output and you can also see that theta L 1 plus theta K 1 would be equal to 1, because the total output exhausts all the payments done to the factors of production and this is perfect competition. So, there is a standard result that if you pay the factors according to their marginal productivity, the total output gets exhausted.

Similarly, you can work on the second equation of change, multiply divide by P 2 and then, multiply and divide by C L 2 and C K 2 and r and w. So, what you get is, this the remaining one would become theta L 2 C L 2 star plus theta L 2 w star plus theta K 2 C K 2 star plus theta K 2 r star is equal to P 2 star. So, you have this particular equation, you have this particular equation. Now, you can further write this as theta L 2 w star plus theta K 2 r star is equal to P 2 star minus theta L 2 C L 2 star plus theta K 2 C K 2 star. Similarly, you can write this equation to be equal to theta L 1 w star plus theta K 1 r star is equal to P 1 star minus theta L 1 C L 1 star plus theta K 1 C K 1 star. So, we going to concentrate on these two equations, which is which are which are these.

So now, let us go back try to understand, what determines the unit labor requirements and the unit capital requirements. So, I am going to rub this portion and I am going to then write what explains the input output requirements.

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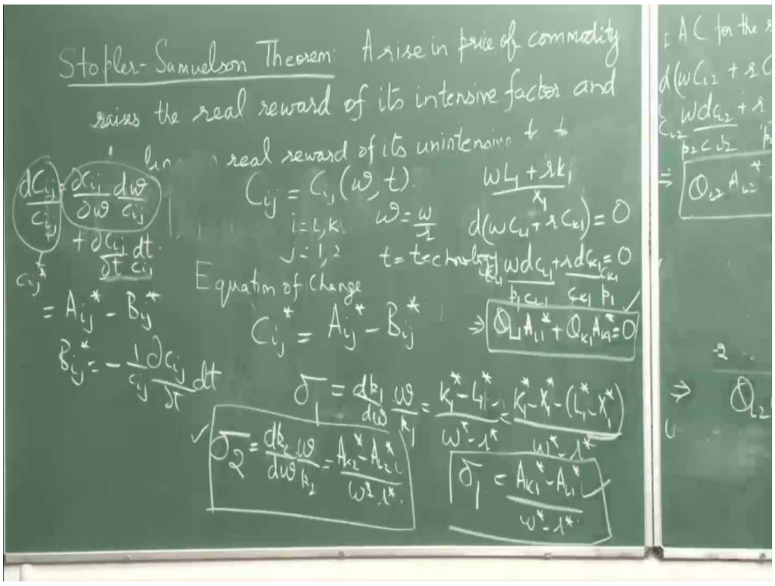

Now, please recall from our previous lecture, the input output requirement i is equal to l, k; j is 1 and 2. l and k are the two factors of production, there are two industries. This is a function of omega and t, where omega is the relative wage rates, t is the technology. Technology has an impact on the production; with a with an improvement in technology, you can use less of capital and labor to produce the same level of output. So, in that way you can save on your labor and capital and the cost of producing the two industries, relative wage rates, if relative wage rate changes then, naturally it will have an impact on the changes in the input output requirements.

So, C i j becomes a function of omega and t. So, after working on the equation of change, here C i j star, proportionate change in the unit labor requirements or the unit capital requirements or the input output requirement, whatever you call it, is equal to A i j star minus B i j star.

Now, what is A i j star please see that d C i j del omega d omega plus del C i j del t d t divide by C i j throughout. Now, this is A i j star minus B i j star, where B i j star is minus 1 by C i j del C i j del t d t. See that, it comes with negative sign because improvement in technology tends to reduce the usage of labor and capital. So, it comes with a negative sign. So, this C i j star can be written as A i j star minus B i j star. Another thing is sigma 1, which is d K 1 by d omega; omega by K 1 is K 1 star minus L

1 star by w star, this is K 1 star minus X 1 star. So, this becomes A K 1 star minus A L 1 star w star minus r star.

