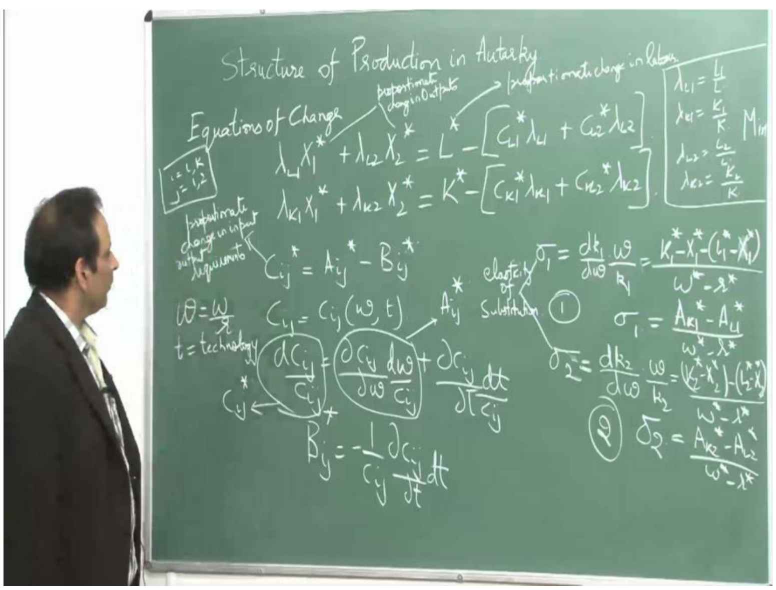
International Economics
Prof. S. K. Mathur
Department of Humanities and Social Science
Indian Institute of Technology, Kanpur

Lecture No. #34

We will continue with what we were discussing in the last lecture. That is, we are trying to prove the Rybczynski theorem, which says that an increase in supply of a factor, keeping the relative prices constant increases the output of the commodity which uses the expanding factor relatively intensively, and decreases the output of the other commodity.

(Refer Slide Time: 01:01)



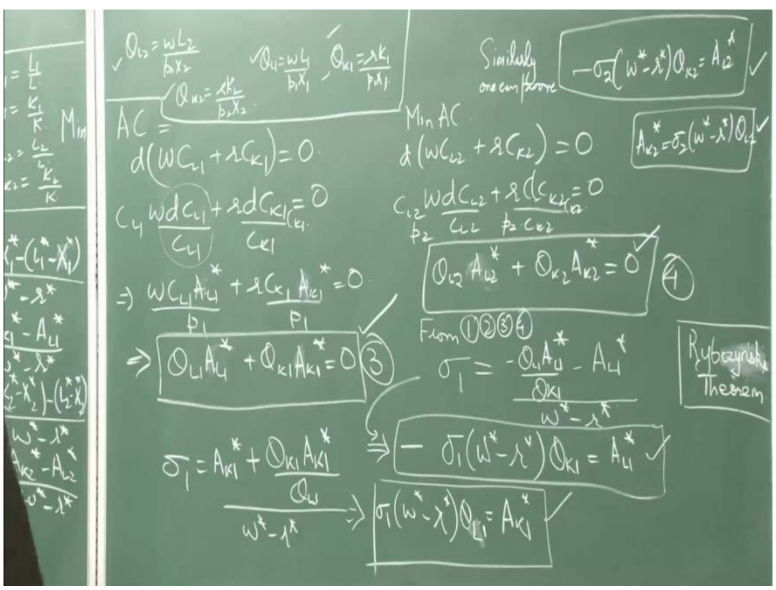
So, it is a relationship, Rybczynski theorem is about the relationship between the factors, and the outputs. Now, for this you need the equations of change, which we derived from the full employment conditions, and they turned out to be this, and this.

So, further now, we need to replace the values of the input output requirements, where C i j star is A i j star minus B i j star, remember the unit labor requirements, and unit capital requirements. Here i is l and k, j is 1 and 2. So, these input output requirements, the proportionate change in this is A i j star minus B i j star. How I got the this is, input output requirement is a function of omega and technology, where omega is w by r, t is technology. So, the total change in C i j will be del C i j del omega d omega plus del C i j del t dt. Now, if you divide by C i j throughout, you get C i j star equal to A i j star, which is del C i j del omega d omega C i j star, and minus B i j star - B i j star is minus C i j del C i j del t dt a.

