International Economics

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Lecture No. #33

Good afternoon. Today we will continue discussing the structure of production in autarky, and you will see at that at the end we will prove two of the theorems, that is the Stoplersamuelson theorem, and the Ryzbynski theorem. The Stoplersamuelson theorem say is a rise in a in the price of a commodity raises the real reward of its intensive factor, and a decline in real reward of its unintensive factor, that is the Stoplersamuelson theorem, which came way back in 1949 to to prove that how trade has unequal effects on the factor returns. Ryzbynski on the other hand, tells about the relationship between the factor supplies and the outputs. And it says an increase in supply of a factor keeping the relative prices constant raises the output of the commodity, which uses the expanding factor relatively intensively, and a decrease in the production of the other commodity.

Now, to prove this you have to first define the basic structure of the of the model, and so we will start with the full employment conditions, and also assume that there is perfect competition in the product markets. Now, if there is perfect competition in the product market, prices will be equal to average cost. So, we will start with the basic model, and then we will discuss the equations of change. So, here the proof is a little long, may be it will extend to the other other lecture tomorrow, but we will start with the very basic model, and then we will do the equations of change, and that will finally give us the proof of the Stoplersamuelson and the Ryzbynski theorem.

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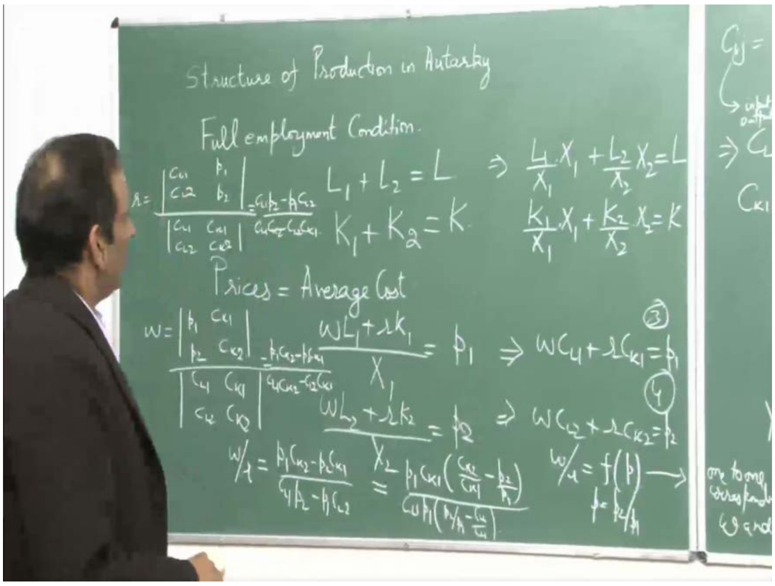

So, then the full employment conditions, L 1 plus L 2 is equal to L, there are two commodities which are produced, one and two and some labour is used in production of good 1, some labour is used in production of good 2, and this totally exhausts the total amount of labour, which is L. There is capital, there is the second factor of production, which is used in the production of good 1, and good 2. So, K 1 plus K 2 is equal to K, then there is the condition that prices are equal to the average cost.

Now, this would mean, this is the total cost divided by the output, this prices are equal to the average cost, for the other good it is equal to this. Now, prices are equal to the average cost, when you assume long run and it is perfect competition, so price are equal to average cost. Now, if you work further on this, you can find out the unit labour requirements, unit capital requirements. So, this gives me, and. So, C i j C i j is the input, this is the input output requirement. You can well understand it as unit labour requirement or unit capital requirement because it can be C L 1, C L 2. C L 1 would be the amount of labour required to produce the first industry the industry the output of the first industry, C L 2 is the amount of labour required to produce one unit of good 2, C K 1 is the amount of capital required to produce one unit of good 1, C K 2 is the amount of capital required to produce one unit of good 2.