So, sigma 1 is A K 1 star minus A L 1 star w star minus r star and then sigma 2, which is the elasticity of substitution is d K 2 by d omega; omega by K 2 will be equal to A K 2 star minus A L 2 star divided by w star minus r star. Further, because this is a case of perfect competition and in the long run firm operates at the minimum of the average cost. So, this is the average cost. So, w C L 1 plus r C K 1, this would be minimize. So, the first order condition would be equal to 0. So, you will get w d C L 1 plus r d C K 1 equal to 0. So, divide by C L 1, multiply with C L 1, divide by C K 1, multiply it with C K 1, divide by P 1, divide by p 1. So, you get w theta L 1 A L 1 star plus theta K 1 A K 1 star equal to 0. So, you have this, you have sigma 2, you have sigma 1.

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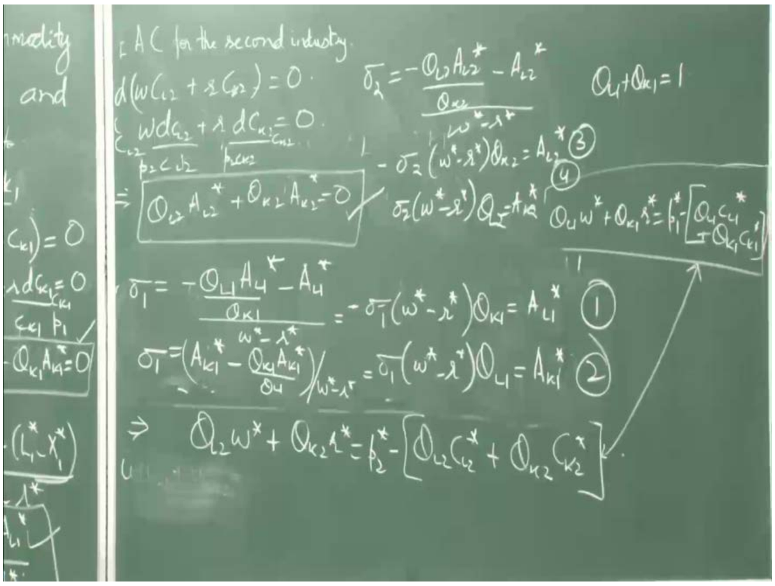

And then similarly, if you work out, this is the average cost for the second industry. So, average cost for the second industry, this has to be minimized. So, you get w d C L 2 r d C K 2 equal to 0, divide by p 2. So, you get theta L 2 A L 2 star plus theta K 2 A K 2 star equal to 0. So, then we got these 4 equations 1 and 2 and 3 and 4 and we need to replace the value of C L 1 star and C K 1 star, C L 2 star and C K 2 star, noting that C i j star is A i j star minus B i j star. So then, say if you work on sigma 1, which is A K 1 star minus A L 1 star, A K 1 star from here works out to be A K 1 star from here works out to be (No audio from 22:45 to 23:22) noting that theta L 1 plus theta K 1 is 1. Similarly, sigma

1 is A K 1 star minus, from A L 1 star from here works out to be (No audio from 23:46 to 24:21).

So, there is a negative sign here minus sigma 1 w star minus r star theta K 1 is equal to A L 1 star sigma 1 w star minus r star theta 1 is equal to A K 1 star. Similarly, you can work on the second one, sigma 2 is sigma 2 is A K 2 star. Instead of A K 2 star, you can replace the value minus theta L 2 A L 2 star by theta K 2 minus A L 2 star divided by w star minus r star. So, sigma 2 minus w star minus r star theta K 2 is equal to A L 2 star. Similarly, if you work on it, sigma 2 star w star minus r star, theta L 2 will work out to be A K 2 star. So then, you can call this as, say if you have not numbered it 1, 2, 3 and 4. So, you have 1, 2, 3 and 4. Now, what you can do is replace the value of C L 1, star C K 1 star, C L 2 star by and C K 2 star by appropriately putting up the values of A i j star and and B i j star.

So then, I am rubbing the this portion and we going to work on the two equations noting the values of.