So, there are two ways, in which this input output labor, input output requirement can change, this is this can change because of the changes in omega, that is the w by r ratio and because of the technology. So, if the relative wage rate change, then firms tend to employ more or less of capital, or more or less of labor. So, that is the relationship between omega and C i j and technology, when technology improves, it assume that they the firms with less units of labor and capital can produce the same level of output. So, we will replace C L 1 star C L 2 star C K 1 starc K 2 star with with with this because C i j star is this, A i j star minus B i j star.

Now, further elasticity of substitution, sigma 1 is d K 1 d omega; omega by K 1, where K 1 is the capital labor ratio used in industry 1. This can be proved to the equal to A K 1 star minus A L 1 star w star minus r star and sigma 2 would be equal to A K 2 star minus A L 2 star w star minus r star. Further, in perfect competition firms tend to produce in the long run at the minimum of the average cost curve.

(Refer Slide Time: 04:33)



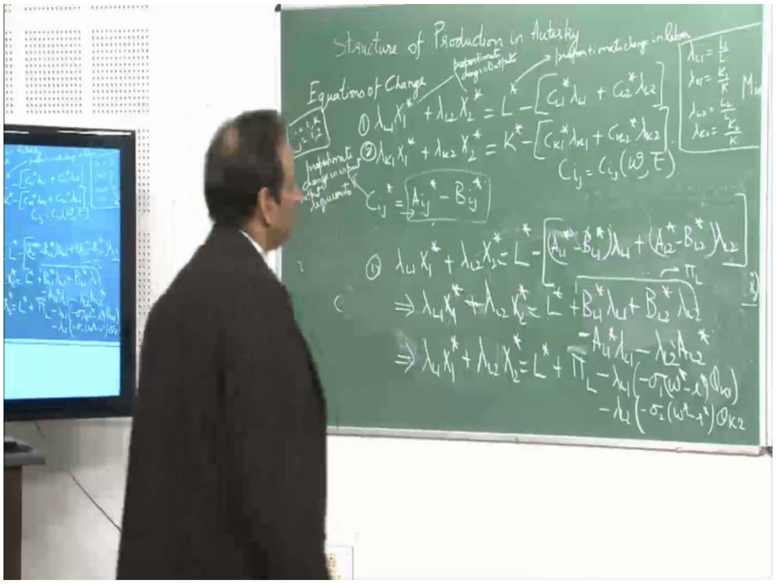
So, here if this is the average cost, then you minimize average cost d A C is equal to 0. So, if you worked on this and then divide by p 1, the relationship that you get is 3 and 4. Well, how you can you can see that this is w d C L 1 plus r d C K 1? Divide by C L 1 and multiply by C L 1, divide by C K 1 and multiply by C K 1. So, you get w C L 1 and you call the change, the proportionate change in the unit labor requirement. For the first industry to be A L 1 star, the proportionate change in the unit capital requirement for industry 1 as A K 1 star divide by p 1. So, you get theta L 1 A L 1 star plus theta K 1 A K 1 star equal to 0; theta L 1 is w L 1 by p 1 X 1; theta K 1 is r into K 1 p 1 into X 1.

Similarly, you can get another relationship, which is 4. So, what you get from 1 2 3 and 4 are values of A L 1 star A K 1 star A K 2 star and A L 2 star. How, look at sigma 1, sigma 1 is A K 1 star minus A L 1 star w star minus r star. Now, if you have to get the value of A L 1 star, you can replace the value of A K 1 star from here. So, that will be minus theta L 1 A L 1 star by theta K 1 minus A L 1 star. So, if you solve it you will get A L 1 star to be equivalent to this. So, it is expressed in terms of elasticity of substitution.

Similarly, A K 1 star; again, sigma 1 is A K 1 star, replace the value of A L 1 star from here and put it here. So, so you would get A K 1 star plus theta K 1 A K 1 star by theta L 1. So, sigma 1, w star minus r star. Now, what here you have assumed that or what have what relationship you get is theta L 1 theta plus theta K 1 is equal to 1, theta L 2 plus theta K 2 is equal to 1. You can see that theta L 1 is w L 1 p 1 X 1, theta L 2 is w L 2 p 2 K 2 and theta K 1 is r K 1 p 1 X 1. So, if you add this up, w L 1 plus r K 1 divided by p 1 X 1, you will get 1, because in perfect competition. If you pay factors according to their marginal productivity, the total output gets exhausted, this is a proven result in a perfect competitive setting.