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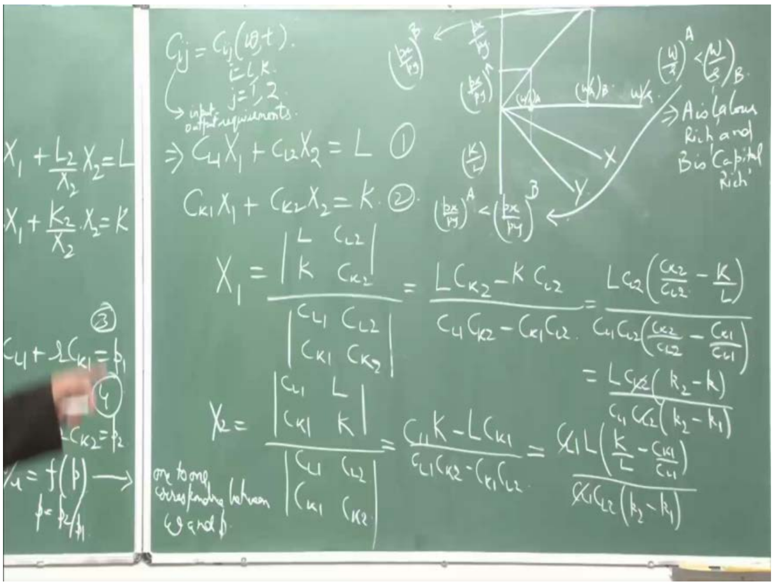

So, you also call it input output requirements or you can say unit labour requirements. So, then you have this 1 and 2 and 3 and 4. If you can solve it using the Cramer method, it would give you (No audio from 6:25 to 8:17). So, if you solve for X 1 and X 2, you can see that that X 1 is a function of L K 2 minus K; K 2 minus K 1, K is the overall capital labour ratio in this country, K 2 is the capital intensity of good 2 K 1 is the capital intensity of good 1. So, if you devote your entire capital and labour into the production of K 2, then there will be no production of good 1 or if you devote entire capital labour ratio in good 1, there will be no production of good 2. So, your over all capital labour ratios have to lie between K 1 to K K 2.

Then only there will be something called incomplete specialization. There will be In other words, there will be production of both goods, X 1 and X 2. So, if the assumption is this, then it leads to incomplete specialization, wherein there is a production of both goods, X 1 and X 2. So, that is an important assumption that we are not assuming that X 1 is 0 or X 2 is 0 or both are 0. We are assuming that in this economy there is a production of both both goods . So, and the overall capital labour ratio lies from this K 1 to K 2, where K 1 and K 2 are the capital intensities in good 1 and good 2. Now, this is after solving 1 and 2; similarly, you can solve for w and r from 3, using 3 and 4 . So, from 3 and 4 (No audio from 10:27to11:24). So, then from here w and r, you can always get w by r ratio, which is this can be written as (No audio from 11:45 to 12:07) p 1 C K 1; C K 2 by C K 1 minus p 2 by p 1; C L 1 p 1; p 2 by p 1 minus C L 2 by C L 1.

Now, what is to be seen is that w by r eventually will become a function of p only, where p is p 2 by p 1. Reason being that this C i j, that is input output requirement is a function of the w by r ratio and technology. So, these are a function of omega C K 2, C K 1, C L 2, C L 1, they are all functions of omega. Now, if you assume technology to be constant, you will see that w by r will become a function of p only because C this input labour requirements or input capital requirements, they are a function of omega. Why because if omega changes, if the w by r ratio in a firm changes, then the capital labour requirements also changes. So, then there is one to one correspondence between correspondence between between omega and and p and when we prove the Heckscher Ohlin theorem, this was one of the basic assumptions that there is one to one correspondence between omega and p. Please recall the diagram that we made for proving the Heckscher Ohlin theorem using the factor prize definition.