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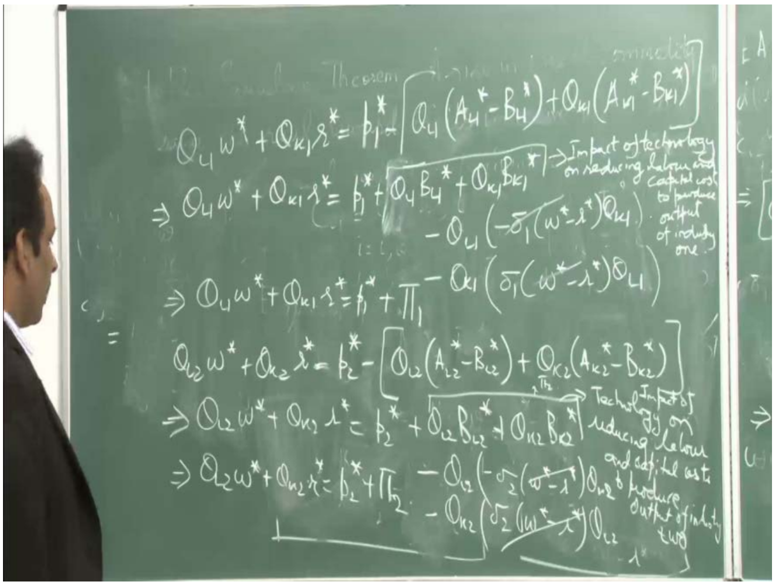

So, theta L 1 w star plus theta K 1 r star is equal to P 1 star minus theta L 1 C L 1 star is A L 1 star minus B L 1 star plus theta K 1 C K 1 star is A K 1 star minus B K 1 star. So, this would become theta L 1 w star. So, with the negative sign, if you work on the negative sign, you get theta L 1 B L 1 star plus theta K 1 B K 1 star minus theta L 1 A L 1 star, A L 1 star is minus sigma 1 w star minus r star theta K 1 and then, minus theta K

1, A K 1 star is sigma 1 w star minus r star theta L 1. Now, you can see if you work on, if you can see from here, these terms minus theta L 1 and there is a negative sign.

So, theta L 1 theta K 1 sigma 1 w star minus r star and minus theta K 1 theta L 1 sigma 1 w star minus r star, they will cancel each other, because there is a this is plus this is minus. So, you left with theta L 1 w star plus theta K 1 r star is equal to P 1 star and we call this term the impact of technology as pi 1. Why is this is an impact of technology? (No audio from 28:40 to 29:12) This theta L 1 B L 1 star plus theta K 1 B K 1 star is the impact of technology on reducing labor and capital costs to produce output of industry 1. So, that is the impact of technology, the other term gets cancelled. So, you have this equation, which relates the prices with the returns, factor returns and you have the impact of technology. Similarly, you can work on the second equation, which is theta L 2 w star plus theta K 2 r star is equal to P 2 star minus theta L 2; C L 2 is A L 2 star minus B L 2 star plus theta K 2 A K 2 star minus B K 2 star.

So, if you work on this, you get theta L 2 w star plus theta K 2 r star is equal to P 2 star plus theta L 2 B L 2 star plus theta K 2 B K 2 star minus theta L 2, instead of L 2 star, you can note that L 2 star is minus sigma 2 w star minus r star theta K 2 and you have minus theta K 2 A K 2 star, which is sigma 2 w star minus r star theta L 2. So, you can see that this term cancels, this is the impact of technology. This is the impact of technology on reducing labor and capital cost to produce output of industry 2. So, impact of technology on reducing on reducing labor and capital cost of producing output, on reducing labor and we call this as pi 2. So then, our equations will become theta L 2 w star plus theta K 2 r star is equal to P 2 star plus pi 2, this is the pi 2 term this is the pi 2 term. As in the first, this is the pi 1 term.

So now, we have got our equations. We have got two equations in two unknowns, we have to solve for w star and r star.