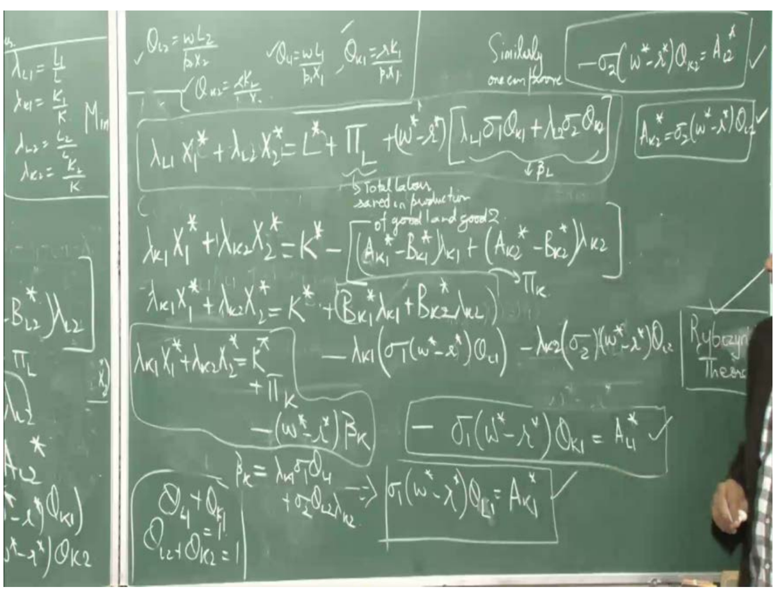
So, so this holds. If this holds, then you get the value of A L 1 star A K 1 star A K 2 star A L 2 star. Now, once we have this, now we have to feed in the value of this in this. So, what I would do is that I would rub the portions which, are not needed and then we will see what finally we get. Finally, our objective is to prove the Rybczynski theorem. Once this is done, then we will prove this Stopler-Samuels. So we have A L 1 star A K 1 star A K 2 star A L 2 star.

(Refer Slide Time: 09:25)



So, working on the first, now replace the value of C L 1 star, it will be A L 1 star minus B L 1 star lambda L 1 plus A L 2 star minus B L 2 star lambda L 2 (No audio from 10:00 to 11:09) Now, A L 1 star from there, A L 2 star from (( )).

(Refer Slide Time: 11:51)



So, then (No audio from 11:49 to 12:44) So, you get lambda L 1 X 1 star plus lambda L 2 X 2 star, L star plus pi L, I will just explain what pi L means and you have minus and minus. So, you take you can take w star minus r star, sigma 1 lambda L 1 theta K 1 plus lambda L 2 sigma 2 theta K 2. Now, what you need to understand is that these proportionate change in outputs and the inputs they are related, but they this is also equal to pi L. Now, pi L is B L 1 star lambda L 1 plus B L 2 star lambda L 2. Now, this is a reflection of the savings done as far as labor is required to produce commodity 1 and commodity 2.

So, it is the total labor saved for producing good 1 and good 2. So, remember when I talked about technological progress, here technological progress means that with less number of labor and capital, you can produce the same output. So, this is a reflection of the labor saved in the production of good 1 and good 2. So, this is a reflection of total labor saved in production of good 1 and good 2 and then this beta L, which is this is this is a reflection of how omega, that is w by r ratio, how this effects the input output requirements. So, you have this and then you have the total labor saved in production of good 1 and good 2 and then beta L, which is this w star minus r star.

This is a reflection of the changes in the input output ratio, input output requirements due to change in the technology; due to sorry, due to change in the omega. So, you have these two components coming here in the equations of change. Now, similarly you can work with the second equation, which is lambda K 1 X 1 star plus lambda K 2 X 2 star, which is equal to K star plus; now again, if you have to solve it you you would get a term, which would be K star minus C K 1 would be A K 1 star minus B K 1 star lambda K 1 plus A K 2 star minus B K 2 star lambda K 2.