(No audio from 14:04 to 14:35) Now, the assumption if you have to prove the Heckscher Ohlin theorem using the factor prize definition. The factor prize definition is that if w by r in A is less than w by r in B, this means A is labour rich and B is capital rich. A is labour rich because it has a lower relative wage rate, B is capital rich because it has a higher relative wage rate. Now, because there is one two one correspondence between relative prices and the relative wage rates, you can see that it is very easy to prove the Heckscher Ohlin theorem, which says that the country which is rich in capital will export capital intensive product and country which is rich in labour will export labour intensive product. This is crude form of the Heckscher Ohlin theorem. A refined form is the of Heckscher Ohlin is that a country exports the good which uses intensively its abundant factor and imports the good which uses intensively its scarce factor.

Now, to prove the Heckscher Ohlin theorem, you need to show that there is one to one correspondence between p x p y and w by r. Now, you can see that we had proved that there is w by r is a function of p. So, see if w by r increases, you would see that the relative price of x, which is x is a labour intensive product, it also goes up because x is a labour intensive product. If the relative wage rates go up, the prices will go up. Now, that is enough to prove the Heckscher Ohlin theorem. You can see good x is labour good x is labour intensive, good y is always capital intensive. So then, if this is the before trade situation p x by p y in A would be less than p x by p y in B. So, the relative wage rates in A is less than relative wage rates in B, this implies that the relative prices in A is less than the relative prices in B. This is because there is one to one correspondence between relative prices and relatives wage rates. If this is so, A has a comparative advantage in production of good x because it can produce x at a lower relative price, lower relative price means lower opportunity cost.

So, A will export good x, A is labour rich. So, it will export good x, x is a labour intensive product. So, this proves the Heckscher Ohlin theorem. A which was labour rich will export the labour intensive product, B which is capital rich will export the capital capital intensive product. Why because, the international prices will be determined and they will be somewhere in the middle of p x by p y A and p x by p y B and you can see eventually what will happen will that, with trade you would have the same relative prices and you would have the same relative wage rates. This is also called the factor prize equalization, this is another off shoot of the Heckscher Ohlin theorem.

Besides the Stoplersamuelson and the and the Ryzbynski, the another off shot of the Heckscher Ohlin model is, that with trade you will see that the relative wage rates will be the same across countries because the relative prices will be the same and this is all happening because there is one to one correspondence between relative prices and relative wages. So, that is where mathematically, that is where it is coming and what is important is that to this to happen, there needs to be incomplete specialization, there is needs to be production of both goods x and y, that is important. You you you have this only when there is production of both goods, good 1, good 2; is then only there relative prices are determined. So, incomplete specialization will lead to such results that there is one to one correspondence between relative prices and relative wage rates.

So, this is the this is how you this is how one can get the values of X 1 and X 2 and w and r. Now, let us work on the equations of change because those equations for change will ultimately lead us to the proof of the Stoplersamuelson theorem and the Ryzbynski theorem. What Stoplersamuelson, arise in price of a commodity raises the real reward of its intensive factor and a decline in real reward of its unintensive factor. The equations which will be needed will be these, that is 1, 2, 3 and 4.

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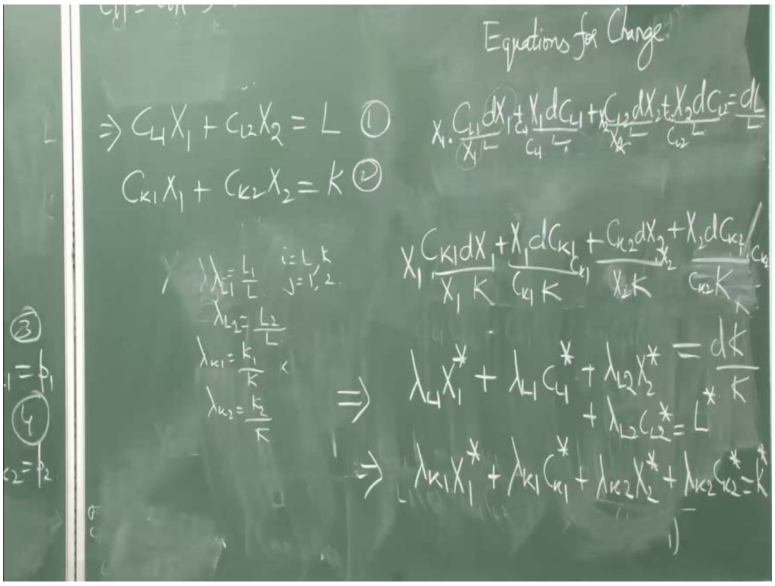