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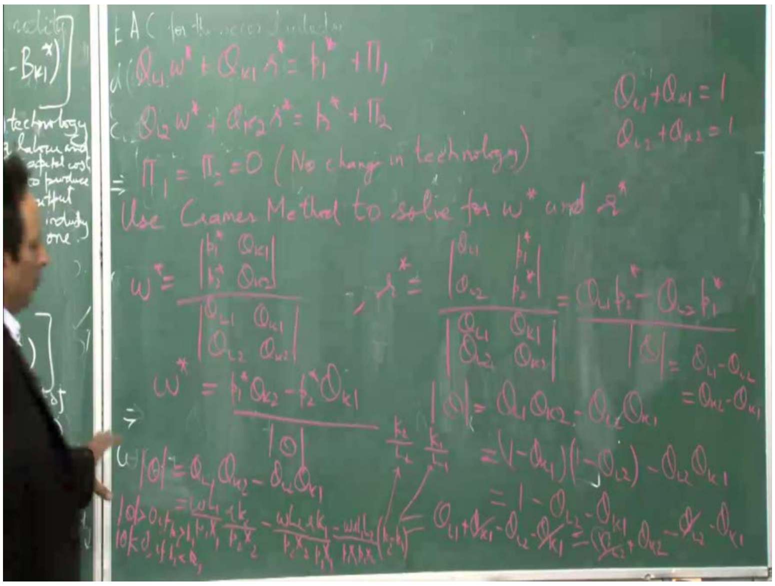

So, let us solve for w star and r star. So, theta L 1 w star plus theta K 1 r star is equal to P 1 star plus pi 1, theta L 2 w star plus theta K 2 r star is equal to P 2 star plus pi 2. Now, to prove the Stopler-Samuelson theorem, you have to assume that pi 1 is equal to pi 2 is equal to 0, that is there is no change in the technology. Now, use Cramer method to solve for w star and r star. So, w star would be determinant, (No audio from 34:15 to34:41) and r star would be (No audio from 34:43 to 35:41). So, we got w star and r star.

Now, this determinant theta is theta L 1 theta K 2 minus theta L 2 theta K 1. Now, theta L 1 plus theta K 1 is equal to 1 and theta L 2 plus theta K 2 is equal to 1. So, theta L 1 is 1 minus theta K 1, theta K 2 is 1 minus theta L 2 minus theta L 2 theta K 1. So, this becomes and this term will cancel. So then, there are two things which come out. One is, one can be written as theta L 1 plus theta K 1 minus theta L 2 minus theta K 1. So, this and this cancels. So, you get theta L 1 minus theta L 2 and this is also equal to theta L 2 plus theta K 2 minus theta L 2 minus theta K 1. So, this and this cancels.

So, determinant theta is theta L 1 minus theta L 2, which is equal to theta K 2 minus theta K 1. So, this determinant is equal to theta L 1 minus theta L 2, this is also equal to theta K 2 minus theta K 1. Further, this determinant theta is theta L 1 theta K 2 minus theta L 2 theta K 1, open this up. So, you get w L 1 P 1 X 1 plus r K 2 P 2 X 2 minus w L 2 P 2 X 2 r K 1 P 1 X 1. So then, this becomes w r L 1 L 2 P 1 X 1 P 2 X 2 and this becomes K 2 minus K, where small K 2 is K 2 L 2 K 1 is K 1 by L 1. So, determinant theta is greater than 0. If K 2 is greater than K 1, determinant theta is less than 0, if K 2 is less than K 1. So, one please note, that determinant theta is theta L 1 minus theta L 2 is equal to theta K 2 minus theta K 1 and also note, that determinant theta is greater than 0, if K 2 is greater than K 1 and determinant theta is less than 0, if K 2 is less than K 1 and we have got this result, that w star is equal to P 1 star theta K 2 minus P 2 star theta K 1, determinant theta r star is theta L 1 P 2 star minus theta L 2 P 1 star determinant theta.

Now, we have got all the ingredients to prove the Stopler-Samuelson theorem. Now, namely that the determinant theta is equivalent to these two component, and determinant theta is greater than 0, if K 2 is greater than K 1; determinant theta is less than 0, if K 2 is less than K 1. So, we will work on it tomorrow, and we will prove the Stopler-Samuelson theorem, and the magnification effect. Thank you.