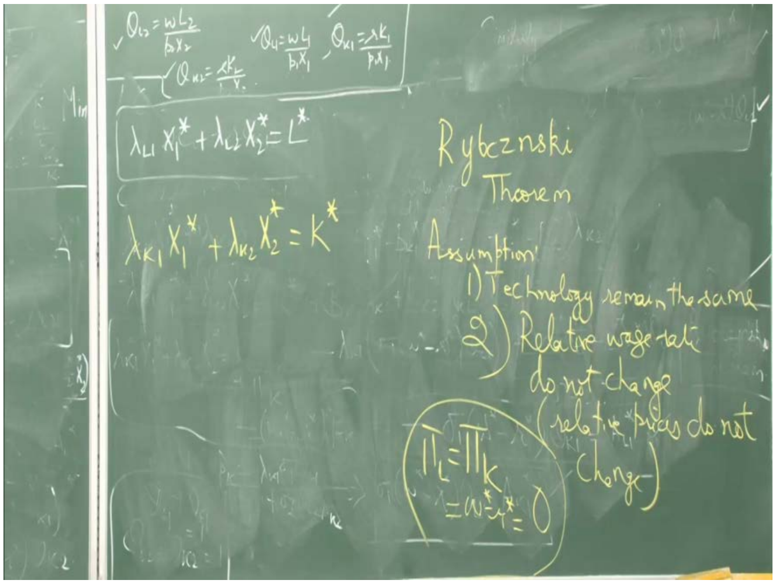
So, you get lambda K 1 X 1 star. So, this lambda K 2 X 2 star K star plus B K 1 star lambda K 1 plus B K 2 star lambda K 2 and you have minus lambda K 1 instead of A K 1 star, you can replace the value here, sigma 1 w star minus r star theta L 1 minus lambda K 2; instead of A K 2 star, you can replace the value of this. So, then what you get is lambda K 1 X 1 star plus lambda K 2 X 2 star K star plus pi K minus w star minus r star and you have a term, which is B k, where B K is, where B K is equal to, where B K is equal to lambda K 1 sigma 1 theta L 1 plus sigma 2 theta L 2 lambda K 2.

So, now you need to, we are finally there. One is this, the other is this. We have 2 equations, where you have pi L was the total labor saved in production of good 1 and good 2; pi K which is which is this; this is pi k, this is the total capital saved in the production of good 1 and good 2, like like pi L was the total labor saved in production of good 1 and good 2. This would be the total labor saved in total capital saved total capital saved in production of good 1 and good 2, that is called pi K and then there will be a term, which will be beta k, which will be this, this will be a reflection of what would be the change in the unit capital requirements due to changes in the omega.

So now, once we have these equations, we can go ahead and prove the Rybczynski theorem. Now, let me start with the Rybczynski theorem again. Rybczynski theorem relates factor inputs with factor outputs and the assumption is that the technology does not change. If the technology does not change, then pi L and pi K terms becomes 0 and second, there is no change in the commodity prices; if there is no change in the commodity prices there will be no change in the relative wage rates; if the relative wage rates do not change then this term works out to be 0.

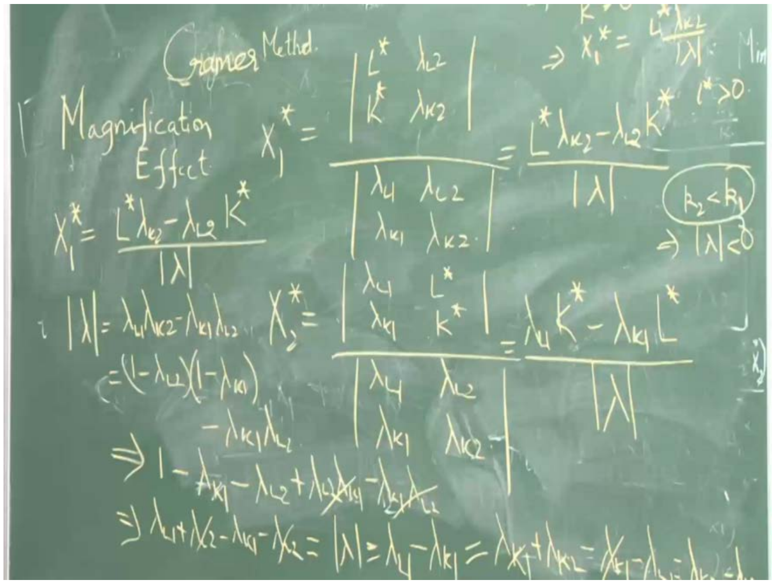
So, Rybczynski theorem says that an increase in supply of a factor, keeping relative prices constant increases the output of the commodity, which uses intensively the expanding factor and decreases the output of the other commodity. So, the first thing that we will do is, that we will assume that technology is same and there are no changes in the relative wage rates. If that is so, the equation will turn out to be only this, lambda L 1 X 1 star plus lambda L 2 X 2 star is equal to L star, lambda K 1 X 1 star plus lambda K 2 X 2 star is equal to K star right .