Now, the equations for change. (No audio from 20:28 to21:22) So, the equations for change would be C L 1 d X 1 plus X 1 d C L 1 plus C L 2 d X 2 plus X 2 d C L 2 is equal to d L, C K 1 d X 1 plus X 1 d C K 1 plus C K 2 d X 2 plus X 2 d C K 2 is equal to d K. Now, divide by L in the numerator, in the denominator throughout (No audio from 21:49 to 22:22) and here by capital in the second equation. Further, multiply and divide this term by X 1, multiply and divide this term with C L 1, multiply this term by and divide by X 2, this by C L 2, C L 2, this term by X 1, multiply it with X 1 and divide by X 1; C K 1, C K 1, C K 2 ;sorry X 2, X 2; C K 2 , C K 2.

So, see what you get from the first. Treat this as d X 1 by X 1, this is the proportionate changed in X 1. So, you left with X 1 L 1 by L, C L 1 by L, C L 1 is the unit labour requirement to produce the first industry. So, L 1 by X 1 into X 1 by L 1 L. So, you get L 1 by L, the proportionate amount of labour, which is used in the first industry. Call that as lambda L 1 X 1 star plus; now, dc L 1 by C L 1 is the proportionate change in C L 1. So, this would become lambda L 1 C L 1 star, this would become lambda L 2 X 2 star, this would become lambda L 2 C L 2 star and this is L star. So, you have this equation, from here you would get lambda K 1 X 1 star plus lambda K 1 C K 1 star plus lambda K 2 plus lambda K 2 X 2 star plus lambda K 2 C K 2 star is equal to K star. Now, what is lambda i j; i is L, K; j is 1 and 2.

So, say for example, if it is lambda L 1 this is L 1 by L; lambda L 2, this is L 2 by L; lambda K 1 is K 1 by K; lambda K 2 is K 2 by K is the proportionate share, L 1 by L is a proportionate share, L 2 by L is the proportionate share.

So, you have lambda L 1, lambda L 2, lambda K 1, lambda K 2 right. So, we will work on equation 3 and 4 also, similarly equations of change, if you work on this 3 and 4, you will finally prove Stoplersamuelson. If you work on this you, will prove Ryzbynski theorems. In fact, if you have read about linear programming, there is something like a primal problem and a dual problem. These two are related in that way, one is a primal and the other is the dual of that. In any case and then remember the the different theorems of L p, which shows that the the value of outputs or the value of the objective functions, whether you solve a primal or a dual works out to be the same. Any case we are not going into it, we will do lot of algebra here.

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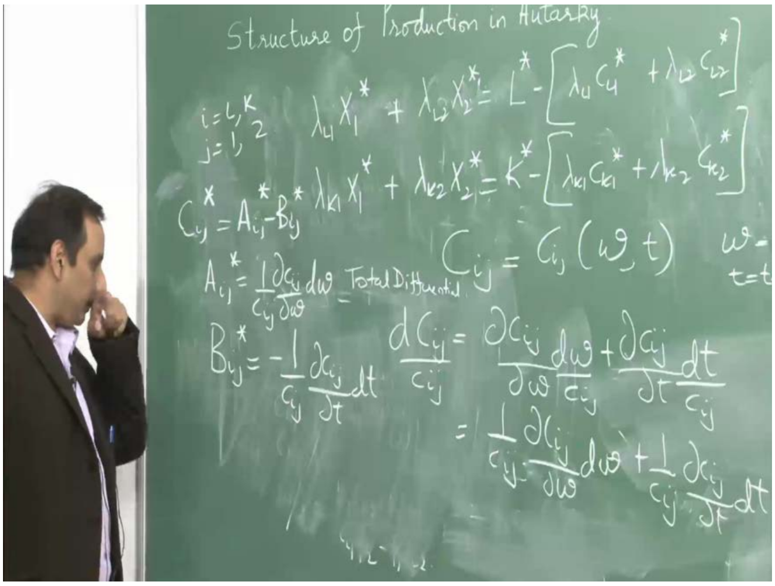