(Refer Slide Time: 21:48)



So, now let us remove the other part. (No audio from 21:47 to 22:20) So, the basic assumption is assumption is technology remains the same, and second relative wage rate do not change or the relative prices; that is the basic assumption in the Rybczynski theorem. What what does it mean, it means pi L is equal to pi K is equal to w star minus r star, they are all equal to 0. If that is so, then our equation is this lambda L 1 X 1 star plus lambda L 2 X 2 star is equal to L star. Lambda K 1 X 1 star plus lambda K 2 X 2 star is equal to K star.

(Refer Slide Time: 23:56)



Now, these are 2 equations in 2 unknowns. So, you can always solve using the Taylor's method, sorry the Cramer method. We use the Cramer method to solve 1 and 2. we use the Cramer method to solve 1 and 2. So, X 1 star, this is the determinant (No audio from 24:33 to 25:59) Now, something more on the determinant of lambda, determinant of lambda is lambda L 1 lambda K 2 minus lambda K 1 lambda L 2 . So, this is L 1 by L, this is K 2 by k, this is K 1 by k, this is L 2 by L. Now, if you take L 1 L 2 common, L K in the denominator, you will get K 2 by L 2 minus K 1 by L 1.

So, this becomes L 1 L 2 L K small k 2 minus small k 1. Now, come back to what we have derived X 1 star is equal to L star L lambda K 2 minus lambda L 2 K star determinant of lambda. Now, if K 2 is greater than K 1, if K 2 is greater than K 1 and K star is 0, you get X 1 star to be equal to L star lambda K 2 determinant lambda. This implies determinant lambda is greater than 0. So, you get a relationship, where X 1 star is equal to L star lambda K 2 determinant lambda, this unit capital requirement, this is positive, this is positive.

So, if this is positive, L star, if L star goes up, then X 1 star will go up. This proves the Rybczynski theorem, that an increase in output, an increase in supply of a factor, keeping relative prices constant increases the output of the commodity, which uses the expanding factor relatively intensively and a decline in the output of the other commodity. So, you can see X 1 is a labor intensive good. So, if you increase labor, output of the labor intensive good goes up, keeping relative prices constant, keeping relative wage rates constant and from here, you can see that if K star is 0, if L star is greater than 0 if L star is greater than 0 and you have a negative sign and if K 2 is greater than K 1, it would mean that X 2 star will go down.

So, on one hand, it increases the output of the labor intensive good; on the other hand, it decreases the output of the capital intensive good. That is the proof of the Rybczynski theorem. More than this, there is something else that you can prove, which is called the magnification effect. All right and if K 2 was less than K 1, if K 2 was less than K 1, then determinant of lambda would have been negative. If determinant of lambda would have been negative, then in that case good 1 will be the capital intensive good, good 2 will be the labor intensive good.

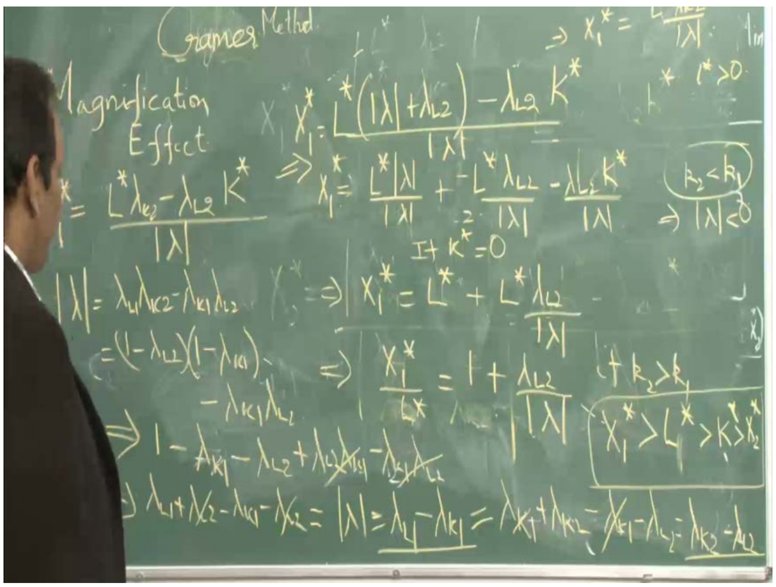
So, in that case, you can see that if supply of capital increases and determinant of lambda is negative; if if this goes up and determinant of lambda is negative and there is no change in labor, it will increase the output of the commodity, which uses capital intensively, which will now be X 1 because X 1 is now capital intensive. So, if capital increases output of the capital intensive, good goes up; if if labor goes if labor goes up supply of labor goes up the supply of labor intensive commodity goes up, while the output of the other commodity decreases. This is happening, the significance of the result is that there prices remain the same, relative wage rates remain the same, the capital intensities in the industries remain the same.

Why is this significance, because this is generally seem to explain one of the empirical facts of migration of labor into the other countries. It is generally felt by a common man that the supply of labor or migration of labor will depress wages, but one can one can take a Q out of this Rybczynski theorem and prove that when the migration of labor takes place, it is not necessarily true that the wages will come down. If say for example, if the relative wage rates in the long run, there is a possibility, that the relative wage rates do not change, the relative prices do not change, prices are anyway little sticky.

So, you may see an increase in output of the labor intensive good and a decline in the output of the other good. So, it balances out there is no change in the relative wage rates, there is no change in the relative prices, there is only a relationship between inputs and outputs. Over and above this, there is something called the magnification effect. Now, if you had worked further on this equation if you had worked further on this equation and observed that determinant lambda is lambda L 1 lambda K 2 minus lambda K 1 lambda L 2, this can be written as 1 minus lambda L 2, this can be written as 1 minus lambda K 1 minus lambda K 1 lambda L 2.

So, you get. So, determinant lambda is (No audio from 34:36 to 35:13). You can see that determinant lambda is equal to lambda L 1 minus lambda K 1 and it is also equal to lambda K 2 minus minus lambda L 2.

(Refer Slide Time: 35:31)



If this is true, then you can see if you work on this L star, instead of lambda K 2; I write determinant lambda plus lambda L 2 minus lambda L 2 K star determinant lambda. So, X 1 star equal to this, X 1 star equal to L star determinant lambda determinant lambda plus L star lambda L 2 determinant lambda minus lambda L 2 K star determinant lambda. If K star is 0, you get X 1 star is equal to L star plus L star lambda L 2 determinant lambda.

Now, divide by L star throughout, you get X 1 star by L star is equal to 1 plus lambda L 2 determinant lambda. Now, if K 2 is greater than K 1, that is good 1 is labor intensive, good 2 is capital intensive. Now, you can see what happens with an increase in labor. This would be a positive quantity and this will be greater than 1; if this is greater than 1 then X 1 star is greater than L 1 star and you can further prove that this would be. So, here is the magnification the magnification theorem, that labor if it labor increases output of the labor intensive good goes up, but by it increases by by a factor greater than the increase in labor. Output of the labor the labor intensive good output of the labor intensive good increases, but this increase is greater than the increase in the labor. This is greater than the increase in capital, increase in capital is greater than the decline in the production of good good 2.

This is the magnification effect, and it is one of the off shoots of the Rybczynski theorem. So, today I will end by restating the Rybczynski theorem, which says that an increase in the supply of a factor, keeping relative prices constant increases, the output of the commodity which uses intensively the expanding factor, and decreases the output of the other commodity. That is the Rybczynski theorem. When we come for the next class, we will prove the Stopler-Samuelson theorem, which will use the other two set of equations, which relates relative prices with relative wage rates; that means, it is a link between two prices - factor prices, and product prices, and then we will prove the Stopler-Samuelson theorem, which says a rise in the price of a commodity, raises the real award, real reward of its intensive factor, and a decline in real reward of its un intensive factor. So, these are things that we will be discussing in the next lecture. Thank you so much.