So, lambda L 1 X 1 star plus lambda L 2 X 2 star is L star minus lambda L 1 C L 1 star plus lambda L 2 C L 2 star and this is lambda K 1 X 1 star plus lambda K 2 X 2 star, this is K star minus lambda K 1 C K 1 star plus lambda K 2 C K 2 star.

Now further, this C i j, recall this is the input output requirement, the unit labour requirement, the unit capital requirement. This is a function of omega and t, t is the technology, omega is w by r, t is technology. So then, if you take d C i j, the total differential, (No audio from 28:50 to 29:14) this becomes, this divide by C i j. Now, what a depict says that this input output requirements, this changes because of the changes in omega keeping technology constant, this input output requirement changes because of the technology keeping the relative wage rates constant. Yesterday, when we were discussing the impact of technology, we saw that with technology you have less units of labour and capital producing the same output. So then, we define technology in three ways, it was labour using, another was capital using and the third was hicks neutral.

In labour using, we saw that the marginal productivity of labour increases by a factor greater than the marginal productivity of capital, marginal productivity of labour is the increase in m p L is greater than the increase in the m p K. As a result, w by r ratio increases. That is called labour using technology. There is something called capital using technology, when the m p K the increase in m p K is greater than the increase in m p L. If the increase in m p L is equal to the increase in m p K, we call it hicks neutral technological progress. Whatever it may be, whichever technology it is, at the end you have less units of both capital and labour producing the same level of output. So, technology tends to have an impact on the labour capital requirements to produce one unit of output and then this also changes because if the relative wage rate change, they also have an impact on C i j.

So, this is reflected in this. Now finally, what comes out is that C i j star, which is the proportionate change, this is equal to a i j star minus b i j star, i is L ,K j is 1 and 2, where a i j star is 1 by C i j del C i j del omega d omega and b i j star is minus C i j del C i j del t d t.

Now, you can see that this b i j star, which is a reflection of how technology effects the input output requirements, it comes with a negative sign. Remember like the law of demand, which says that price and demand and inversely related. So, when you write a formula for elasticity price elasticity of demand, you write it is the proportionate change in demand divided by proportionate change in price, but there is a negative sign, implying that price and demand are inversely related.

So, here also technology tends to have an impact on input output requirement, but there is a negative sign, implying that with an improvement in technology, you imply less of labour and capital to produce the same output. So therefore, there is a negative sign. So, then C i j star proportionate change in the input output requirements is A i j star minus B i j star. Now, you can see this, why I am writing C i j C i j star, A i j star minus B i j star because i want to replace this C L 1 star with A L 1 star minus B L 1 star C L 2 star with A L 2 star minus B L 2 star C K 1 star with A K 1 star minus B K 1 star C K 2 star with A K 2 star minus B K 2 star. But, further we need to work on also the fact couple of things that sigma 1 that is the elasticity of substitution and sigma 2 (No audio from 34:09 to35:35).

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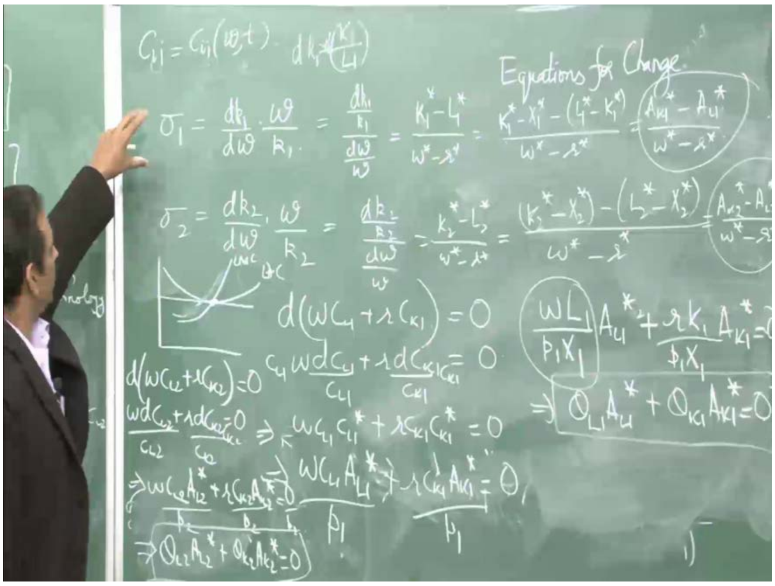

Now, you need to understand this elasticity of substitution, which says what is the change in the capital labour ratio as the relative wage rate changes. Now, you can write this as d K 1 by K 1, where small K 1 is K 1 big K 1 divided by L 1. Now, you do d K 1 d K 1 by L 1 divide by K 1 and use the quotient rule of derivative. Please recall the quotient rule of derivative, if you have to find d K d K 1, if you have to find d K 1, if you use the quotient rule it will be L 1 d K 1 minus K 1 d L 1 divided by L 1 square and then if you divide by K 1, you will get proportionate change, you will get a proportionate change in K 1, proportionate change in labour.

So, K 1 star minus L 1 star, w star minus r star. Now, you can write this as K 1 star minus X 1 star minus L 1 star minus K K 1 star. So, this is A K 1 star. Please recall A K 1 star, it is the change of the input output requirements due to changes in the omega, omega is the w by r ratio. So, here keeping technology constant this becomes A K 1 star, this is A L 1 star this is A K 2 star minus A L 2 star divided by w star minus r star.

. So, we will remember this relationship that sigma 1 is equal to that, sigma 2 is equal to A K 2 star minus A L 2 star divided by w star minus r star. Further, if you recall in perfect competition, you operate at the minimum of the long run average cost. So, this requires the w C L 1 plus r C K 2 r C K 1, differential of this should be equal to 0. Why? If you are operating at the minimum of this, then d average cost d x, the differential of this would be equal to 0, because remember at this point the slope is 0 and this is your total cost of producing good 1, this is the total average cost of producing good 1, where is C L 1 is L 1 by X 1; C k 1 is K 1 by X 1. So, you get w L 1 plus r K 1 divided by X 1. So, this is the average cost. Now you are operating at the minimum of this average cost. So, this should hold, if this holds, then w d C L 1 plus r d C K 1 is equal to 0.

Now, you can always divide by C L 1, multiply by C L 1, divide by C K 1, multiply by C K 1. So, you get w C L 1 C L 1 star plus r C K 1 C K 1 star. Now, here we are assuming technology to be same. So, if you want to study (()) the d C L 1 by C L 1 star when technology remains the same, then this a C L 1 star will be replaced by a L 1 star and this would be replaced by a K 1 star. Further, divide by p 1. So, what you get is, from here w L 1 p 1 X 1 A L 1 star plus r K 1 p 1 X 1 A K 1 star equal to 0.

Now, this is theta L 1 A L 1 star plus theta K 1 A K 1 star equal to 0. Now, do a similar thing do a similar thing by minimizing the average cost of producing good 2. So, in that case, d w C L 2 plus r C K 2 is equal to zero. So, you will get w d C L 2 plus r d C K 2 equal to zero. So, C L 2, C L 2; C K 2, C K 2. So, you get w C L 2 A L 2 star plus r C K 2 A K 2 star equal to 0, divide by p 2. So, you get theta L 2 A L 2 star plus theta K 2 A K 2 star equal to 0 .

So, you have 1 and 2 and you have this 3 and you have this 4. Now, what you need to do, is to replace the value of A K 1 star from here and see what you get a sigma 1, then you need to replace the value of A L 1 star from this, then you need to replace the value of A L 2 star from here, then you need to replace the value of A K 2 star from here and then we need to get what would be A K 1 star, A L 1 star, A K 2 star, A L 2 star in terms of this elasticity of substitution. Once we have that, then we will put it in this, C L 1 star, C L 2 star, C K 1 star, C K 2 star and then using this, that C L 1star is A L 1 star minus B L 1 star A L 1 star A K 1 star A K 2 star A L 2 star will be expressed all in terms of the elasticity of substitution. Once we have that, then we will we will see that what these two final equations will look like.

So, what you will get on the right hand side will be proportionate change in labour proportionate change in capital, but there will be some term, which will express two things, because remember C i j star this is input output requirement, this is a function of omega and t and you are saying that this is changing. So, this can change because of the change in omega and this can change because of the change in technology. So, when you talk of Ryzbynski, you have two assumptions. If Ryzbynski has to hold; that means, if you have to establish relationship between output X 1 and L X 1 X 2 and K, you have to assume there are two important assumptions. One technology does not change, relative wage rates do not change. Because, then only you will get a relationship between x is and labour and capital, if these things do not change. If these things do not change, it means omega does not change, technology does not change.

So, many of the interviews they ask what is Ryzbynski. So, you have to say, that is a relationship between outputs and factor supplies. Let me repeat, what Ryzbynski is? An increase in supply of a factor keeping relative prices constant, relative prices constant means relative wage rates constant, increases the output of the commodity, which uses intensively the expanding factor and decreases the production of the other commodity. Now, you may ask, where is it used this Ryzbynski? Now, people say that when people from here migrate to US, the US people say that it will depress wages there, because so many people are going there, but here it will tell you, Ryzbynski tells us that if you increase labour, output of the expanding factor increases keeping relative prices constant, if relative prices are constant, relative wage rates are constant.

So, if you can see debates in the parliaments are that they say that migration any way is a very tricky issue. In Europe, in US they say lot of migration will depress wages or some illegal migrants are coming from Mexico or from Cuba, that will depress wages, in say Florida. If you test for Ryzbynski, you can prove that with an increase in labour in the long run output of the commodity, which uses intensively that labour that increases, prices remain the same, wages remain the same. In fact, there is a empirical evidence, which shows that, as as soon as the Cubans migrated illegally moved to florida, they had they had they had assumed that the wages will go go go down, that did not happen. In fact, the output went up of the labour intensive product. Similarly, when there is skilled people moved from say Russia to Israel, it was feared that it will depress wages of the skilled workers, it did not, outputting expanded. So, there is an empirical evidence to show.

So, if you go and read the literature on Ryzbynski, you will see that there are so many interesting applications of Ryzbynski. So, we will tomorrow we will see, how once we have established this? Once we have established this, How A K 1 star A L 1 star A K 2 star A L 2 star will become a function of sigma 1 sigma 2. Once it is here, then you can replace it here, you will get in equation, which will reflect, not only this relationship between X 1 and L star, but you will have some term, which will show the change of C i j due to change in omega, a change in C i j due to due to change in t. So, for Ryzbynski then you have to further assume that those are constant, then if you work on it, you just have to find using Cramer method value of X 1 star and X 2 star.

You can easily prove the Ryzbynski, that with an increase in supply of factor output of the good, which uses the expanding factor, intensively will increase and there will be a decline in production of the other commodities. So, this is with if you work of with this equation you will prove the Ryzbynski, if you work with equation 3 and 4, which is a relationship between relative wage rates and relative price, you will get this Stoplersamuelson theorem. Again on the right hand side, you will see a term, which will reflect technology. So, if you have to prove Stoplersamuelson theorem, you have to prove that the technology remains the same, then you will find a link between the relative wage rates and the relative prices.

So, again you will find the Cramer method, we will the value of w star r star, and then it will be very easy to prove the linkage between the two prices - product prices, and the factor prizes, that Stoplersamuelson, physical relationship between output and labour, that is Ryzbynski. So, we will meet tomorrow. Thank you so much.